Honors Calculus: Unit 6 Integration Rules Practice Test   Name ____________

Find the antiderivative.

1. $\int -9\cos \pi x \, dx$

$\boxed{\dfrac{-9\sin \pi x}{\pi} + C}$

2. $\int e^{-12x}\, dx$

$\boxed{-\dfrac{e^{-12x}}{12} + C \text{ or } \dfrac{-1}{12e^{12x}} + C}$

3. $\int \csc(8x)\cot(8x)\, dx$

$\boxed{-\dfrac{\csc(8x)}{8} + C}$

4. $\int 6\sqrt{2x-5}\, dx$

$\dfrac{2\cdot 6(2x-5)^{3/2}}{3\cdot 2} = \boxed{2(2x-5)^{3/2} + C}$

or $2\sqrt{2x-5}^{\,3} + C$

5. $\int (\cos^3 x)(\sin x)\, dx$

$u = \cos x$
$du = \sin x\, dx$
$dx = -\dfrac{du}{\sin x}$

$\int u^3 (\cancel{\sin x}) \cdot \dfrac{-du}{\cancel{\sin x}}$

$-\dfrac{u^4}{4} = \boxed{-\dfrac{\cos^4 x}{4} + C}$

6. $\int \dfrac{3x^2+3}{(x^3+3x+2)^4}\, dx$

$u = x^3+3x+2$
$du = 3x^2+3\, dx$
$dx = \dfrac{du}{3x^2+3}$

$\int \dfrac{\cancel{3x^2+3}}{u^4} \cdot \dfrac{du}{\cancel{3x^2+3}}$

$\int u^{-4} = \dfrac{u^{-3}}{-3} = \dfrac{1}{-3u^3}$

$\boxed{\dfrac{-1}{3(x^3+3x+2)^3} + C}$

7. $\int \cos(2x)\cdot(5+\sin(2x))^6\, dx$

$u = 5+\sin(2x)$
$du = 2\cos(2x)\, dx$
$dx = \dfrac{du}{2\cos(2x)}$

$\int \cancel{\cos(2x)}\, u^6 \dfrac{du}{2\cancel{\cos(2x)}}$

$\dfrac{1}{2}\int u^6 du$

$\boxed{\dfrac{(5+\sin 2x)^7}{14} + C}$

8. $\int \dfrac{x^4}{\sqrt[3]{x^5+8}}\, dx$

$u = x^5+8$
$du = 5x^4\, dx$
$dx = \dfrac{du}{5x^4}$

$\int \dfrac{\cancel{x^4}}{u^{1/3}} \dfrac{du}{5\cancel{x^4}} = \dfrac{1}{5}\int u^{-1/3} du$

$\dfrac{1}{5}\cdot\dfrac{3u^{2/3}}{2} = \boxed{\dfrac{3(x^5+8)^{2/3}}{10} + C}$

9. $\int (6^{x^2})5x\, dx$

$u = x^2$
$du = 2x\, dx$
$dx = \dfrac{du}{2x}$

$\int 6^u 5\cancel{x} \dfrac{du}{2\cancel{x}}$

$\dfrac{5}{2}\cdot\dfrac{6^u}{\ln 6} = \boxed{\dfrac{5(6)^{x^2}}{2\ln 6} + C}$

10. $\int \cos(3x)\cdot 5x^2\, dx$

T A

u	dV
$+5x^2$	$\cos 3x$
$-10x$	$\frac{\sin 3x}{3}$
$+10$	$-\frac{\cos 3x}{9}$
0	$-\frac{\sin 3x}{27}$

$\boxed{\dfrac{5x^2\sin(3x)}{3} + \dfrac{10x\cos(3x)}{9} - \dfrac{10\sin(3x)}{27} + C}$

11. $\int (x^3+3x)e^{2x}\, dx$

A E

u	dV
$+\ x^3+3x$	e^{2x}
$-\ 3x^2+3$	$e^{2x}/2$
$+\ 6x$	$e^{2x}/4$
$-\ 6$	$e^{2x}/8$
0	$e^{2x}/16$

$\boxed{\dfrac{(x^3+3x)e^{2x}}{2} - \dfrac{(3x^2+3)e^{2x}}{4} + \dfrac{3xe^{2x}}{4} - \dfrac{3e^{2x}}{8} + C}$

12. $\int 2x^3\cdot \ln x\, dx$

A L

$uv - \int v\,du$

$u = \ln x \quad dV = 2x^3$
$du = \dfrac{1}{x}dx \quad V = \dfrac{2x^4}{4} = \dfrac{x^4}{2}$

$\dfrac{x^4\ln x}{2} - \int \dfrac{x^4}{2}\cdot\dfrac{1}{x}\, dx$

$\dfrac{x^4\ln x}{2} - \dfrac{1}{2}\int x^3 dx$

$\boxed{\dfrac{x^4\ln x}{2} - \dfrac{x^4}{8} + C}$

13.

$\int \dfrac{e^{5x}}{1+e^{5x}}$

$u = 1+e^{5x}$
$du = 5e^{5x}dx$
$dx = \dfrac{du}{5e^{5x}}$

$\int \dfrac{e^{5x}}{u}\cdot\dfrac{du}{5e^{5x}}$

$\dfrac{1}{5}\int \dfrac{1}{u} du$

$\boxed{\dfrac{1}{5}\ln|1+e^{5x}| + C}$

Find the definite integrals.

14. $\int_1^e -4x^{-1}\,dx$

$-4\ln|x|\Big|_1^e$

$-4\ln e + 4\ln 1 = \boxed{-4}$

$-4(1)+4(0)$

15. $\int_0^2 \frac{2x}{\sqrt{3+4x^2}}\,dx$

$u_2 = 19 \quad u_0 = 3$

$u = 3+4x^2$

$du = 8x\,dx$

$dx = \frac{du}{8x}$

$\int_0^2 \frac{\cancel{2x}}{u^{1/2}}\frac{du}{\cancel{8x}_4} = \frac{1}{4}\int_0^2 u^{-1/2}$

$\frac{1}{\cancel{4}2}\cancel{2}u^{1/2}\Big|_3^{19} = \boxed{\frac{\sqrt{19}}{2} - \frac{\sqrt{3}}{2}}$

16. $\int_0^1 2x\cdot e^{3x}\,dx$

u	dv
$+2x$	e^{3x}
-2	$\frac{e^{3x}}{3}$
$+0$	$\frac{e^{3x}}{9}$

$\frac{2xe^{3x}}{3} - \frac{2e^{3x}}{9}\Big|_0^1$

$\left(\frac{3\cdot 2e^3}{3\cdot 3} - \frac{2e^3}{9}\right) + \left(0 + \frac{2}{9}\right)$

$\boxed{\frac{4e^3}{9} + \frac{2}{9}}$

17. $\int_0^\pi 3x^2\cos(x)\,dx$

u	dV
$+3x^2$	$\cos x$
$-6x$	$\sin x$
$+6$	$-\cos x$
-0	$-\sin x$

$3x^2\sin x + 6x\cos x - 6\sin x\Big|_0^\pi$

$3\pi^2\sin\pi + 6\pi\cos\pi - (6\sin\pi) - (0+0-6\sin 0)$

$0 + 6\pi(-1) - 0 - 0 - 0 + 0 = \boxed{-6\pi}$

18. $\int_{.5}^1 5^{2x-1}\,dx$

$\frac{5^{2x-1}}{2\ln 5}\Big|_{.5}^1 = \frac{5}{2\ln 5} - \frac{1}{2\ln 5} = \boxed{\frac{2}{\ln 5}}$

Find the average value of the functions over the given interval.

19. $f(x) = \sin x; \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$\frac{1}{\frac{3\pi}{2} - \frac{\pi}{2}}\int_{\pi/2}^{3\pi/2} \sin x = -\frac{1}{\pi}\cos x\Big|_{\pi/2}^{3\pi/2}$

$-\frac{\cos 3\pi/2}{\pi} - -\frac{\cos \pi/2}{\pi} = 0 + 0 = \boxed{0}$

20. $f(x) = 2e^{x-1}$; ~~[3, 2]~~ [2, 3]

$\frac{1}{3-2}\int_2^3 2e^{x-1} = 2e^{x-1}\Big|_2^3$

$\boxed{2e^2 - 2e}$

Find the general solution of the differential equation.

21. $\frac{dy}{dx} = \frac{2x}{y^3}$

$\int y^3\,dy = \int 2x\,dx$

$\frac{y^4}{4} = \frac{\cancel{2}x^2}{\cancel{2}} + C$

$\sqrt[4]{y^4} = \sqrt[4]{4x^2+4C}$

$\boxed{y = \pm\sqrt[4]{4x^2+4C}}$

22. $\frac{dy}{dx} = 6yx$

$\frac{1}{y}dy = 6x\cancel{y}\,dx$

$\ln y = \frac{6x^2}{2} + C$

$\log_e y = 3x^2 + C$

$e^{3x^2+C} = y$

$y = e^{3x^2}\cdot e^C$

$\boxed{y = Ce^{3x^2}}$

Find the particular solution of the differential equation.

23. $\frac{dy}{dx} = 10x+5$; y = -4 when x = -3

$dy = 10x + 5\,dx$

$y = \frac{10x^2}{2} + 5x + C$

$y = 5x^2 + 5x + C$

$-4 = 5(-3)^2 + 5(-3) + C$

$C = -34$

$\boxed{y = 5x^2 + 5x - 34}$

24. $\frac{dy}{3x} = \frac{4\,dx}{2y}$; $y(-1) = 3$

$\int 2y\,dy = \int 12x\,dx$

$y^2 = 6x^2 + C$

$y = \pm\sqrt{6x^2 + C}$

or plug in here to find C

$3 = \sqrt{6(1) + C}$

$9 = 6 + C \quad C = 3$

$\boxed{y = \pm\sqrt{6x^2+3}}$