

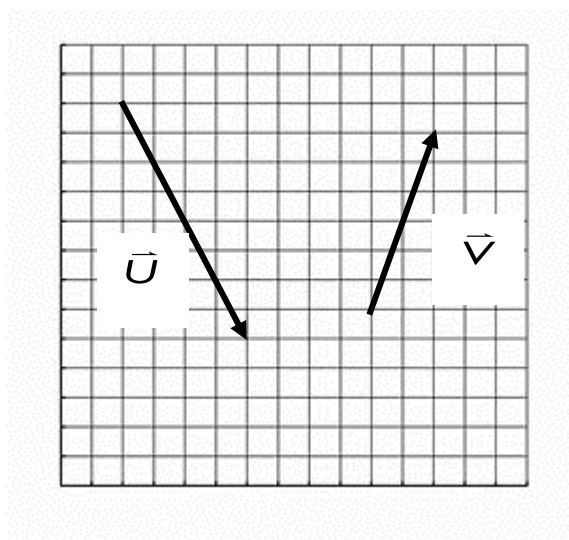
Unit 7 Vectors & Complex Numbers

Date	Topic(s)	Assignment/HW	Credit
Mon. 11/15	Geometric Vectors		
Tues. 11/16	Algebraic Vectors		
Wed. 11/17	Algebraic Vectors		
Thurs. 11/18	Vector Applications		
Fri. 11/19	Vector Applications		
	THANKSGIVING BREAK		
Mon. 11/29	Vector Review		
Tues. 11/30	Vector Quiz		
Wed. 12/1	Complex Numbers		
Thurs. 12/2	Absolute Value & Direction of Complex Numbers		
Fri. 12/3	Distance & Midpoint of Complex Numbers		
Mon. 12/6	Converting Between Rectangular Form & Polar Form of Complex Numbers		
Tues. 12/7	Unit Review		
Wed. 12/8	UNIT 7 TEST		

Name _____

WS #1 Geometric Approach to Vectors

Given vectors \vec{U} and \vec{V} , find their component form and magnitude.



Component form magnitude

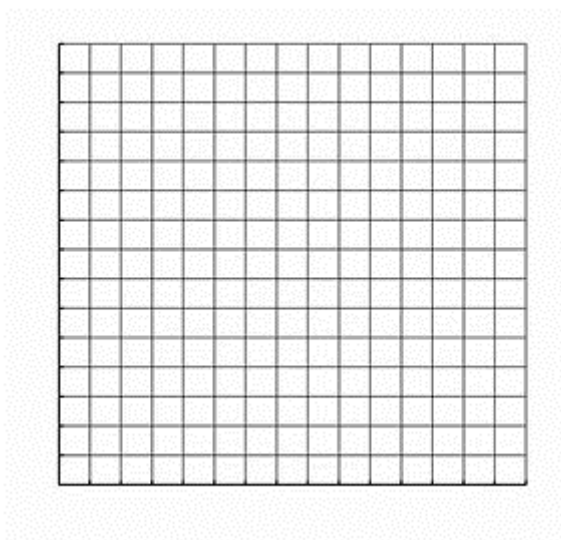
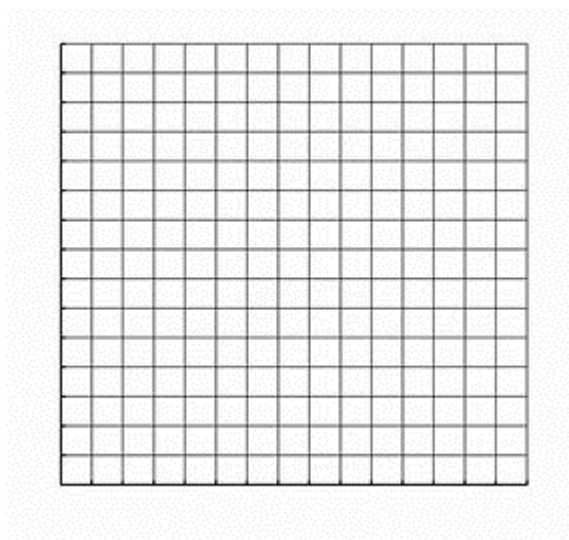
1. \vec{U} _____, _____

2. \vec{V} _____, _____

Sketch the following. Then find the component form of the resultant.

3. $2\vec{U}$

4. $\vec{U} + \vec{V}$

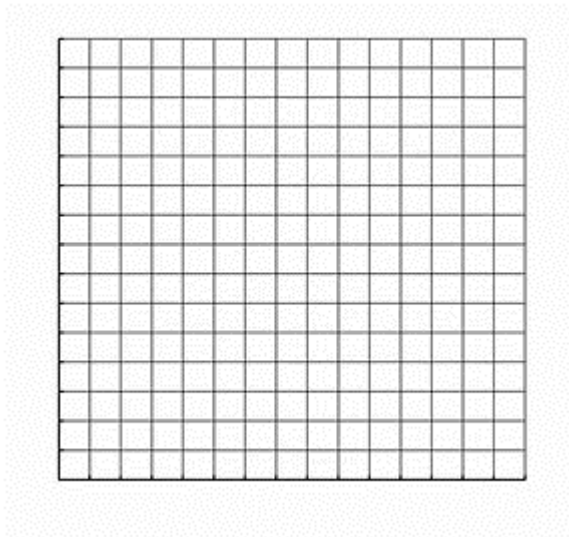


Copy the component form from #1-2 to continue to use for #5-8

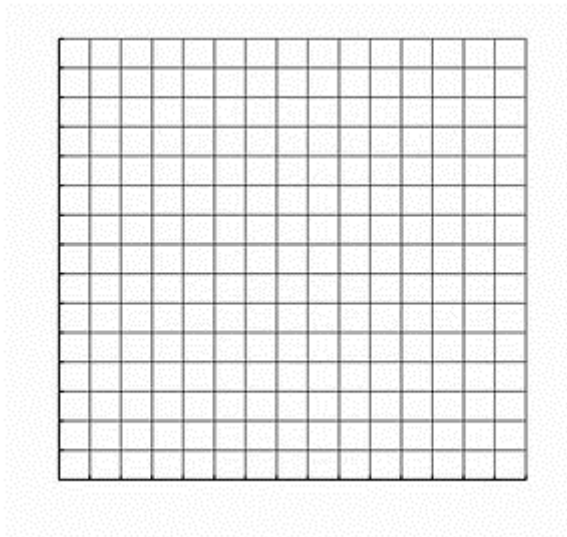
$$\vec{U} = \underline{\hspace{2cm}}$$

$$\vec{V} = \underline{\hspace{2cm}}$$

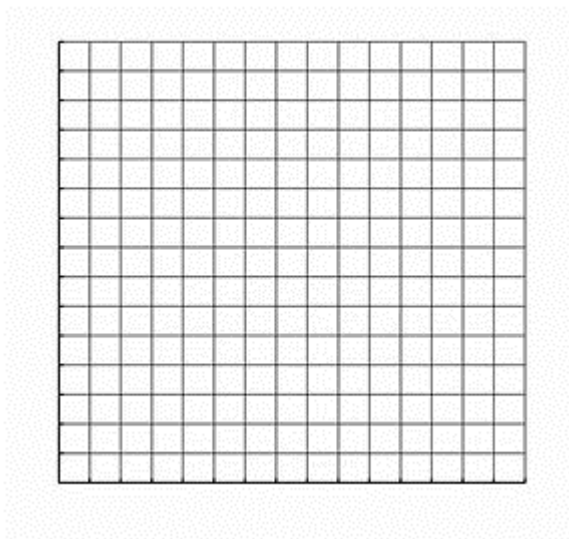
5. $\vec{U} - \vec{V}$



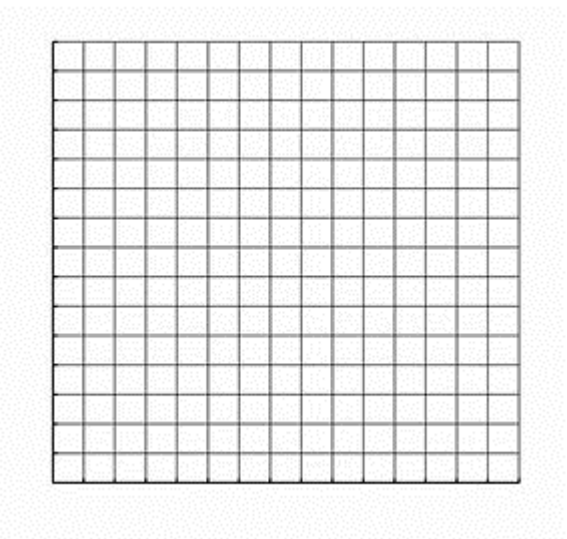
6. $\vec{V} - \vec{U}$



7. $\vec{U} + 2\vec{V}$

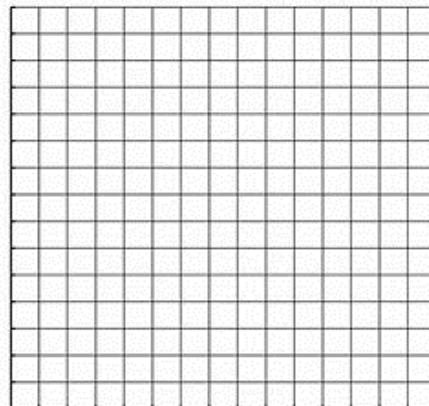
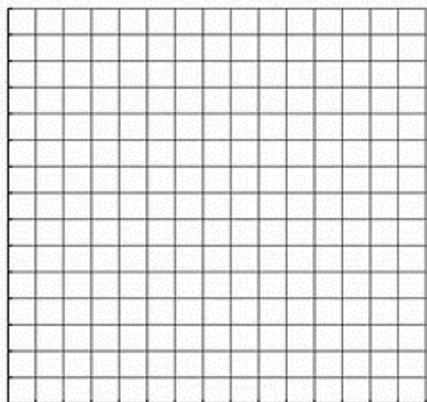


8. $2\vec{U} - \frac{1}{2}\vec{V}$



9. $m = \langle 14, 7 \rangle$

10. Vector CD , where $C = (-10, 10)$ $D = (9, 3)$

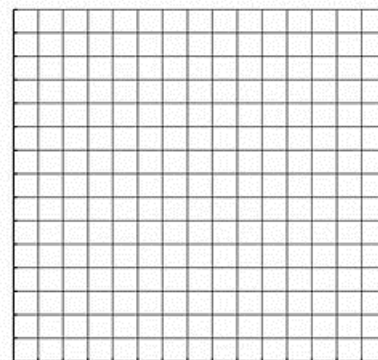
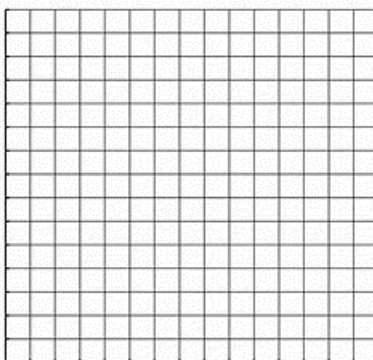
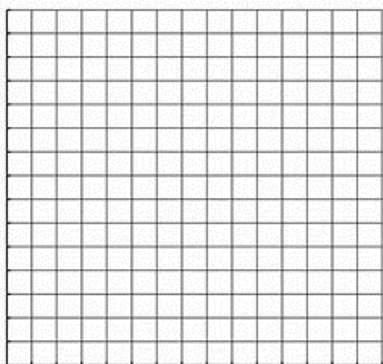


Sketch and find the component form of the resultant vector.

11. $\vec{u} = \langle 1, -1 \rangle$, $\vec{v} = \langle 4, -2 \rangle$
Find: $-\vec{u} + \vec{v}$

12. $\vec{f} = \langle -2, -6 \rangle$, $\vec{v} = \langle -3, -4 \rangle$
Find: $\vec{f} + \vec{v}$

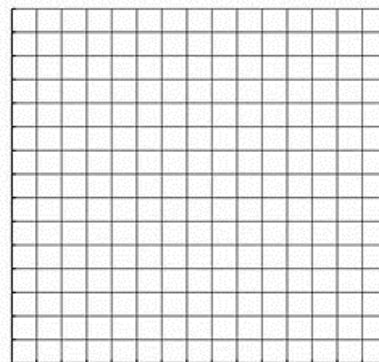
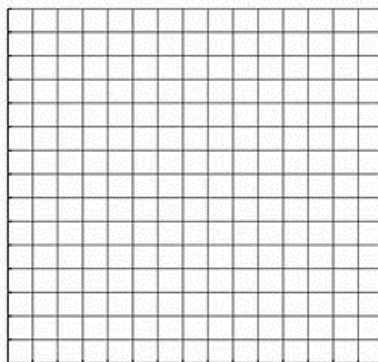
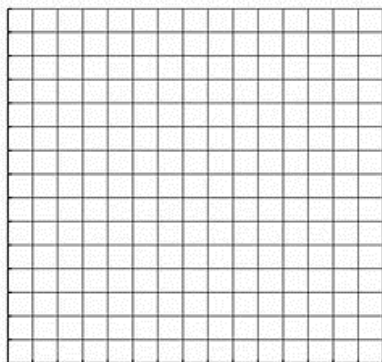
13. $\vec{u} = \langle -3, 4 \rangle$, $\vec{g} = \langle -6, -4 \rangle$
Find: $-\vec{u} - \vec{g}$



14. $\vec{f} = \langle 10, 1 \rangle$, $\vec{g} = \langle -6, 3 \rangle$
Find: $-\vec{f} - \vec{g}$

15. $\vec{f} = \langle -4, -12 \rangle$, $\vec{v} = \langle 2, 3 \rangle$
Find: $\vec{f} + \vec{v}$

16. $\vec{f} = \langle 5, -3 \rangle$
Find: $3\vec{f}$



WS #2 Algebraic Approach to Vectors

Find the following information for each vector: Component form, magnitude.

1. \overrightarrow{PQ} where $P=(0, 5)$, $Q=(1, -3)$

2. \overrightarrow{AB} where $A=(-1, 6)$, $Q=(0, 1)$

3. \overrightarrow{AB} where $A=(-9, -6)$, $Q=(3, 5)$

4. \overrightarrow{RS} where $R=(0, 6)$, $Q=(-5, -1)$

5. \overrightarrow{CD} where $C=(-1, 6)$, $D=(-3, -4)$

6. \overrightarrow{AB} where $A=(-6, 2)$, $Q=(-8, 2)$

7. \overrightarrow{PQ} where $P=(-6, -10)$, $Q=(-6, 6)$

8. \overrightarrow{AB} where $A=(-2, 10)$, $Q=(7, -10)$

9. \overrightarrow{CD} where $C=(-9, -4)$, $Q=(-7, 0)$

10. \overrightarrow{RS} where $R=(-9, -2)$, $S=(8, -6)$

Find the component form of the resultant vector by addition, subtraction, or scalar multiplication.

11. $f = \langle -4, 11 \rangle$, $v = \langle -11, 4 \rangle$
Find: $f + v$

12. $u = \langle -2, 1 \rangle$
Find: $-7u$

13. $u = \langle -10, 9 \rangle$, $v = \langle 12, -10 \rangle$
Find: $u - v$

14. $u = \langle -3, -8 \rangle$
Find: $10u$

15. $u = \langle 1, 9 \rangle$, $v = \langle 10, -7 \rangle$
Find: $u - v$

16. $f = \langle 3, -11 \rangle$, $v = \langle -2, 1 \rangle$
Find: $f + v$

17. $u = \langle 2, -4 \rangle$, $v = \langle 4, -12 \rangle$
Find: $-u - v$

18. $u = \langle 8, -11 \rangle$, $g = \langle -2, 10 \rangle$
Find: $u - 6g$

19. $f = \langle 10, 2 \rangle$, $v = \langle -9, -4 \rangle$
Find: $-5f - 3v$

Write each vector as a linear combination.

21. $\vec{b} = \langle -29, 35 \rangle$

22. $\vec{v} = \langle -4, 8 \rangle$

23. $\vec{n} = \langle -10, -5 \rangle$

24. $\vec{b} = \langle -5, -27 \rangle$

25. $\vec{m} = \langle -19, 3 \rangle$

26. $\vec{k} = \langle -16, 30 \rangle$

27. $\vec{r} = \langle 1, -3 \rangle$

28. $\vec{b} = \langle -34, -47 \rangle$

29. $\vec{p} = \langle 20, 0 \rangle$

Write each vector in component form. (Be sure the signs of your answer are appropriate for the quadrant where the angles lie.)

30. $|\vec{v}| = 10, 138^\circ$

31. $|\vec{r}| = 93, 162^\circ$

32. $|\vec{r}| = 16, 199^\circ$

33. $|\vec{b}| = 20, 332^\circ$

34. $|\vec{v}| = 20, 203^\circ$

35. $|\vec{a}| = 53, 119^\circ$

36. $|\vec{n}| = 94, 337^\circ$

37. $|\vec{r}| = 75, 73^\circ$

38. $|\vec{v}| = 91, 31^\circ$

Find the dot product of the given vectors. Then state if the 2 vectors are orthogonal (\perp) or not.

39. $\vec{u} = \langle -2, -7 \rangle, \vec{v} = \langle 5, -7 \rangle$

40. $\vec{u} = \langle 6, -2 \rangle, \vec{v} = \langle 1, 3 \rangle$

41. $\vec{u} = \langle 6, 0 \rangle, \vec{v} = \langle 0, -5 \rangle$

42. $\vec{u} = \langle 6, -6 \rangle, \vec{v} = \langle -8, -8 \rangle$

43. $\vec{u} = \langle 9, -4 \rangle, \vec{v} = \langle 9, 8 \rangle$

44. $\vec{u} = \langle -4, 8 \rangle, \vec{v} = \langle -7, -9 \rangle$

45. $\vec{u} = \langle 4, 1 \rangle, \vec{v} = \langle -1, -9 \rangle$

46. $\vec{u} = \langle -4, 0 \rangle, \vec{v} = \langle 0, -4 \rangle$

47. $\vec{u} = \langle -2, 3 \rangle, \vec{v} = \langle 9, -6 \rangle$

WS #3 Vector Applications

1. Two forces of 5 kg and 12 kg act on an object at right angles. Find the magnitude of the resultant and the angle it makes with the smaller force.	2. Two forces of 30 kg and 40 kg act on an object at right angles. Find the magnitude of the resultant and the angle it makes with the smaller force.
3. Forces of 420 kg and 300 kg act on an object. The angle between the forces is 50 degrees. Find the resultant and the angle it makes with the larger force.	4. Forces of 410 kg and 600 kg act on an object. The angle between the forces is 47 degrees. Find the resultant and the angle it makes with the smaller force.
5. A balloon is rising 12 ft/sec while a wind is blowing 18 ft/sec. Find the speed of the balloon and the angle it makes with the ground.	6. A balloon is rising 10 ft/sec while a wind is blowing 5 ft/sec. Find the speed of the balloon and the angle it makes with the ground.

More Applications - Draw and label each vector and angle.

1. A plane flies due west at 250 kilometers per hour while the wind blows south at 70 kilometers per hour. Find the plane's resultant velocity and direction.	2. A plane flies east for 200 km, then 60° South of east for 80 km. Find the plane's distance and direction from its starting point.
3. One force of 100 units acts on an object. Another force of 80 units acts on the object at a 40° angle from the first force. Find the magnitude and direction of the resultant force on the object.	4. Two forces of 15 and 22 newtons (a newton is a unit of force used in physics) act on a point in the plane. If the angle between the forces is 100° , find the magnitude of the resultant force.
5. Forces of 18 and 31 newtons form an angle of 110° . Find the magnitude of the resultant force and the angle between it and the 18-newton force.	6. Forces of 320 and 450 lbs. form an angle of 75° . Find the angle between the 450-lb force and the resultant force and the magnitude of the resultant.

7. Find the magnitude of the equilibrant of forces of 48 and 60 newtons acting on a point A, if the angle between the forces is 50° . Then find the angle between the equilibrant and the 48-newton force.

8. One end of a ramp is raised to the back of a truck 3 meters above the ground. If the length of the ramp is 7 meters, what is the approximate measure of the angle the ramp makes with the ground. Round to the closest 10th.

9. Nina is pushing a lawn mower with a force of 95 newtons along the handle of the mower. The handle makes a 72° angle with the horizontal. What are the magnitudes of the horizontal and vertical components of the force? Round to the closest 10th.

10. Two forces of 18 and 29 newtons act on a point in the plane. If the angle between the forces is 120° , find the magnitude of the resultant force and the angle the resultant makes with the 18-newton force. Round to the closest 10th.

11. An airplane heads in a direction of 80° at 120 km/h airspeed while a wind blows east 60 km/h. Find the speed of the airplane over the ground and the direction of its track over the ground. Round to the closest 10th.

WS #4 Complex Number Review

Solve for x.

1. $x^2 + 64 = 0$

2. $4x^2 + 3 = 2x^2 + 4$

3. $-2(x + 1)^2 = 72$

Simplify. Write in standard form, $a + bi$.

4. $(3 + 2i) + (-5 + 8i)$

5. $(-2 - 4i) - (-1 + 5i)$

6. $2i(3 - 4i)$

7. $(5 + 3i)(2 - 4i)$

8. $(2 + 3i)^2$

9. $\frac{5}{1 + 2i}$

10. $\frac{4 + i}{3 - 2i}$

11. $\frac{1 + i}{\sqrt{2} - i}$

12. $(1 - 5i)(2 + i) - i(3 - 4i)$

13. $2(2 + i) - (1 + i)^2$

Evaluate the following.

14. i^9

15. i^{36}

16. $i^{1,347}$

More Practice on Complex Numbers

Simplify completely. Write in standard form.

1. $(1-i) - (-6-7i)$

2. $(-5+6i) + (-7-7i)$

3. $4 + (i) - (4 + 6i)$

4. $(3-8i) - (-1+5i)$

5. $(-4+6i) + (-8-5i)$

6. $(-5-2i) - (2-5i)$

7. $(-3-3i) + (7+3i)$

8. $(5-8i) + (-6-3i)$

9. $(3+5i) - (-8-6i)$

10. $(-1-2i) + (8-4i)$

11. $(4i)(2i)(-4-5i)$

12. $(7-2i)(6-7i)$

13. $(3i)(7i)(-4-8i)$

14. $(-6-2i)(2+7i)$

15. $(7+8i)(3+4i)$

Simplify completely.

$$16. \frac{6}{5i}$$

$$17. \frac{-8}{-7i}$$

$$18. \frac{2i}{-4-8i}$$

$$19. \frac{8}{-10+10i}$$

$$20. \frac{-2-3i}{-2+4i}$$

$$21. \frac{-5+6i}{1+5i}$$

$$22. \frac{-1-i}{8+7i}$$

$$23. \frac{6i}{-4-2i}$$

$$24. \frac{3+i}{3-8i}$$

WS #5 Graphing Abs. Value/Complex #/Distance & Midpoint

Find the absolute value of each complex number.

1. $|2 - 10i|$

2. $|-9 + 3i|$

3. $|7 + 2i|$

4. $|3 - 5i|$

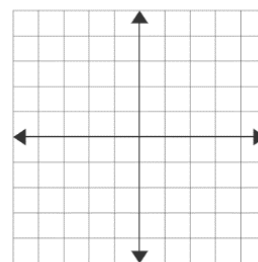
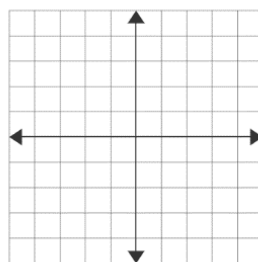
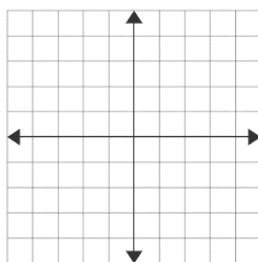
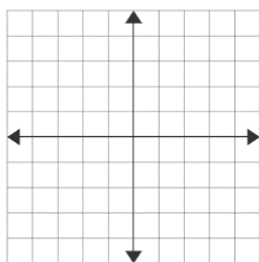
Graph each number in the complex plane. Then find the direction.

5. $-i$

6. $-3 - 4i$

7. $1 + 5i$

8. -2



Direction: _____

Direction: _____

Direction: _____

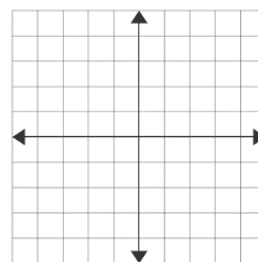
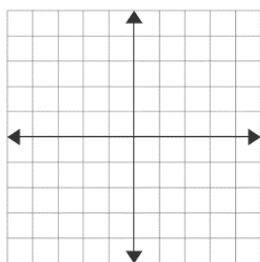
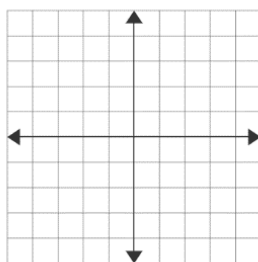
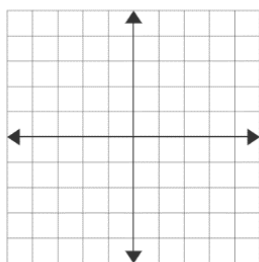
Direction: _____

9. $-1 - 4i$

10. $4 + 4i$

11. $-5 + 5i$

12. $2 + 4i$



Direction: _____

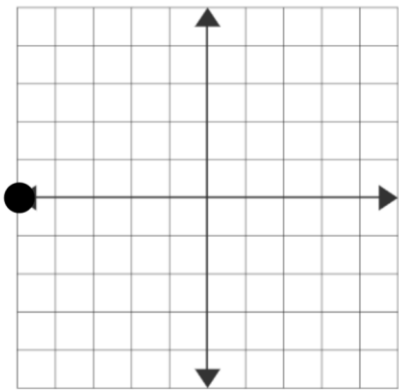
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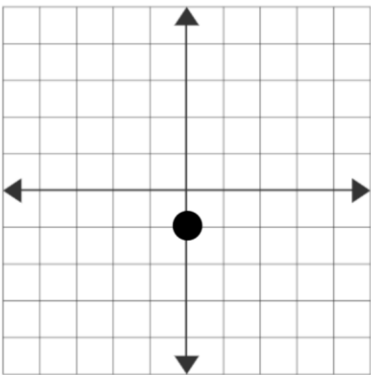
Direction: _____

Identify each complex number graphed. Write in standard form.

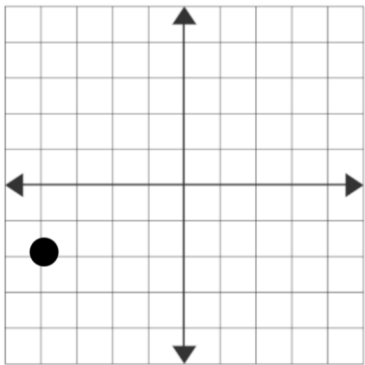
13. _____



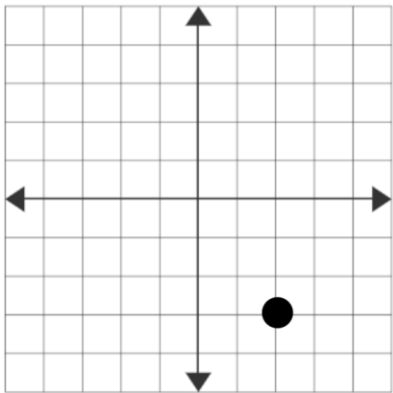
14. _____



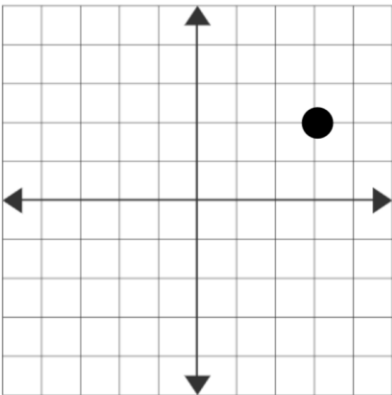
15. _____



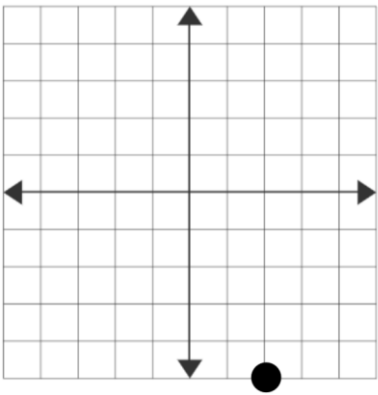
16. _____



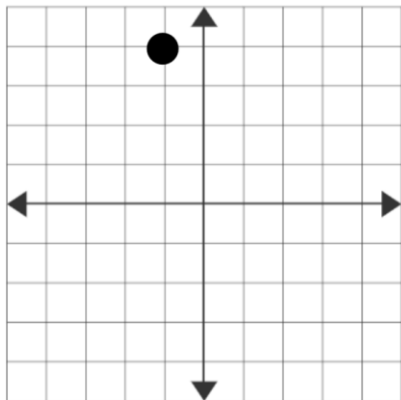
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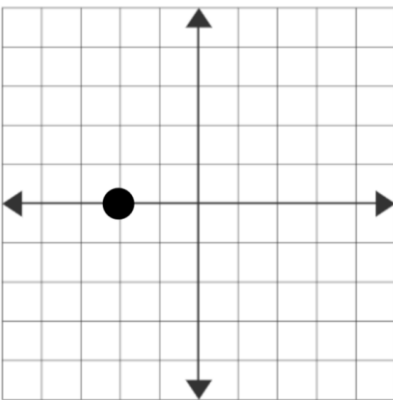
18. _____



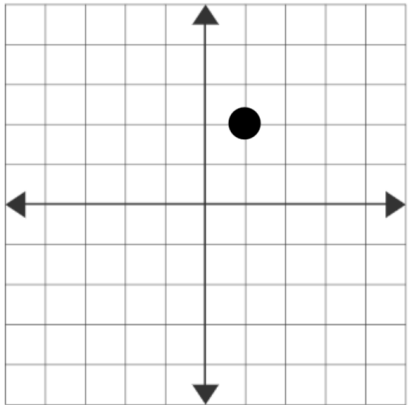
15. _____



16. _____



17. _____



Calculating Distance and Midpoint in the complex plane - Matching!

Match the word problem to their answers. Write the answer that matches the problem for each.

Distance/midpoint

- | | | |
|-------|-------|---|
| _____ | _____ | 1. Find the distance and midpoint between $(-3 - 5i)$ and $(3 + 4i)$ on the complex plane. |
| _____ | _____ | 2. Calculate the distance and midpoint between $(-4 + 2i)$ and $(3 + 4i)$ on the complex plane. |
| _____ | _____ | 3. Find the distance and midpoint between $(-3 - 5i)$ and $(-3 + 5i)$. |
| _____ | _____ | 4. Calculate the distance and midpoint between $(-4 - 4i)$ and $(-4 + 4i)$. |
| _____ | _____ | 5. What is the distance and midpoint between $(-5 - 4i)$ and $(-4 + 2i)$ on the complex plane? |
| _____ | _____ | 6. What is the distance and midpoint between $(-2 - 2i)$ and $(2 + 2i)$ on the complex plane? |
| _____ | _____ | 7. Calculate the distance and midpoint between $(-1 + i)$ and $(-5 + 6i)$. |
| _____ | _____ | 8. Find the distance and midpoint between $(2 - 4i)$ and $(-4 + i)$. |
| _____ | _____ | 9. What is the distance and midpoint between $(-4 - 4i)$ and $(2 + i)$ on the complex plane? |
| _____ | _____ | 10. Find the distance and midpoint between $(-2 + 3i)$ and $(4 + 5i)$? |

Some answers may be used twice.

- | | | | | | | |
|------------------------|---------------------------------|------------------------|-----------------------|------------------------|-------------------|---------|
| a. 8 | b. 10 | c. 10.82 | d. 5.65 | e. 6.08 | f. 7.28 | g. 6.32 |
| h. 7.80 | i. 7.41 | j. 6.40 | k. $1 + 4i$ | l. 0 | m. $\frac{-i}{2}$ | n. -3 |
| o. $\frac{-1}{2} + 3i$ | p. $\frac{-7}{2} - \frac{i}{2}$ | q. $-3 + \frac{7i}{2}$ | r. $\frac{-9}{2} - i$ | s. $-1 - \frac{3}{2}i$ | t. -4 | |

WS #6 Polar and Rectangular Form

Express each complex number in polar form.

1. $3 + 3\sqrt{3}i$

2. $7i$

3. $-\sqrt{3} + i$

4. 4

5. $1 - i$

6. $-1 + i$

7. $\sqrt{2} - \sqrt{2}i$

8. $-\sqrt{2} + \sqrt{2}i$

9. $2\sqrt{3} + 2i$

10. $1.5 + 1.5i$

11. $5 - 5\sqrt{3}i$

12. $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$

Express each complex number in rectangular form. Write with both exact values and decimals to the closest 10th.

13. $2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

14. $\sqrt{2}\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

15. $1.5\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

16. $4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

17. $4\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

18. $3\left(\cos\frac{11\pi}{4} + i\sin\frac{11\pi}{4}\right)$

WS #7 Test Review

1. Draw the vector with the given magnitude and direction. Then find the magnitude of the horizontal and vertical components.

a. $r = 1.4 \text{ cm}$ $\theta = 50^\circ$

b. $r = 3 \text{ cm}$ $\theta = 10^\circ$

2. Find the ordered pair (in component form) that represents the vector from A to B. Then find the magnitude of vector AB.

a. A (2, 3) B (7, 15)

b. A (-2, 8) B (4, 12)

3. Find the ordered pair in component form to represent \vec{u} if $\vec{v} = \langle 2, -5 \rangle$ and $\vec{w} = \langle 3, -1 \rangle$,

a. $\vec{u} = \vec{v} + \vec{w}$

b. $\vec{u} = 3\vec{v} - \vec{w}$

c. $\vec{u} = 3\vec{w} - 2\vec{v}$

4. Find the component form of \vec{v} with the given magnitude and direction angle.

a. $|\vec{v}| = 6$ and $\theta = 113^\circ$

b. $|\vec{v}| = 16$ and $\theta = 330^\circ$

5. Find the direction angle of each vector to the nearest tenth of a degree.

a. $2i - 6j$

b. $\langle -6, -4 \rangle$

6. Find the inner (dot) product of the 2 vectors and state whether the vectors are orthogonal.

a. $\langle 5, -1 \rangle \bullet \langle -2, 6 \rangle$

b. $\langle 3, -2 \rangle \bullet \langle 4, 6 \rangle$

7. Perform transformations on the vector $\langle -1, 4 \rangle$ for the transformation matrices. Graph the original vector and the transformed vector on the same graph. State whether the vector's magnitude and/or direction has changed.

a. $A = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

b. $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

8. A 32-pound force and a 26-pound force act on the same object at the same time. Find the magnitude of their resultant if the angle between the two forces is 82° .



9. An airplane flies east for 23 km before turning 35° south and flying for 15 km. Find the distance and direction of the plane from its starting point.



Simplify using each operation.

10. $(2 + 3i) + (4 - 4i)$

11. $(-3 - i) - (2 + 7i)$

12. $(2 - 4i)(5 + 2i)$

13. $\frac{4 + i}{5 - 2i}$

14. $i^{10} \bullet i^{25}$

15. Find the absolute value and direction of the following complex number.

a. $|6 + 8i|$

b. $|-2 + 3i|$

16. Express the polar form of the complex number into the rectangular form of a complex number.

Write as an exact value and then round to the closest tenth.

a. $4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

b. $8\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

17. Express the rectangular form of the complex number into the polar form of a complex number.

a. $-2 + 2i\sqrt{3}$

b. $4 - 3i$

18. Find the distance and midpoint between the complex numbers.

a. $(2 + 3i)$ and $(4 + 4i)$

b. $(-3 - i)$ and $(-2 + 7i)$

19. Two ropes are pulling on a sign. The ropes make a 26° angle with each other. If one is pulling with a force of 12 pounds and the other is pulling with a force of 30 pounds, what is the magnitude and direction of the net force on the sign?

