Unit 5 Identities of Trig Functions

KEY STANDARDS -

- MGSE9-12.F.IF.7e Graph trigonometric functions, showing period, midline, and amplitude. Extend the domain of trigonometric functions using the unit circle
- MGSE9-12.F.TF.2 Model periodic phenomena with trigonometric functions
- MGSE9-12.F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- MGSE9-12.F.TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Date	Topic(s)	Assignment/HW	Credit
Fri. 10/15	Basic Trig identities		
Mon. 10/18	Verifying Trig Identities		
Tues. 10/19	**Identities Memorization Quiz** Verifying Trig Identities		
Wed. 10/20	Verifying Trig Identities Solve Trig Equations		
Thurs. 10/21	Solve Trig Equations Review		
Fri. 10/22	**Quiz**		
Mon. 10/25	Sum & Difference Identities		
Tues. 10/26	Double & Half Angle Identities		
Wed. 10/27	Unit 5 Review		
Thurs. 10/28	Unit 5 Review		
Fri. 10/29	**Unit 5 Trig Identities Test**		

Name _____

WS #1 Fundamental Trig Identities

Use identities (NOT RIGHT TRIANGLES) to solve the following. Show all steps.

1. If tan x = 2, find cot x.

2. If $\csc x = -\frac{5}{2}$, find sin x.

3. If $\sin x = \frac{2}{3}$, find cos x.

4. If $\cos x = \frac{1}{5}$, find $\csc x$.

5. If tan x = 3, find sec x.

6. If $\cot x = \frac{7}{2}$, find $\csc x$.

7. If $\cos x = \frac{1}{4}$, find tan x.

8. If $\sin x = \frac{7}{10}$, find cot x.

9. If $\tan x = \frac{7}{2}$, find sin x.

10. If cot x = -5 and $\sin x = -\frac{\sqrt{26}}{26}$, find the remaining trig function values.

Simplify.

11. $\cot\theta \bullet \sin\theta$

12. $\dfrac{\tan\theta}{\sec\theta}$

13. $\sec^2\theta \bullet \cot^2\theta$

14. $\cos\theta \bullet \csc\theta \bullet \sin\theta \bullet \sec\theta$

15. $1-\cos^2\theta$

16. $\csc^2\theta - 1$

17. $\dfrac{\sin^2\theta}{1-\cos^2\theta}$

18. $\dfrac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}$

19. $\dfrac{1+\tan^2\theta}{\tan^2\theta}$

20. $\dfrac{\sec^2\theta-1}{\sec^2\theta}$

21. $\cot\theta \bullet \tan\theta - \sec\theta \bullet \cos\theta$

WS #2 Verifying Trig Identities

Verify each identity. Work with only one side of the equation.

1. $\sin\theta \bullet \sec\theta = \tan\theta$

2. $\sin\theta \cot\theta = \cos\theta$

3. $\tan^2\theta - \sec^2\theta = -1$

4. $\cos\theta \csc\theta \tan\theta = 1$

5. $\sec^2\theta - \sec^2\theta \sin^2\theta = 1$

6. $\dfrac{\sin\theta}{1-\cos^2\theta} = \csc\theta$

7. $1+\sec^2\theta\ \sin^2\theta=\sec^2\theta$

8. $\cot\theta\,(\tan\theta\ +\cot\theta)=\csc^2\theta$

9. $\tan\theta\ \cot\theta+\sin^2\theta+\cos^2\theta=2$

10. $(1-\cos^2\theta)\ \csc\theta\ \cot\theta=\cos\theta$

11. $\dfrac{\sin\theta}{1-\sin^2\theta}=\sec\theta\ \tan\theta$

12. $\sec^2\theta\ \csc^2\theta=\csc^2\theta+\sec^2\theta$

13. $\dfrac{\cot\theta}{1+\cot^2\theta}=\cos\theta\ \sin\theta$

14. $\dfrac{\cot\theta}{\sec\theta}=\dfrac{1}{\sin\theta}-\sin\theta$

WS #3 More - Verifying Trig Identities

Verify each identity. Work with only one side of the equation.

1. $\sin^2 x \ \cot^2 x + \cos^2 x \ \tan^2 x = 1$

2. $\cos x \ \csc x = \cot x$

3. $\sin x(\sec x - \csc x) = \tan x - 1$ (distribute)

4. $2\cos^2 x - \sin^2 x + 1 = 3\cos^2 x$

5. $\dfrac{\sin^2 x}{1+\cos x} + \cos x = 1$

6. $\dfrac{1+\tan^2 x}{\tan^2 x} = \csc^2 x$

7. $\cos x(\csc x - \sec x) = \cot x - 1$ (distribute)

8. $\dfrac{1-\sin^2 x}{1-\cos^2 x} = \cot^2 x$

9. $\dfrac{\sin x \ \cot x + \cos x}{\sin x} = 2\cot x$

10. $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$ (factor)

11. $\cos^2 x = \csc x \sin x - \sin^2 x$ (factor)

12. $\tan^2 x = \sec^2 x - \cos x \sec x$

13. $\tan^2 x + \sin^2 x + \cos^2 x = \sec^2 x$

14. $\tan^2 A\ (1 + \cot^2 A) = \sec^2 A$

15. $(1 - \sin^2 x)(\sec^2 x - 1) = \sin^2 x$

16. $\sin^2 x\ + \sin^2 x\ \tan^2 x\ = \tan^2 x$

17. $\cos x = \dfrac{\cot x}{\csc x}$

18. $\cot x = \dfrac{\csc x}{\sec x}$

(work both sides #19 & #20)

19. $\dfrac{\sec x}{\tan x} = \dfrac{\cot x}{\cos x}$

20. $\dfrac{\tan x}{\sec x} = \dfrac{\cos x}{\cot x}$

21. $(1-\cos x)(1+\cos x)=\dfrac{1}{\csc^2 x}$

22. $(1-\sec^2 x)\cot x=-\tan x$

23. $\dfrac{2-\sec^2 x}{\sec^2 x}=2\cos^2 x-1$ (split left into 2 fractions)

24. $\dfrac{(1+\sin x)^2}{\cos^2 x}=\dfrac{1+\sin x}{1-\sin x}$

25. $\dfrac{\sin x\ \csc x}{\tan x}=\cot x$

WS #4 Solving Trig Equations

Solve for θ on the interval $0 \le \theta \le 2\pi$.

1. $2\sin\theta - 1 = 0$

2. $\cos\theta + 2 = 2$

3. $2\sin\theta + \sqrt{3} = 0$

4. $\csc\theta - 2 = 0$

5. $\sin^2\theta = \frac{3}{4}$

6. $5\sin\theta + 5 = 0$

7. $\cot\theta - \sqrt{3} = 0$

8. $\sqrt{3}\sec\theta + 2 = 0$

9. $\sin^2\theta = 1$

10. $4\cos^2\theta - 1 = 0$

11. $4\sin^2\theta = 1$

12. $2\cos^2\theta = \cos\theta$

13. $2\sin^2\theta = \sin\theta + 1$

14. $2\cos^2\theta + 7\cos\theta = 4$

15. $(4\cos^2\theta - 3)(\sin^2\theta + 2) = 0$

16. $2 - \sin\theta = 2\cos^2\theta$

17. $2\cos^2\theta + 3\cos\theta + 1 = 0$

18. $2\sin^2\theta + 5\sin\theta = 3$

19. $4\sin^2\theta + 4\sin\theta + 1 = 0$

20. $2\sin^2\theta - 3\cos\theta - 3 = 0$

WS #5 Quiz Review

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1. If $\cos\theta = -\frac{2}{5}$ and θ lies in quadrant II, find $\csc\theta$.

2. If $\tan\beta = 3$ and θ lies in quadrant I, find $\sec\beta$.

3. If $\sin\theta = \frac{2}{3}$ and θ lies in Quadrant II, find $\cos\theta$.

4. If $\cot\theta = -2$ and θ lies in Quadrant IV, find $\cos\theta$.

Simplify.

5. $\sin^2\theta + \cos^2\theta + \tan^2\theta$

6. $\sin\theta\csc\theta + \tan\theta\cot\theta$

7. $\tan^2\theta \bullet \cos^2\theta$

8. $\dfrac{\cos^2\theta + \sin^2\theta}{\sin^2\theta}$

9. $\csc\theta - \cos\theta\cot\theta$

10. $\cot\theta\tan\theta - \sec\theta\cos\theta$

11. $(1+\cos\theta)(\csc\theta - \cot\theta)$

12. $(1-\sin\theta)(\sec\theta + \tan\theta)$

Verify each identity.

13. $\sin\theta\cot\theta\sec\theta = 1$

14. $\dfrac{\sin\theta}{1-\cos^2\theta} = \csc\theta$

15. $\dfrac{1}{\sin^2\theta} - \dfrac{1}{\tan^2\theta} = 1$

16. $\cos^2\theta\tan^2\theta + \sin^2\theta\cot^2\theta = 1$

17. $\sin\theta(1+\cot^2\theta) = \csc\theta$

18. $\dfrac{\cot\theta}{1+\cot^2\theta} = \cos\theta\sin\theta$

19. $\tan\theta\sin\theta + \cos\theta = \sec\theta$

20. $\dfrac{\tan^2\theta}{\sec\theta+1} = \dfrac{1-\cos\theta}{\cos\theta}$

WS #6 Sum and Difference Identities

Use the sum and difference identities to find the exact value of each function.

1. $\cos 75^\circ$

2. $\cos 375^\circ$

3. $\sin(-165^\circ)$

4. $\sin(-105^\circ)$

5. $\sin 95^\circ \cos 55^\circ + \cos 95^\circ \sin 55^\circ$

6. $\cos 160^\circ \cos 40^\circ + \sin 160^\circ \sin 40^\circ$

7. $\tan(135^\circ + 120^\circ)$

8. $\tan 345^\circ$

If u and v are the measures of two first quadrant angles, find the exact value of each function.

9. If $\sin u = \frac{12}{13}$ *and* $\cos v = \frac{3}{5}$, find cos (u - v)

10. If $\cos u = \frac{12}{13}$ *and* $\cos v = \frac{12}{37}$, find tan (u - v)

11. If $\cos u = \frac{8}{17}$ *and* $\tan v = \frac{5}{12}$, find cos (u + v)

12. If $\csc u = \frac{13}{12}$ *and* $\sec v = \frac{5}{3}$, find sin (u - v)

WS #7 Double-Angle and Half-Angle Identities

If $\sin A = \frac{12}{13}$ and A is in the first quadrant, find each value.

1. cos 2A

2. sin 2A

3. tan 2A

4. $\cos\frac{A}{2}$

5. $\sin\frac{A}{2}$

6. $\tan\frac{A}{2}$

Given that $\cos B = \frac{7}{25}$ and B is in Quadrant IV, find the exact value for each.

7. sin 2B

8. $\tan\frac{B}{2}$

Use a half-angle identity to find each value.

9. tan 22.5^0

10. Cos 112.5^0

11. Sin 67.5^0

WS #8 Test Review

Find the following using trigonometric identities.

1. If $\sin\theta = \frac{2}{3}$, find cos θ.

2. If $\sin\theta = \frac{7}{10}$, find cot θ.

3. If $\tan\theta = \frac{12}{5}$, find $\sin\theta$.

4. If $\cot\theta = \frac{5}{9}$, find tan θ.

Verify the following identities. Remember to only work with ONE side.

5. $\tan\beta\csc\beta = \sec\beta$

6. $\frac{1}{\sec^2\beta} + \frac{1}{\csc^2\beta} = 1$

7. $\cos x(\csc x - \sec x) = \cot x - 1$

8. $\frac{\sin x \bullet \cot x + \cos x}{\sin x} = 2\cot x$

9. $(1-\cos x)(1+\cos x) = \frac{1}{\csc^2 x}$

10. $\sin x \tan x = \sec x - \cos x$

If a and b are measures of two first quadrant angles, find the exact value of each function.

11. If $\sin a = \frac{12}{13}$ and $\cos b = \frac{3}{5}$, find cos (a + b)

12. If $\cos a = \frac{12}{13}$ and $\cos b = \frac{12}{37}$, find tan (a - b)

13. If $\csc a = \frac{13}{12}$ and $\sec b = \frac{5}{3}$, find sin (a - b)

Find the exact value.

14. tan 75^0

15. sin (-15^0)

If $\sin A = -\frac{12}{13}$ and <A is an angle in quadrant IV, find the exact value.

16. cos 2A

17. tan 2A

18. $\sin\frac{A}{2}$

19. $\cos\frac{A}{2}$

Solve for values on θ such that $0 \le \theta < 2\pi$.

20. $2\sin^2\theta - 1 = 0$

21. $(2\sin\theta - 1)(2\cos\theta + \sqrt{3}) = 0$

22. $2\sin^2 x - 5\sin x = -2$

23. $\tan^2 x - \sqrt{3}\tan x = 0$

Write the expression as the sine, cosine or tangent of an angle.

24. $\cos 25^\circ \cos 15^\circ - \sin 25^\circ \sin 15^\circ$

25. $\dfrac{\tan 68^\circ - \tan 115^\circ}{1 + \tan 68^\circ \tan 115^\circ}$