

Advanced Algebra/Trig
Chapter 6 Review

Name Answer Key

Find the following using trigonometric identities.

1. If $\sin \theta = \frac{2}{3}$, find $\cos \theta$.

$\cos \theta = \frac{\sqrt{5}}{3}$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\left(\frac{2}{3}\right)^2 + \cos^2 \theta = 1$$

$$\frac{4}{9} + \cos^2 \theta = 1$$

$$\cos^2 \theta = \frac{5}{9}$$

$$\cos \theta = \frac{\sqrt{5}}{3}$$

2. If $\sin \theta = \frac{7}{10}$, find $\cot \theta$.

$\csc \theta = \frac{10}{7}$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$1 + \cot^2 \theta = \left(\frac{10}{7}\right)^2$$

$$-1 + \cot^2 \theta = \frac{100}{49} - 1$$

$$\sqrt{\cot^2 \theta} = \sqrt{\frac{51}{49}}$$

$\cot \theta = \frac{\sqrt{51}}{7}$

3. If $\tan \theta = \frac{12}{5}$, find $\sin \theta$.

$\cot \theta = \frac{5}{12}$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$1 + \left(\frac{5}{12}\right)^2 = \csc^2 \theta$$

$$1 + \frac{25}{144} = \csc^2 \theta$$

$$\frac{169}{144} = \csc^2 \theta$$

$\csc \theta = \frac{13}{12}$

$\sin \theta = \frac{12}{13}$

If $\cot \theta = \frac{5}{9}$, find $\tan \theta$.

$\tan \theta = \frac{9}{5}$

Verify the following identities.

5. $\tan \beta \csc \beta = \sec \beta$

$$\frac{\sin \beta \cdot \frac{1}{\sin \beta}}{\cos \beta} = \frac{1}{\cos \beta}$$

$$\frac{1}{\cos \beta} = \sec \beta$$

6. $\frac{1}{\sec^2 \beta} + \frac{1}{\csc^2 \beta} = 1$

$$\cos^2 \beta + \sin^2 \beta = 1$$

7. $\cos x (\csc x - \sec x) = \cot x - 1$

$$\cos x \cdot \csc x - \cos x \cdot \sec x$$

$$\cos x \cdot \frac{1}{\sin x} - \cos x \cdot \frac{1}{\cos x}$$

$$\frac{\cos x}{\sin x} - \frac{\cos x}{\cos x}$$

$$\cot x - 1$$

8. $\frac{\sin x \cdot \cot x + \cos x}{\sin x} = 2 \cot x$

$$\frac{\sin x \cdot \frac{\cos x}{\sin x} + \cos x}{\sin x}$$

$$\frac{2 \cos x}{\sin x}$$

$$2 \cot x$$

9. $(1 - \cos x)(1 + \cos x) = \frac{1}{\csc^2 x}$

FOIL

$$1 + \cos x - \cos x - \cos^2 x$$

$$1 - \cos^2 x$$

$$\frac{\sin^2 x}{1} = \frac{1}{\csc^2 x}$$

10. $\sin x \tan x = \sec x - \cos x$

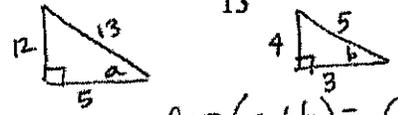
$$\frac{1}{\cos x} - \frac{\cos x}{1} = \frac{\cos x}{\cos x}$$

$$\frac{1}{\cos x} - \frac{\cos x}{\cos x}$$

$$\frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$$

If a and b are measures of two first quadrant angles, find the exact value of each function.

11. If $\sin a = \frac{12}{13}$ and $\cos b = \frac{3}{5}$, find $\cos(a + b)$.

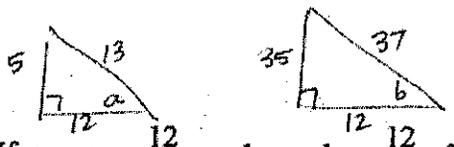


$$\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$$

$$= \left(\frac{5}{13}\right) \left(\frac{3}{5}\right) - \left(\frac{12}{13}\right) \left(\frac{4}{5}\right)$$

$$= \frac{15}{65} - \frac{48}{65} = \frac{-33}{65}$$

$$\frac{\sin x \sin x}{\cos x} = \sin x \tan x$$



12. If $\cos a = \frac{12}{13}$ and $\cos b = \frac{12}{37}$, find $\tan(a - b)$.

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b} = \frac{\frac{5}{12} - \frac{35}{12}}{1 + \frac{5}{12} \cdot \frac{35}{12}} = \frac{-\frac{30}{12}}{1 + \frac{175}{144}} = \frac{-\frac{30}{12}}{\frac{319}{144}}$$

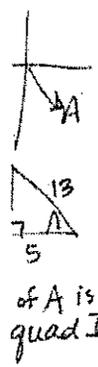
13. If $\csc a = \frac{13}{12}$ and $\sec b = \frac{5}{3}$, find $\sin(a - b)$.

$$\sin(a-b) = \sin a \cdot \cos b - \cos a \cdot \sin b = \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) = \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$$

Find the exact value.

14. $\tan 75^\circ = \tan \frac{150^\circ}{2} = \frac{1 - \cos 150^\circ}{\sin 150^\circ} = \frac{1 - (-\frac{\sqrt{3}}{2})}{\frac{1}{2}} = \frac{1 + \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 + \sqrt{3}$

15. $\sin(-15^\circ) = \sin(30^\circ - 45^\circ) = \sin 30^\circ \cos 45^\circ - \cos 30^\circ \sin 45^\circ = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$



If $\sin A = -\frac{12}{13}$ and $\angle A$ is an angle in quadrant IV, find the exact value.

16. $\cos 2A = 1 - 2\sin^2 A = 1 - 2\left(-\frac{12}{13}\right)^2 = 1 - 2\left(\frac{144}{169}\right) = \frac{-119}{169}$

17. $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2\left(-\frac{12}{5}\right)}{1 - \left(-\frac{12}{5}\right)^2} = \frac{-\frac{24}{5}}{1 - \frac{144}{25}} = \frac{-\frac{24}{5}}{\frac{-119}{25}} = \frac{-24}{5} \cdot \frac{25}{-119} = \frac{120}{119}$

18. $\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}} = \pm \sqrt{\frac{1 - (-\frac{119}{169})}{2}} = \pm \sqrt{\frac{1 + \frac{119}{169}}{2}} = \pm \sqrt{\frac{\frac{188}{169}}{2}} = \pm \sqrt{\frac{94}{169}} = \pm \frac{\sqrt{94}}{13}$

19. $\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}} = \pm \sqrt{\frac{1 + (-\frac{119}{169})}{2}} = \pm \sqrt{\frac{1 - \frac{119}{169}}{2}} = \pm \sqrt{\frac{\frac{50}{169}}{2}} = \pm \sqrt{\frac{25}{169}} = \pm \frac{5}{13}$

Solve for values of θ such that $0 \leq \theta < 2\pi$.

20. $2\sin^2 \theta - 1 = 0 \Rightarrow \sin^2 \theta = \frac{1}{2} \Rightarrow \sin \theta = \pm \frac{\sqrt{2}}{2}$
 $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

21. $(2\sin \theta - 1)(2\cos \theta + \sqrt{3}) = 0$
 $2\sin \theta - 1 = 0 \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$
 $2\cos \theta + \sqrt{3} = 0 \Rightarrow \cos \theta = -\frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

22. $2\sin^2 x - 5\sin x + 2 = 0$
 $(2\sin x - 1)(\sin x - 2) = 0$
 $2\sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

23. $\tan^2 x - \sqrt{3} \tan x = 0$
 $\tan x(\tan x - \sqrt{3}) = 0$
 $\tan x = 0 \Rightarrow x = 0, \pi$
 $\tan x = \sqrt{3} \Rightarrow x = \frac{\pi}{3}, \frac{4\pi}{3}$

Write the expression as the sine, cosine, or tangent of an angle.

24. $\cos 25^\circ \cos 15^\circ - \sin 25^\circ \sin 15^\circ = \cos(25^\circ + 15^\circ) = \cos 40^\circ$

25. $\frac{\tan 68^\circ - \tan 115^\circ}{1 + \tan 68^\circ \tan 115^\circ} = \tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \cdot \tan v}$
 $\tan(68^\circ - 115^\circ) = \tan(-47^\circ)$