

Unit 6

Integration Rules

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- ❖ Quick Indefinite Integrals
- ❖ U Substitution of Indefinite Integrals
- ❖ U Substitution of Definite Integrals
- ❖ Integration by Parts
- ❖ Differential Equations

Quiz is _____

Test is _____

Name: _____

Quick Integrals Practice

1. $\int \sec 5x \tan 5x \, dx$

2. $\int \sec^2 2x \, dx$

3. $\int \sin 15x \, dx$

4. $\int \cos 7x \, dx$

5. $\int e^{4x} \, dx$

6. $\int e^{-x} \, dx$

7. $\int \csc^2 11x \, dx$

8. $\int 3^{8x} \, dx$

9. $\int e^{12x} \, dx$

10. $\int (3x + 1)^4 \, dx$

11. $\int (5x + 11)^6 \, dx$

12. $\int (3 - 7x)^{12} \, dx$

13. $\int \csc 2x \cot 2x \, dx$

14. $\int \sin ex \, dx$

15. $\int \cos \pi x \, dx$

16. $\int \csc(2 - 11x) \cot(2 - 11x) \, dx$

17. $\int 5^{12x+3} \, dx$

18. $\int e^{1-2x} \, dx$

19. $\int \sqrt{4x - 1} \, dx$

20. $\int \frac{3}{x} \, dx$

21. $\int -6x^{-1} \, dx$

22. $\int \sin(5 + 17x) \, dx$

23. $\int 3^{2-8x} \, dx$

24. $\int e^{5x+12} \, dx$

25. $\int \frac{1}{5x} \, dx$

26. $\int (x - 7)^{80} \, dx$

27. $\int e^{5-x} \, dx$

Integration by Substitution: Part I

Find the Antiderivative

1. $\int y(y^2 + 5)^8 dy$

2. $\int t^2(t^3 - 3)^{10} dt$

3. $\int x(x^2 - 4)^{\frac{7}{2}} dx$

4. $\int \frac{1}{\sqrt{4-x}} dx$

5. $\int x(x^2 + 3)^2 dx$

6. $\int \frac{dy}{y+5}$

7. $\int (2t - 7)^{73} dt$

8. $\int 16x(2x^2 + 1)^2 dx$

9. $\int \sin \theta (\cos \theta + 5)^7 d\theta$

10. $\int \sqrt{\cos(3t)} \sin 3t dt$

11. $\int x e^{-x^2} dx$

12. $\int \sin^6 x \cos x dx$

13. $\int \sin^6(5x) \cos(5x) dx$

14. $\int x^2 e^{x^3+1} dx$

15. $\int \sin^3 x \cos x dx$

16. $\int \frac{(\ln x)^2}{x} dx$

17. $\int \frac{e^t + 1}{e^t + t} dt$

18. $\int \frac{y}{y^2 + 4} dy$

$$19. \int \tan 2x \, dx$$

$$20. \int \frac{\cos \sqrt{x}}{\sqrt{x}} \, dx$$

$$21. \int \frac{e^{\sqrt{y}}}{\sqrt{y}} \, dy$$

$$22. \int \frac{1 + e^x}{\sqrt{x + e^x}} \, dx$$

$$23. \int \frac{e^x}{2 + e^x} \, dx$$

$$24. \int \frac{x + 1}{x^2 + 2x + 19} \, dx$$

$$25. \int x^2(1 + 2x^3)^2 \, dx$$

$$26. \int \frac{x \cdot \cos(x^2)}{\sqrt{\sin(x^2)}} \, dx$$

$$27. \int \frac{t}{1 + 3t^2} \, dt$$

$$28. \int \frac{e^x - e^{-x}}{e^x + e^{-x}} \, dx$$

Integration by Substitution: Part II

Find the Antiderivative

1. $\int_0^1 \frac{x}{1+x^2} dx$

2. $\int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx$

3. $\int_0^{\pi} \cos(x + \pi) dx$

4. $\int_0^{\frac{1}{2}} \cos(\pi x) dx$

5. $\int_0^{\frac{\pi}{2}} e^{-\cos \theta} \sin \theta d\theta$

6. $\int_1^2 2xe^{x^2} dx$

7. $\int_1^8 \frac{e^{\sqrt[3]{x}}}{\sqrt[3]{x^2}} dx$

8. $\int_{-1}^{e-2} \frac{1}{t+2} dt$

9. $\int_1^4 \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

10. $\int_0^2 \frac{x}{(1+x^2)^2} dx$

11. $\int_{-1}^3 (x^3 + 5x) dx$

12. $\int_0^2 \frac{4y}{1+y^2} dy$

13. $\int_1^3 \frac{1}{x} dx$

14. $\int_1^3 \frac{dt}{(t+7)^2}$

15. $\int_{-1}^2 \sqrt{x+2} dx$

16. $\int_{-2}^0 \frac{2x+4}{x^2+4x+5} dx$

$$17. \int_0^1 x(1+x^2)^{20} dx$$

$$18. \int_0^\pi \sin x (\cos x + 5)^7 dx$$

$$19. \int_0^{\frac{\pi}{12}} \sin 3x dx$$

$$20. \int_0^1 \frac{x}{1+5x^2} dx$$

$$21. \int_1^2 \frac{x^2+1}{x} dx$$

$$22. \int_0^1 \frac{x+2}{x^2+4x+1} dx$$

$$23. \int_0^1 \frac{x+2}{(x+2)^2+1} dx$$

$$24. \int_{-1}^1 x\sqrt{x^2+4} dx$$

Integration by Parts: Part I – No table

$$\int u dv = uv - \int v du$$

Find the Antiderivative

1. $\int t e^{5t} dt$

2. $\int x \ln 5x dx$

3. $\int p e^{-0.1p} dp$

4. $\int t \sin t dt$

5. $\int y \ln y dy$

6. $\int x^3 \ln x dx$

7. $\int (z + 1) e^{2z} dz$

8. $\int \frac{z}{e^z} dz$

Integration By Parts - Table

$$1) \int x e^{-x} dx$$

$$2) \int x e^{3x} dx$$

$$3) \int x^2 e^x dx$$

$$4) \int x^2 e^{-2x} dx$$

$$5) \int x \sin 2x dx$$

$$6) \int x \cos 3x dx$$

$$7) \int x^2 \cos x dx$$

$$8) \int x^2 \sin x dx$$

Differential Equations

Find the general solution of each differential equation.

1. $\frac{dy}{dx} = 3 \cos x$

2. $\frac{dy}{dx} = 2x + 3$

3. $\frac{dy}{dx} = \frac{5}{x}$

4. $\frac{dy}{dx} = \frac{e^x}{y^2}$

5. $\frac{dy}{dx} = \frac{x}{e^y}$

6. $\frac{dy}{dx} = y \cos x$

7. $\frac{dy}{dx} = \frac{2x}{y^2}$

8. $\frac{dy}{dx} = x^2(y + 1)$

For each problem, find the particular solution of the differential equation that satisfies the initial condition.

9. $\frac{dy}{dx} = 2x + 3, y(-1) = 0$

10. $\frac{dy}{dx} = 2 \sin x, y\left(\frac{\pi}{4}\right) = -\sqrt{2}$

11. $\frac{dy}{dx} = 4x + 1, y(1) = 2$

12. $\frac{dy}{dx} = 3 \cos x, y\left(\frac{\pi}{2}\right) = 0$

13. $\frac{dy}{dx} = 2xy^2, y(3) = -\frac{1}{12}$

14. $\frac{dy}{dx} = \frac{2x^3}{y^2}, y(-2) = 3$

Unit 6 Integration Rules Unit Review

1. $\int \frac{x^2}{\sqrt{x^3 + 3}} dx$

2. $\int \frac{2x}{\sqrt{x^2 + 9}} dx$

3. $\int x(1 - 3x^2)^4 dx$

4. $\int \frac{\cos x}{\sqrt{\sin(x)}} dx$

5. $\int \frac{x + 3}{x^2 + 6x - 5} dx$

6. $\int (x^2 - 1)e^x dx$

7. $\int x^2 \sqrt{x^3 + 3} dx$

8. $\int x^2 \sin 2x dx$

9. $\int \sin^3 x \cos x dx$

10. $\int xe^{-2x} dx$

11. $\int x \sin(3x^2) dx$

12. $\int x^3 e^x dx$

13. $\int \frac{\sin x}{\sqrt{1 - \cos x}} dx$

14. $\int x^2 e^{x^3} dx$

15. $\int \sec 2x \tan 2x dx$

16. $\int (x^2 - 1)e^x dx$

17. $\int (1 + \sec \pi x)^2 \sec \pi x \tan \pi x dx$

18. $\int x \cos x dx$

19. $\int \cot^4 x \csc^2 x dx$

20. $\int_3^6 \frac{x}{3\sqrt{x^2 - 8}} dx$

21. $\int_0^3 \frac{1}{\sqrt{1+x}} dx$

22. $\int x e^{x^2} dx$

$$23. \int_0^1 x^2(x^3 + 1)^3 dx$$

$$24. \int_0^\pi x \sin 2x dx$$

$$25. \int_0^\pi \cos\left(\frac{x}{2}\right) dx$$

$$26. \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin(2x) dx$$

$$27. \int_0^1 x e^{-x^2} dx$$

$$28. \int x \cos x dx$$

$$29. \int_0^{\frac{\pi}{4}} \cos(2x) dx$$

$$30. \int_0^1 x^2 e^x dx$$

$$31. \int_0^{\pi} \sin^2 x \cos x \, dx$$

$$32. \int x^4 e^{-x} \, dx$$

Solve the differential equation:

$$33. \frac{dy}{dx} = \frac{x^2+3}{x}$$

$$34. \frac{dy}{dx} = xy^2$$

$$35. \frac{dy}{dx} - e^y \sin x = 0$$

$$36. xy^2 \frac{dy}{dx} = x + 1$$

Find the particular solution of the differential equation that satisfies the initial condition:

$$37. \frac{dy}{dx} = 2xy^2, y(-1) = -\frac{1}{4}$$

$$38. \frac{dy}{dx} = \frac{x}{y}, y(0) = -3$$