Unit 6

Integration Rules

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- ❖ Quick Indefinite Integrals
- ❖ U Substitution of Indefinite Integrals
- ❖ U Substitution of Definite Integrals
- ❖ Integration by Parts
- ❖ Average Function Value & Mean Value Theorem
- ❖ Differential Equations

Quiz is ____________________

Test is ____________________

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F19

Quick Integrals Practice

1. $\int \sec 5x \tan 5x \, dx$

$\frac{\sec(5x)}{5} + C$

2. $\int \sec^2 2x \, dx$

$\frac{\tan 2x}{2} + C$

3. $\int \sin 15x \, dx$

$\frac{-\cos 15x}{15} + C$

4. $\int \cos 7x \, dx$

$\frac{\sin 7x}{7} + C$

5. $\int e^{4x} \, dx$

$\frac{e^{4x}}{4} + C$

6. $\int e^{-x} \, dx$

$\frac{e^{-x}}{-1}$ or $-e^{-x} + C$

$-\frac{1}{e^x} + C$

7. $\int \csc^2 11x \, dx$

$\frac{-\cot 11x}{11} + C$

8. $\int 3^{8x} \, dx$

$\frac{3^{8x}}{8 \ln 3} + C$

9. $\int e^{12x} \, dx$

$\frac{e^{12x}}{12} + C$

10. $\int (3x+1)^4 \, dx$

$\frac{(3x+1)^5}{15} + C$

11. $\int (5x+11)^6 \, dx$

$\frac{(5x+11)^7}{35} + C$

12. $\int (3-7x)^{12} \, dx$

$\frac{(3-7x)^{13}}{-91} + C$

13. $\int \csc 2x \cot 2x \, dx$

$\frac{-\csc 2x}{2} + C$

14. $\int \sin ex \, dx$

$\frac{-\cos ex}{e} + C$

15. $\int \cos \pi x \, dx$

$\frac{\sin \pi x}{\pi} + C$

16. $\int \csc(2-11x)\cot(2-11x)dx$

$\frac{+\csc(2-11x)}{+11} + C$

17. $\int 5^{12x+3} \, dx$

$\frac{5^{12x+3}}{12 \ln 5} + C$

18. $\int e^{1-2x} dx$

$\frac{e^{1-2x}}{-2} + C$

19. $\int \sqrt{4x-1} \, dx$ $(4x-1)^{1/2}$

$\frac{2(4x-1)^{3/2}}{3 \cdot 4} = \frac{(4x-1)^{3/2}}{6} + C$

20. $\int \frac{3}{x} \, dx$

$3 \ln|x| + C$

21. $\int -6x^{-1} \, dx$

$-6 \ln|x| + C$

22. $\int \sin(5+17x)dx$

$\frac{-\cos(5+17x)}{17} + C$

23. $\int 3^{2-8x} dx$

$\frac{3^{2-8x}}{-8 \ln 3} + C$

24. $\int e^{5x+12} \, dx$

$\frac{e^{5x+12}}{5} + C$

25. $\int \frac{1}{5x} \, dx$

$\frac{1}{5} \ln|x| + C$

26. $\int (x-7)^{80} dx$

$\frac{(x-7)^{81}}{81} + C$

27. $\int e^{5-x} \, dx$

$\frac{e^{5-x}}{-1} + C$

$-e^{5-x} + C$

Integration by Substitution: Part I

Find the Antiderivative

1. $\int y(y^2+5)^8 dy$

$u = y^2+5$
$du = 2y\,dy$
$dy = \frac{du}{2y}$

$\int y \cdot u^8 dy$
$\frac{1}{2}\int y \cdot u^8 \frac{du}{\cancel{2y}}$
$\frac{1}{2} \cdot \frac{u^9}{9} = \frac{u^9}{18} + C$
$\frac{(y^2+5)^9}{18} + C$

2. $\int t^2(t^3-3)^{10} dt$

$u = t^3 - 3$
$du = 3t^2 dt$
$dt = \frac{du}{3t^2}$

$\int t^2 \cdot u^{10} dt$
$\frac{1}{3}\int t^2 \cdot u^{10} \frac{du}{\cancel{3t^2}}$
$\frac{1}{3} \cdot \frac{u^{11}}{11} = \frac{u^{11}}{33} + C$ $\frac{(t^3-3)^{11}}{33} + C$

3. $\int x(x^2-4)^{\frac{7}{2}} dx$

$u = x^2 - 4$
$du = 2x\,dx$
$dx = \frac{du}{2x}$

$\int x \cdot u^{7/2} dx$
$\frac{1}{2}\int x \cdot u^{7/2} \frac{du}{\cancel{2x}}$
$\frac{1}{2} \cdot \frac{2u^{9/2}}{9} = \frac{u^{9/2}}{9} + C$
$\frac{(x^2-4)^{9/2}}{9} + C$

4. $\int \frac{1}{\sqrt{4-x}} dx$

$u = 4 - x$
$du = -1dx$
$dx = -1du$

$\int \frac{1}{u^{1/2}} \cdot -1du$
$-1\int u^{-1/2} du = -1\,u^{1/2} \cdot 2 + C$
$-2(4-x)^{1/2} + C$ or $-2\sqrt{4-x} + C$

5. $\int x(x^2+3)^2 dx$

$u = x^2+3$
$du = 2x\,dx$
$dx = \frac{du}{2x}$

$\int x \cdot u^2 dx$
$\frac{1}{2}\int x \cdot u^2 \frac{du}{\cancel{2x}}$
$\frac{1}{2}\frac{u^3}{3} = \frac{u^3}{6} + C = \frac{(x^2+3)^3}{6} + C$

6. $\int \frac{dy}{y+5}$

$u = y+5$
$du = 1dy$
$dy = du$

$\int \frac{1}{u} dy$
$\int \frac{1}{u} du$
$\ln|u| + C$
$\ln|y+5| + C$

★This is also a quick interval.

7. $\int (2t-7)^{73} dt$

$u = 2t - 7$
$du = 2dt$
$dt = \frac{du}{2}$

$\int u^{73} dt$
$\frac{1}{2}\int u^{73} \frac{du}{\cancel{2}}$
$\frac{1}{2} \cdot \frac{u^{74}}{74} + C = \frac{u^{74}}{148} + C$
$\frac{(2t-7)^{74}}{148} + C$

8. $\int 16x(2x^2+1)^2 dx$

$u = 2x^2 + 1$
$du = 4x\,dx$
$dx = \frac{du}{4x}$

$\int 16x \cdot u^2 dx$
$4\int \cancel{16x} \cdot u^2 \cdot \frac{du}{\cancel{4x}}$
$\frac{4u^3}{3} + C$
$\frac{4(2x^2+1)^3}{3} + C$

9. $\int \sin\theta\,(\cos\theta + 5)^7\,d\theta$

$u = \cos\theta + 5$

$du = -\sin\theta\,d\theta$

$d\theta = \frac{du}{-\sin\theta}$

$\int \sin\theta \cdot u^7\,d\theta$

$\int \cancel{\sin\theta} \cdot u^7 \cdot \frac{du}{-\cancel{\sin\theta}}$

$-\frac{u^8}{8} + C = -\frac{(\cos\theta+5)^8}{8} + C$

10. $\int \sqrt{\cos(3t)}\,\sin 3t\,dt$

$u = \cos(3t)$

$du = -\sin(3t)\cdot 3\,dt$

$dt = \frac{du}{-3\sin(3t)}$

$\int u^{1/2}\sin 3t\,dt$

$\int u^{1/2}\cancel{\sin 3t}\,\frac{du}{-3\cancel{\sin 3t}}$

$-\frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$

$-\frac{2(\cos(3t))^{3/2}}{9} + C$

11. $\int xe^{-x^2}dx$

$u = -x^2$

$du = -2x\,dx$

$dx = \frac{du}{-2x}$

$\int x e^u\,dx$

$\int \cancel{x} e^u \frac{du}{-2\cancel{x}}$

$-\frac{1}{2}e^u + C = -\frac{1}{2}e^{-x^2} + C$ or $\frac{-1}{2e^{x^2}} + C$

12. $\int \sin^6 x \cos x\,dx$

$u = \sin x\,dx$

$du = \cos x\,dx$

$dx = \frac{du}{\cos x}$

$\int u^6 \cancel{\cos x}\,\frac{du}{\cancel{\cos x}}$

$\frac{u^7}{7} + C$

$\frac{\sin^7 x}{7} + C$

13. $\int \sin^6(5x)\cos(5x)dx$

$u = \sin(5x)$

$du = 5\cos(5x)dx$

$dx = \frac{du}{5\cos(5x)}$

$\frac{1}{5}\int u^6 \cancel{\cos(5x)}\cdot\frac{du}{\cancel{5\cos(5x)}}$

$\frac{1}{5}\cdot\frac{u^7}{7} + C = \frac{u^7}{35} + C$

$\frac{\sin^7(5x)}{35} + C$

14. $\int x^2 e^{x^3+1}\,dx$

$u = x^3 + 1$

$du = 3x^2\,dx$

$dx = \frac{du}{3x^2}$

$\int x^2 e^u\,dx$

$\frac{1}{3}\int \cancel{x^2} e^u \frac{du}{\cancel{3x^2}}$

$\frac{1}{3}e^u + C = \frac{1}{3}e^{x^3+1} + C$

15. $\int \sin^3 x \cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$dx = \frac{du}{\cos x}$

$\int u^3 \cos x\,dx$

$\int u^3 \cancel{\cos x}\,\frac{du}{\cancel{\cos x}}$

$\frac{u^4}{4} + C = \frac{\sin^4 x}{4} + C$

16. $\int \frac{(\ln x)^2}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$dx = x\,du$

$\int \frac{u^2}{\cancel{x}}\,\cancel{x}dx$

$\frac{u^3}{3} + C$

$\frac{(\ln x)^3}{3} + C$

17. $\int \frac{e^t + 1}{e^t + t}\,dt$

$u = e^t + t$

$du = e^t + 1\,dt$

$dt = \frac{du}{e^t + 1}$

$\int \frac{\cancel{e^t+1}}{u}\,\frac{du}{\cancel{e^t+1}}$

$\int \frac{1}{u}\,du$

$\ln|u| + C$

$\ln|e^t + t| + C$

18. $\int \frac{y}{y^2+4}\,dy$

$u = y^2 + 4$

$du = 2y\,dy$

$dy = \frac{du}{2y}$

$\frac{1}{2}\int \frac{\cancel{y}}{u}\cdot\frac{du}{\cancel{2y}}$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u| + C$

$\frac{1}{2}\ln|y^2+4| + C$

*Rewrite with trig identities

19. $\int \tan 2x \, dx = \int \frac{\sin 2x}{\cos 2x}$

$u = \cos(2x)$

$du = -2\sin(2x)dx$

$dx = \frac{du}{-2\sin(2x)}$

$\int \frac{\cancel{\sin(2x)}}{u} \cdot \frac{du}{-2\cancel{\sin(2x)}}$

$-\frac{1}{2}\int \frac{1}{u} du = -\frac{1}{2}\ln|u| + C$

$-\frac{1}{2}\ln|\cos 2x| + C$

20. $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$

$u = \sqrt{x}$

$u = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}dx$

$du = \frac{1}{2\sqrt{x}}dx$

$dx = 2\sqrt{x}\, du$

$\int \frac{\cos u}{\cancel{\sqrt{x}}} \cdot 2\cancel{\sqrt{x}}\, du$

$2\int \cos u \, du$

$2\sin u + C$

$2\sin\sqrt{x} + C$

21. $\int \frac{e^{\sqrt{y}}}{\sqrt{y}} dy$

$u = \sqrt{y}$

$u = y^{1/2}$

$du = \frac{1}{2\sqrt{y}}dy$

$dy = 2\sqrt{y}\, du$

$\int \frac{e^u}{\cancel{\sqrt{y}}} \cdot 2\cancel{\sqrt{y}}\, du$

$2e^u + C$

$2e^{\sqrt{y}} + C$

22. $\int \frac{1+e^x}{\sqrt{x+e^x}} dx$

$u = x + e^x$

$du = 1 + e^x dx$

$dx = \frac{du}{1+e^x}$

$\int \frac{\cancel{1+e^x}}{u^{1/2}} \frac{du}{\cancel{1+e^x}}$

$\int u^{-1/2} du$

$2u^{1/2} + C$

$2\sqrt{x+e^x} + C$

23. $\int \frac{e^x}{2+e^x} dx$

$u = 2 + e^x$

$du = e^x dx$

$dx = \frac{du}{e^x}$

$\int \frac{\cancel{e^x}}{u} \cdot \frac{du}{\cancel{e^x}}$

$\int \frac{1}{u} du$

$\ln|u| + C = \ln|2+e^x| + C$

24. $\int \frac{x+1}{x^2+2x+19} dx$

$u = x^2 + 2x + 19$

$du = 2x + 2\, dx$

$dx = \frac{du}{2(x+1)}$

$\frac{1}{2}\int \frac{\cancel{x+1}}{u} \cdot \frac{du}{\cancel{2(x+1)}}$

$\frac{1}{2}\int \frac{1}{u} du$

$\frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+2x+19| + C$

25. $\int x^2(1+2x^3)^2 dx$

$u = 1 + 2x^3$

$du = 6x^2 dx$

$dx = \frac{du}{6x^2}$

$\int x^2 u^2 dx$

$\frac{1}{6}\int \cancel{x^2} u^2 \frac{du}{\cancel{6x^2}}$

$\frac{1}{6}\frac{u^3}{3} + C = \frac{(1+2x^3)^3}{18} + C$

26. $\int \frac{x \cdot \cos(x^2)}{\sqrt{\sin(x^2)}} dx$

$u = \sin(x^2)$

$du = \cos(x^2) \cdot 2x\, dx$

$dx = \frac{du}{2x\cos(x^2)}$

$\int \frac{\cancel{x \cdot \cos(x^2)}}{u^{1/2}} \cdot \frac{du}{\cancel{2x\cos(x^2)}}$

$\frac{1}{2}\int u^{-1/2} du$

$\frac{1}{\cancel{2}} \cdot \cancel{2}u^{1/2} = \sqrt{u} + C = \sqrt{\sin(x^2)} + C$

27. $\int \frac{t}{1+3t^2} dt$

$u = 1 + 3t^2$

$du = 6t\, dt$

$dt = \frac{du}{6t}$

$\frac{1}{6}\int \frac{\cancel{t}}{u} \frac{du}{\cancel{6t}}$

$\frac{1}{6}\int \frac{1}{u} du$

$\frac{1}{6}\ln|u| + C$

$\frac{1}{6}\ln|1+3t^2| + C$

28. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

$u = e^x + e^{-x}$

$du = e^x - e^{-x} dx$

$dx = \frac{du}{e^x - e^{-x}}$

$\int \frac{\cancel{e^x - e^{-x}}}{u} \cdot \frac{du}{\cancel{e^x - e^{-x}}}$

$\int \frac{1}{u} du$

$\ln|u| + C$

$\ln|e^x + e^{-x}| + C$

Integration by Substitution: Part II

Find the Antiderivative

1. $$\int_0^1 \frac{x}{1+x^2}\,dx$$

$u = 1+x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int_0^1 \frac{x}{u}\,\frac{du}{2x}$

$\frac{1}{2}\int_0^1 \frac{1}{u}\,du = \frac{1}{2}\ln|1+x^2|\Big|_0^1$

$\frac{1}{2}\ln|1+1| - \frac{1}{2}\ln|1+0| = \frac{1}{2}\ln 2$

$\underset{0}{\frac{1}{2}\ln 1}$

2. $$\int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$dx = \frac{du}{-\sin x}$

$\int_0^{\pi/4} \frac{\sin x}{u}\cdot\frac{du}{-\sin x}$

$-\ln|u|\Big|_1^{\sqrt{2}/2}$

$u_{\frac{\pi}{4}} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$u_0 = \cos 0 = 1$

$-\ln\frac{\sqrt{2}}{2} + \underset{0}{\ln 1} = -\ln\frac{\sqrt{2}}{2}$

3. $$\int_0^{\pi} \cos(x+\pi)\,dx$$

$u = x+\pi$

$du = 1\,dx$

$dx = 1\,du$

$\int_0^{\pi} \cos u\,du$

$\sin u\Big|_{\pi}^{2\pi}$

$u_{\pi} = 2\pi$

$u_0 = \pi$

$\sin 2\pi - \sin\pi$

$0 - 0 = 0$

4. $$\int_0^{\frac{1}{2}} \cos(\pi x)\,dx$$ Quick Integral

$\frac{\sin(\pi x)}{\pi}\Big|_0^{1/2}$

$\frac{\sin\frac{\pi}{2}}{\pi} - \frac{\sin 0}{\pi}$

$\frac{1}{\pi} - \frac{0}{\pi} = \frac{1}{\pi}$

5. $$\int_0^{\frac{\pi}{2}} e^{-\cos\theta}\sin\theta\,d\theta$$

$u = -\cos\theta$

$du = \sin\theta\,d\theta$

$d\theta = \frac{du}{\sin\theta}$

$\int_0^{\pi/2} e^u \cancel{\sin\theta}\,\frac{du}{\cancel{\sin\theta}}$

$e^u\Big|_{-1}^{0}$

$u_{\pi/2} = -\cos\frac{\pi}{2} = 0$

$u_0 = -\cos 0 = -1$

$e^0 - e^{-1} = 1 - \frac{1}{e}$

6. $$\int_1^2 2xe^{x^2}\,dx$$

$u = x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int_1^2 \cancel{2x}e^u\,\frac{du}{\cancel{2x}}$

$e^u\Big|_1^4$

$u_2 = 4 \quad u_1 = 1$

$e^4 - e^1 = e^4 - e$

7. $$\int_1^8 \frac{e^{\sqrt[3]{x}}}{\sqrt[3]{x^2}}\,dx$$

$u = \sqrt[3]{x} = x^{1/3}$

$du = \frac{1}{3}x^{-2/3}\,dx$

$du = \frac{1\,dx}{3x^{2/3}}$

$dx = 3\sqrt[3]{x^2}\,du$

$\int_1^8 \frac{e^u}{\sqrt[3]{x^2}}\,3\sqrt[3]{x^2}\,du$

$3e^u\Big|_1^2$

$u_8 = \sqrt[3]{8} = 2$

$u_1 = \sqrt[3]{1} = 1$

$3e^2 - 3e$

8. $$\int_{-1}^{e-2} \frac{1}{t+2}\,dt$$

$u = t+2$

$du = dt$

$\int_{-1}^{e-2} \frac{1}{u}\,du$

$\ln|u|\Big|_1^e$

$u_{e-2} = e-2+2 = e$

$u_{-1} = -1+2 = 1$

$\ln e - \ln 1$

$1 - 0 = 1$

9. $\displaystyle\int_1^4 \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

$u=\sqrt{x}$

$du=\frac{1}{2\sqrt{x}}dx$

$dx=2\sqrt{x}\,du$

$\displaystyle\int_1^4 \frac{\cos u}{\cancel{\sqrt{x}}}\,2\cancel{\sqrt{x}}\,du$

$u_4=\sqrt{4}=2$

$u_1=\sqrt{1}=1$

$2\sin u\Big|_1^2$

$2\sin 2-2\sin 1$

10. $\displaystyle\int_0^2 \frac{x}{(1+x^2)^2}\,dx$

$u=1+x^2$

$du=2x\,dx$

$dx=\frac{du}{2x}$

$\displaystyle\frac{1}{2}\int_0^2 \frac{\cancel{x}}{u^2}\,\frac{du}{\cancel{2x}}$

$\displaystyle\frac{1}{2}\int_0^2 u^{-2}\,du$

$u_2=1+2^2=5$

$u_0=1+0^2=1$

$\frac{1}{2}\cdot\frac{-1}{u}\Big|_1^5=\frac{-1}{2u}\Big|_1^5$

$-\frac{1}{10}-\frac{-1}{2}=\frac{4}{10}=\frac{2}{5}$

11. $\displaystyle\int_{-1}^3 (x^3+5x)\,dx$ Quick Integral

$\frac{x^4}{4}+\frac{5x^2}{2}\Big|_{-1}^3$

$\left(\frac{81}{4}+\frac{45}{2}\right)-\left(\frac{1}{4}+\frac{5}{2}\right)$

40

12. $\displaystyle\int_0^2 \frac{4y}{1+y^2}\,dy$

$u=1+y^2$

$du=2y\,dy$

$dy=\frac{du}{2y}$

$\displaystyle 2\int_0^2 \frac{4y}{u}\,\frac{du}{2y}$

$\displaystyle 2\int_0^2 \frac{1}{u}\,du$

$u_2=5 \quad u_0=1$

$2\ln|u|\Big|_1^5$

$2\ln 5-2\ln 1=2\ln 5$

13. $\displaystyle\int_1^3 \frac{1}{x}\,dx$ Quick Integral

$\ln|x|\Big|_1^3$

$\ln 3-\ln 1$

$\ln 3-0=\ln 3$

14. $\displaystyle\int_1^3 \frac{dt}{(t+7)^2}$ Quick Integral or u-sub

$\displaystyle\int_1^3 (t+7)^{-2}\,dt$

$\frac{(t+7)^{-1}}{-1}\Big|_1^3=\frac{-1}{t+7}\Big|_1^3$

$\left(\frac{-1}{3+7}\right)-\left(\frac{-1}{1+7}\right)=\frac{-1}{10}+\frac{1}{8}=\frac{1}{40}$

15. $\displaystyle\int_{-1}^2 \sqrt{x+2}\,dx$ Quick Integral or usub

$\displaystyle\int_{-1}^2 (x+2)^{1/2}\,dx$

$\frac{2(x+2)^{3/2}}{3}\Big|_{-1}^2$

$\left(\frac{2}{3}\sqrt{2+2}^{\,3}\right)-\left(\frac{2}{3}\sqrt{-1+2}^{\,3}\right)$

$\frac{16}{3}-\frac{2}{3}=\frac{14}{3}$

16. $\displaystyle\int_{-2}^0 \frac{2x+4}{x^2+4x+5}\,dx$

$u=x^2+4x+5$

$du=2x+4\,dx$

$dx=\frac{du}{2x+4}$

$\displaystyle\int_{-2}^0 \frac{\cancel{2x+4}}{u}\,\frac{du}{\cancel{2x+4}}$

$\displaystyle\int_{-2}^0 \frac{1}{u}\,du$

$u_0=5 \quad u_{-2}=1$

$\ln|u|\Big|_1^5$

$\ln 5-\ln 1=\ln 5$

17. $\int_0^1 x(1+x^2)^{20}\,dx$

$u=1+x^2$

$du=2x\,dx$

$dx=\frac{du}{2x}$

$\int_0^1 \cancel{x}\, u^{20}\frac{du}{2\cancel{x}}$

$\frac{1}{2}\int_0^1 u^{20}\,du$

$u_1=2$

$u_0=1$

$\frac{1}{2}\cdot\frac{u^{21}}{21}\Big|_1^2=\frac{u^{21}}{42}\Big|_1^2$

$\frac{2^{21}}{42}-\frac{1^{21}}{42}=\frac{2097{,}151}{42}$

18. $\int_0^{\pi}\sin x\,(\cos x+5)^7\,dx$

$u=\cos x+5$

$du=-\sin x\,dx$

$dx=\frac{du}{-\sin x}$

$\int_0^{\pi}\cancel{\sin x}\,u^7\frac{du}{-\cancel{\sin x}}$

$-\frac{u^8}{8}\Big|_6^4$

$u_\pi=\cos\pi+5=4$

$u_0=\cos 0+5=6$

$-\frac{(4)^8}{8}-\left(-\frac{(6)^8}{8}\right)=201{,}760$

19. $\int_0^{\frac{\pi}{12}}\sin 3x\;dx$ Quick Integral

$\frac{-\cos(3x)}{3}\Big|_0^{\pi/12}$

$\left(\frac{-\cos \pi/4}{3}\right)-\left(\frac{-\cos 0}{3}\right)$

$-\frac{1}{3}\cdot\frac{\sqrt{2}}{2}-\frac{-1}{3}=-\frac{\sqrt{2}}{6}+\frac{1}{3}$

20. $\int_0^1\frac{x}{1+5x^2}$

$u=1+5x^2$

$du=10x\,dx$

$dx=\frac{du}{10x}$

$\int_0^1\frac{\cancel{x}}{u}\,\frac{du}{10\cancel{x}}$

$\frac{1}{10}\int_0^1\frac{1}{u}\,du$

$u_1=6\quad u_0=1$

$\frac{1}{10}\ln|u|\Big|_1^6$

$\frac{\ln 6}{10}-\frac{\ln 1}{10}=\frac{\ln 6}{10}$

21. $\int_1^2\frac{x^2+1}{x}\,dx$ Rewrite

$\int_1^2\frac{x^2}{x}+\frac{1}{x}\,dx$

$\int_1^2 x+\frac{1}{x}\,dx$

$\left(\frac{x^2}{2}+\ln|x|\right)\Big|_1^2$

$(2+\ln 2)-(\frac{1}{2}+\ln 1)$

$\frac{3}{2}+\ln 2$

22. $\int_0^1\frac{x+2}{x^2+4x+1}\,dx$

$u=x^2+4x+1$

$du=2x+4\,dx$

$dx=\frac{du}{2x+4}$

$\int_0^1\frac{\cancel{x+2}}{u}\,\frac{du}{2(\cancel{x+2})}$

$\frac{1}{2}\int_0^1\frac{1}{u}\,du$

$u_1=6\quad u_0=1$

$\frac{1}{2}\ln|u|\Big|_1^6$

$\frac{\ln 6}{2}-\frac{\ln 1}{2}=\frac{\ln 6}{2}$

23. $\int_0^1\frac{x+2}{(x+2)^2+1}\,dx$

$u=(x+2)^2+1$

$du=2(x+2)\,dx$

$dx=\frac{du}{2(x+2)}$

$\int_0^1\frac{\cancel{x+2}}{u}\,\frac{du}{2(\cancel{x+2})}$

$\frac{1}{2}\ln|u|\Big|_5^{10}$

$u_1=10$

$u_0=5$

$\frac{1}{2}\ln 10-\frac{1}{2}\ln 5$

24. $\int_{-1}^1 x\sqrt{x^2+4}\,dx$

$u=x^2+4$

$du=2x\,dx$

$dx=\frac{du}{2x}$

$\int_{-1}^1 \cancel{x}\,u^{1/2}\frac{du}{2\cancel{x}}$

$\frac{\cancel{1}}{\cancel{2}}\frac{\cancel{2}u^{3/2}}{3}\Big|_5^5$ same so 0!

$u_1=5$

$u_{-1}=5$

$\frac{(5)^{3/2}}{3}-\frac{(5)^{3/2}}{3}=0$

Integration by Parts: Part I – No table

L~~I~~ATE $\int u\,dv = uv - \int v\,du$

Find the Antiderivative

1. $\int te^{5t}\,dt$

$u = t \quad dv = e^{5t}$

$du = 1dt \quad v = \frac{e^{5t}}{5}$

$t\frac{e^{5t}}{5} - \int \frac{e^{5t}}{5}\cdot 1dt$

$t\frac{e^{5t}}{5} - \frac{1}{5}\frac{e^{5t}}{5} + C$

$\frac{te^{5t}}{5} - \frac{e^{5t}}{25} + C$

2. $\int x \ln 5x\,dx$

$u = \ln 5x \quad dv = x$

$du = \frac{1}{5x}\cdot 5 \quad v = \frac{x^2}{2}$

$du = \frac{1}{x}dx$

$\ln(5x)\frac{x^2}{2} - \int \frac{x^{\cancel{2}}}{2}\frac{1}{\cancel{x}}dx$

$\frac{x^2\ln(5x)}{2} - \frac{1}{2}\int x\,dx$

$\frac{x^2\ln(5x)}{2} - \frac{1}{2}\frac{x^2}{2} + C = \frac{x^2\ln(5x)}{2} - \frac{x^2}{4} + C$

3. $\int pe^{-0.1p}\,dp$

$u = p$

$du = 1dp$

$dv = e^{-0.1p}$

$v = \frac{e^{-0.1p}}{-0.1}$

$v = \frac{-10}{e^{0.1p}}$

$p\cdot\frac{-10}{e^{0.1p}} - \int \frac{-10}{e^{0.1p}}\cdot 1dp$

$\frac{-10p}{e^{0.1p}} + 10\int e^{-0.1p}dp$

$\frac{-10p}{e^{0.1p}} + 10\frac{e^{-0.1p}}{-0.1} + C$

$\frac{-10p}{e^{0.1p}} - \frac{100}{e^{0.1p}} + C$

4. $\int t\sin t\,dt$

$u = t \quad dv = \sin t$

$du = 1dt \quad v = -\cos t$

$t\cdot -\cos t - \int -\cos t\cdot 1dt$

$-t\cos t + \int \cos t\,dt$

$-t\cos t + \sin t + C$

5. $\int y\ln y\,dy$

$u = \ln y$

$du = \frac{1}{y}dy$

$dv = y$

$v = \frac{y^2}{2}$

$\ln y\cdot\frac{y^2}{2} - \int \frac{y^{\cancel{2}}}{2}\frac{1}{\cancel{y}}dy$

$\frac{y^2\ln y}{2} - \frac{1}{2}\int y\,dy$

$\frac{y^2\ln y}{2} - \frac{1}{2}\cdot\frac{y^2}{2} + C$

$\frac{y^2\ln y}{2} - \frac{y^2}{4} + C$

6. $\int x^3\ln x\,dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$dv = x^3$

$v = \frac{x^4}{4}$

$\ln x\cdot\frac{x^4}{4} - \int \frac{x^{\cancel{4}\,3}}{4}\cdot\frac{1}{\cancel{x}}dx$

$\frac{x^4\ln x}{4} - \frac{1}{4}\int x^3dx$

$\frac{x^4\ln x}{4} - \frac{1}{4}\cdot\frac{x^4}{4} + C =$

$\frac{x^4\ln x}{4} - \frac{x^4}{16} + C$

7. $\int (z+1)e^{2z}\,dz$

$u = z+1$

$du = 1dz$

$dv = e^{2z}$

$v = \frac{e^{2z}}{2}$

$(z+1)\frac{e^{2z}}{2} - \int \frac{e^{2z}}{2}\cdot 1dz$

$\frac{(z+1)e^{2z}}{2} - \frac{1}{2}\int e^{2z}dz$

$\frac{(z+1)e^{2z}}{2} - \frac{e^{2z}}{4} + C$

8. $\int \frac{z}{e^z}\,dz$

$u = z$

$du = 1dz$

$dv = \frac{1}{e^z}$

$dv = e^{-z}$

$v = \frac{e^{-z}}{-1}$

$v = -e^{-z}$

$z\cdot -e^{-z} - \int -e^{-z}\cdot 1dz$

$-e^{-z}z + \frac{e^{-z}}{-1} + C$

$\frac{-z}{e^z} - \frac{1}{e^z} + C$

or

$\frac{-z-1}{e^z} + C$

Integration By Parts - Table

1) $\int xe^{-x}dx$

u	dV
$+x$	e^{-x}
-1	$-e^{-x}$
$+0$	e^{-x}

$-xe^{-x} - 1e^{-x} + C$

$-\frac{x}{e^x} - \frac{1}{e^x} + C$

2) $\int xe^{3x}\,dx$

u	dV
$+x$	e^{3x}
-1	$\frac{e^{3x}}{3}$
$+0$	$\frac{e^{3x}}{9}$

$\frac{xe^{3x}}{3} - \frac{e^{3x}}{9} + C$

3) $\int x^2e^x\,dx$

u	dV
$+x^2$	e^x
$-2x$	e^x
$+2$	e^x
-0	e^x

$x^2e^x - 2xe^x + 2e^x + C$

4) $\int x^2e^{-2x}\,dx$

u	dV
$+x^2$	e^{-2x}
$-2x$	$\frac{e^{-2x}}{-2}$
$+2$	$\frac{e^{-2x}}{4}$
-0	$\frac{e^{-2x}}{-8}$

$-\frac{x^2}{2e^{2x}} - \frac{x}{2e^{2x}} - \frac{1}{4e^{2x}} + C$

5) $\int x\sin 2x\,dx$

u	dV
$+x$	$\sin 2x$
-1	$\frac{-\cos 2x}{2}$
$+0$	$\frac{-\sin 2x}{4}$

$\frac{-x\cos 2x}{2} + \frac{\sin 2x}{4} + C$

6) $\int x\cos 3x\,dx$

u	dV
$+x$	$\cos 3x$
-1	$\frac{\sin 3x}{3}$
$+0$	$\frac{\cos 3x}{9}$

$\frac{x\sin 3x}{3} - \frac{\cos 3x}{9} + C$

7) $\int x^2\cos x\,dx$

u	dV
$+x^2$	$\cos x$
$-2x$	$\sin x$
$+2$	$-\cos x$
-0	$-\sin x$

$x^2\sin x + 2x\cos x - 2\sin x + C$

8) $\int x^2\sin x\,dx$

u	dV
$+x^2$	$\sin x$
$-2x$	$-\cos x$
$+2$	$-\sin x$
-0	$\cos x$

$-x^2\cos x + 2x\sin x + 2\cos x + C$

Differential Equations

Find the general solution of each differential equation.

1. $\frac{dy}{dx} = 3\cos x$

$\int dy = \int 3\cos x \, dx$

$y = 3\sin x + C$

2. $\frac{dy}{dx} = 2x + 3$

$\int dy = \int 2x + 3 \, dx$

$y = x^2 + 3x + C$

3. $\frac{dy}{dx} = \frac{5}{x}$

$\int dy = \int \frac{5}{x} \, dx$

$y = 5\ln|x| + C$

4. $\frac{dy}{dx} = \frac{e^x}{y^2}$

$\int y^2 \, dy = \int e^x \, dx$

$\frac{y^3}{3} = e^x + C$

$y^3 = 3e^x + C$

$y = \sqrt[3]{3e^x + C}$

5. $\frac{dy}{dx} = \frac{x}{e^y}$

$\int e^y \, dy = \int x \, dx$

$e^y = \frac{x^2}{2} + C$

$y = \log_e \frac{x^2}{2} + C$

$y = \ln \frac{x^2}{2} + C$

6. $\frac{dy}{dx} = y\cos x$

$\int \frac{1}{y} dy = \int \cos x \, dx$

$\ln|y| = \sin x + C$

$\log_e y = \sin x + C \xrightarrow{\text{Prop of Exp.}} e^{\sin x} \cdot e^{C}$

Just a constant

So $C \cdot e^{\sin x}$

$e^{\sin x + C} = y$

$y = Ce^{\sin x}$

7. $\frac{dy}{dx} = \frac{2x}{y^2}$

$\int y^2 \, dy = \int 2x \, dx$

$\frac{y^3}{3} = x^2 + C$

$y^3 = 3x^2 + C$

$y = \sqrt[3]{3x^2 + C}$

8. $\frac{dy}{dx} = x^2(y + 1)$

$\int \frac{1}{y+1} dy = \int x^2 \, dx$

$\ln|y+1| = \frac{x^3}{3} + C$

$e^{x^3/3 + C} = y + 1$

$Ce^{x^3/3} = y + 1$

$y = Ce^{x^3/3} - 1$

For each problem, find the particular solution of the differential equation that satisfies the initial condition.

9. $\frac{dy}{dx} = 2x + 3,\ y(\overset{x}{-1}) = \overset{y}{0}$

$\int dy = \int 2x+3\,dx$

$y = x^2 + 3x + C$

$0 = (-1)^2 + 3(-1) + C$

$0 = -2 + C$

$C = 2$

$y = x^2 + 3x + 2$

10. $\frac{dy}{dx} = 2\sin x,\ y\left(\overset{x}{\frac{\pi}{4}}\right) = \overset{y}{-\sqrt{2}}$

$\int dy = \int 2\sin x\,dx$

$y = -2\cos x + C$

$-\sqrt{2} = -2\cos\frac{\pi}{4} + C$

$-\sqrt{2} = -2 \cdot \frac{\sqrt{2}}{2} + C$

$-\sqrt{2} = -\sqrt{2} + C$

$C = 0$

$y = -2\cos x$

11. $\frac{dy}{dx} = 4x + 1,\ y(\overset{x}{1}) = \overset{y}{2}$

$\int dy = \int 4x+1\,dx$

$y = 2x^2 + x + C$

$2 = 2(1)^2 + 1 + C$

$2 = 3 + C$

$C = -1$

$y = 2x^2 + x - 1$

12. $\frac{dy}{dx} = 3\cos x,\ y\left(\overset{x}{\frac{\pi}{2}}\right) = \overset{y}{0}$

$\int dy = \int 3\cos x\,dx$

$y = 3\sin x + C$

$0 = 3\sin\left(\frac{\pi}{2}\right) + C$

$0 = 3 + C$

$C = -3$

$y = 3\sin x - 3$

13. $\frac{dy}{dx} = 2xy^2,\ y(\overset{x}{3}) = \overset{y}{-\frac{1}{12}}$

$\int \frac{1}{y^2} = \int 2x\,dx$

$-y^{-1} = \frac{\cancel{2}x^2}{\cancel{2}} + C$

$\frac{-1}{y} = x^2 + C$

$\frac{-1}{\frac{-1}{12}} = (3)^2 + C$

$12 = 9 + C$

$C = 3$

$\frac{-1}{y} = x^2 + 3$

$y = \frac{-1}{x^2+3}$

14. $\frac{dy}{dx} = \frac{2x^3}{y^2},\ y(\overset{x}{-2}) = \overset{y}{3}$

$\int y^2\,dy = \int 2x^3\,dx$

$\frac{y^3}{3} = \frac{\cancel{2}x^4}{\cancel{4}2} + C$

$y^3 = \frac{3x^4}{2} + C$

$(3)^3 = \frac{3(-2)^4}{2} + C$

$27 = 24 + C$

$C = 3$

$y^3 = \frac{3x^4}{2} + 3$

$y = \sqrt[3]{\frac{3x^4}{2} + 3}$

Unit 6 Integration Rules Unit Review

1. $\int \frac{x^2}{\sqrt{x^3+3}}\,dx$

$u = x^3+3$

$du = 3x^2\,dx$

$dx = \frac{du}{3x^2}$

$\int \frac{\cancel{x^2}}{u^{1/2}} \frac{du}{3\cancel{x^2}}$

$\frac{1}{3}\frac{u^{-1/2+1}}{1/2}\cdot\frac{2}{1} = \frac{2u^{1/2}}{3}+C = \frac{2\sqrt{x^3+3}}{3}+C$

2. $\int \frac{2x}{\sqrt{x^2+9}}\,dx$

$u = x^2+9$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int \frac{\cancel{2x}}{u^{1/2}}\cdot\frac{du}{\cancel{2x}}$

$2u^{1/2}+C$

$2\sqrt{x^2+9}+C$

3. $\int x(1-3x^2)^4\,dx$

$u = 1-3x^2$

$du = -6x\,dx$

$dx = \frac{du}{-6x}$

$\int \cancel{x}\cdot u^4 \frac{du}{-6\cancel{x}}$

$-\frac{1}{6}\frac{u^5}{5}+C$

$-\frac{(1-3x^2)^5}{30}+C$

4. $\int \frac{\cos x}{\sqrt{\sin(x)}}\,dx$

$u = \sin x$

$du = \cos x\,dx$

$dx = \frac{du}{\cos x}$

$\int \frac{\cancel{\cos x}}{u^{1/2}}\frac{du}{\cancel{\cos x}}$

$2u^{1/2}+C$

$2\sqrt{\sin x}+C$

5. $\int \frac{x+3}{x^2+6x-5}\,dx$

$u = x^2+6x-5$

$du = 2x+6\,dx$

$dx = \frac{du}{2x+6}$

$\int \frac{\cancel{x+3}}{u}\frac{du}{\cancel{2x+6}}$ $2(\cancel{x+3})$

$\frac{1}{2}\ln|u|+C$

$\frac{1}{2}\ln|x^2+6x-5|+C$

6. $\int (\overset{A}{x^2-1})\overset{E}{e^x}\,dx$

u	dV
$+x^2-1$	e^x
$-2x$	e^x
$+2$	e^x
-0	e^x

$e^x(x^2-1)-2xe^x+2e^x+C$

7. $\int x^2\sqrt{x^3+3}\,dx$

$u = x^3+3$

$du = 3x^2dx$

$dx = \frac{du}{3x^2}$

$\int \cancel{x^2}u^{1/2}\frac{du}{3\cancel{x^2}}$

$\frac{1}{3}u^{3/2}\cdot\frac{2}{3}+C$

$\frac{2(x^3+3)^{3/2}}{9}+C$

8. $\int \overset{A}{x^2}\,\overset{T}{\sin 2x}\,dx$

u	dV
$+x^2$	$\sin 2x$
$-2x$	$-\frac{\cos 2x}{2}$
$+2$	$-\frac{\sin 2x}{4}$
-0	$\frac{\cos 2x}{8}$

$-\frac{x^2\cos 2x}{2}+\frac{x\sin 2x}{2}+\frac{\cos 2x}{4}+C$

9. $\int \sin^3 x\cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$dx = \frac{du}{\cos x}$

$\int u^3\cancel{\cos x}\frac{du}{\cancel{\cos x}}$

$\frac{u^4}{4}+C$

$\frac{\sin^4 x}{4}+C$

10. $\int \overset{A}{x}\overset{E}{e^{-2x}}\,dx$

u	dV
$+x$	e^{-2x}
-1	$\frac{e^{-2x}}{-2}$
$+0$	$\frac{e^{-2x}}{4}$

$-\frac{x}{2e^{2x}}-\frac{1}{4e^{2x}}+C$

11. $\int x\sin(3x^2)dx$

$u = 3x^2$

$du = 6x\,dx$

$dx = \frac{du}{6x}$

$\int \cancel{x} \sin u \frac{du}{6\cancel{x}}$

$-\frac{1}{6}\cos u + C =$

$-\frac{1}{6}\cos(3x^2) + C$

12. $\int x^3 e^x dx$

A E

u	dv
$+x^3$	e^x
$-3x^2$	e^x
$+6x$	e^x
-6	e^x
$+0$	e^x

$x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

13. $\int \frac{\sin x}{\sqrt{1-\cos x}} dx$

$u = 1-\cos x$

$du = \sin x\,dx$

$dx = \frac{du}{\sin x}$

$\int \frac{\cancel{\sin x}}{u^{1/2}} \cdot \frac{du}{\cancel{\sin x}}$

$\int u^{-1/2} du$

$2u^{1/2} + C = 2\sqrt{1-\cos x} + C$

14. $\int x^2 e^{x^3} dx$

A E

$u = x^3$

$du = 3x^2 dx$

$dx = \frac{du}{3x^2}$

$\int \cancel{x^2} e^u \frac{du}{3\cancel{x^2}}$

$\frac{1}{3}e^u + C = \frac{1}{3}e^{x^3} + C$

15. $\int \sec 2x \tan 2x\ dx$

$\frac{\sec(2x)}{2} + C$

16. $\int (x^2 - 1)e^x\,dx$

A E

u	dv
$+x^2$	e^x
$-2x$	e^x
$+2$	e^x
-0	e^x

$x^2e^x - 2xe^x + 2e^x + C$

17. $\int (1+\sec \pi x)^2 \sec \pi x\ \tan \pi x$

$u = 1 + \sec \pi x$

$du = \sec \pi x \tan \pi x\,dx$

$dx = \frac{du}{\sec \pi x \tan \pi x}$

$\int u^2 \cancel{\sec \pi x \tan \pi x} \cdot \frac{du}{\cancel{\sec \pi x \tan \pi x}}$

$\frac{u^3}{3} + C =$

$\frac{(1+\sec \pi x)^3}{3} + C$

18. $\int x \cos x\,dx$

A T

u	dv
$+x$	$\cos x$
-1	$\sin x$
$+0$	$-\cos x$

$x\sin x + \cos x + C$

19. $\int \cot^4 x \csc^2 x\ dx$

~~$u = \csc^2 x$~~

$u = \cot x$

$du = \csc^2 x\,dx$

$dx = \frac{du}{\csc^2 x}$

$\int u^4 \cancel{\csc^2 x} \frac{du}{\cancel{\csc^2 x}}$

$\frac{u^5}{5} + C = \cot^5 x + C$

20. $\int_3^6 \frac{x}{3\sqrt{x^2-8}} dx$

$u = x^2 - 8$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int_3^6 \frac{\cancel{x}}{3u^{1/2}} \cdot \frac{du}{2\cancel{x}}$

$\frac{1}{6} u^{1/2} \cdot 2 = \frac{\sqrt{u}}{3}\Big|_1^{28}$

$u_6 = 28$

$u_3 = 1$

$\frac{\sqrt{28}}{3} - \frac{1}{3} = \frac{2\sqrt{7}-1}{3}$

21. $\int_0^3 \frac{1}{\sqrt{1+x}} dx$

$u = 1 + x$

$du = dx$

$\int_0^3 \frac{1}{u^{1/2}} dx$

$u_3 = 4$

$u_0 = 1$

$2u^{1/2}\Big|_1^4$

$2\sqrt{4} - 2\sqrt{1} = 2$

$4 - 2$

22. $\int x\,e^{x^2} dx$

$u = x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int \cancel{x} e^u \frac{du}{2\cancel{x}}$

$\frac{1}{2}e^u + C$

u

$\frac{1}{2}e^{x^2} + C$

23. $\int_0^1 x^2(x^3+1)^3\,dx$

$u = x^3+1$

$du = 3x^2\,dx$

$dx = \frac{du}{3x^2}$

$\int_0^1 x^2 u^3 \frac{du}{3x^2}$

$\frac{1}{3}\int_0^1 u^3\,du$

$u_0 = 1$

$u_1 = 2$

$\frac{1}{3}\cdot\frac{u^4}{4}\Big|_1^2$

$\frac{2^4}{12} - \frac{1^4}{12} = \frac{15}{12} = \frac{5}{4}$

24. $\int_0^\pi x \sin 2x\,dx$ A T

u	dV
$+x$	$\sin 2x$
-1	$-\frac{\cos 2x}{2}$
$+0$	$-\frac{\sin 2x}{4}$

$-\frac{x\cos 2x}{2} + \frac{\sin 2x}{4}\Big|_0^\pi = \left(-\frac{\pi\cos 2\pi}{2} + \frac{\sin 2\pi}{4}\right) - \left(\frac{0\cos 0}{2} + \frac{\sin 0}{4}\right) =$

$-\frac{\pi}{2} + 0 - 0 - 0 = \frac{-\pi}{2}$

25. $\int_0^\pi \cos\left(\frac{x}{2}\right)\,dx$

$2\sin\left(\frac{x}{2}\right)\Big|_0^\pi$

$2\sin\frac{\pi}{2} - 2\sin\frac{0}{2}$

$2(1) - 2(0) = 2$

26. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin(2x)\,dx$

$-\frac{\cos(2x)}{2}\Big|_{-\pi/4}^{\pi/4}$

$\frac{-\cos(\pi/2)}{2} - \frac{-\cos(-\pi/2)}{2}$

$\frac{0}{2} + \frac{0}{2} = 0$

27. $\int_0^1 xe^{-x^2}\,dx$

$u = -x^2$

$du = -2x\,dx$

$dx = \frac{du}{-2x}$

$\int_0^1 xe^u \frac{du}{-2x}$

$-\frac{1}{2}\int_0^1 e^u\,du$

$u_1 = -1$

$u_0 = 0$

$-\frac{1}{2}e^u\Big|_0^{-1}$

$-\frac{1}{2}e^{-1} - -\frac{1}{2}e^0 = \frac{-1}{2e} + \frac{1}{2}$

28. $\int x\cos x\,dx$ A T

u	dV
$+x$	$\cos x$
-1	$\sin x$
$+0$	$-\cos x$

$x\sin x + \cos x + C$

29. $\int_0^{\frac{\pi}{4}} \cos(2x)\,dx$

$\frac{\sin(2x)}{2}\Big|_0^{\pi/4}$

$\frac{\sin\pi/2}{2} - \frac{\sin 0}{2}$

$\frac{1}{2} - \frac{0}{2} = \frac{1}{2}$

30. $\int_0^1 x^2 e^x\,dx$ A E

u	dV
$+x^2$	e^x
$-2x$	e^x
$+2$	e^x
-0	e^x

$x^2e^x - 2xe^x + 2e^x\Big|_0^1$

$(1^2e^1 - 2(1)e^1 + 2e^1) - (0^2e^0 - 2(0)e^0 + 2e^0)$

$(e-2+2) - (0-0+2) = e-2$

14

31. $\int_0^{\pi} \sin^2 x \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$dx = \frac{du}{\cos x}$

$\int_0^{\pi} u^2 \cancel{\cos x} \frac{du}{\cancel{\cos x}}$

$u_{\pi} = \sin \pi = 0$

$u_0 = \sin 0 = 0$

$\frac{u^3}{3}\Big|_0^0 = 0$

32. $\int x^4 e^{-x} \, dx$

A E

u	dv
$+x^4$	e^{-x}
$-4x^3$	$-e^{-x}$
$+12x^2$	e^{-x}
$-24x$	$-e^{-x}$
$+24$	e^{-x}
-0	$-e^{-x}$

$\frac{-x^4}{e^x} - \frac{4x^3}{e^x} - \frac{12x^2}{e^x} - \frac{24x}{e^x} - \frac{24}{e^x} + C$

or

$-\frac{x^4 + 4x^3 + 12x^2 + 24x + 24}{e^x} + C$

Solve the differential equation:

33. $\frac{dy}{dx} = \frac{x^2+3}{x}$

$\int dy = \int x + \frac{3}{x} \, dx$

$y = \frac{x^2}{2} + 3\ln|x| + C$

34. $\frac{dy}{dx} = xy^2$

y^{-2}

$\int \frac{1}{y^2} dy = \int x \, dx$

$\frac{y^{-1}}{-1} = \frac{x^2}{2} + C$

$y \cdot -\frac{1}{y} = \frac{y \cdot x^2}{2} + C$

$\frac{2}{x^2} \cdot -1 = \frac{2y}{x^2} \frac{x^2}{2} + 2$

$y = \frac{-2}{x^2} + C$

35. $\frac{dy}{dx} - e^y \sin x = 0$

$\frac{dy}{dx} = e^y \sin x$

$\frac{1}{e^y} dy = \sin x \, dx$

$\int e^{-y} dy = \int \sin x \, dx$

$+e^{-y} = +\cos x + C$

$-1 \cdot \log_e(\cos x + c) = y \cdot -1$

$y = -\ln|\cos x + c|$

36. $xy^2 \frac{dy}{dx} = x + 1$

$y^2 \frac{dy}{dx} = \frac{x}{x} + \frac{1}{x}$

$\int y^2 dy = \int 1 + \frac{1}{x} \, dx$

$\frac{y^3}{3} = x + \ln|x| + C$

$y^3 = 3x + 3\ln|x| + C$

$y = \sqrt[3]{3x + 3\ln|x| + C}$

Find the particular solution of the differential equation that satisfies the initial condition:

37. $\frac{dy}{dx} = 2xy^2$, $y(-1) = -\frac{1}{4}$

y^{-2}

$\int \frac{1}{y^2} dy = \int 2x \, dx$

$\frac{y^{-1}}{-1} = x^2 + C$

$-\frac{1}{y} = x^2 + C \rightarrow y = \frac{-1}{x^2} + C$

$\frac{-1}{\left(\frac{-1}{4}\right)} = (-1)^2 + C$

$c = 3$

$y = \frac{-1}{x^2} + 3$

38. $\frac{dy}{dx} = \frac{x}{y}$, $y(0) = -3$

$\int y \, dy = \int x \, dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C \rightarrow \frac{y^2}{2} = \frac{x^2}{2} + \frac{9}{2}$

$\frac{(-3)^2}{2} = \frac{(0)^2}{2} + C$

$c = \frac{9}{2}$

$y^2 = x^2 + 9$

$y = \pm\sqrt{x^2 + 9}$