Unit 5

Introduction to Integration

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- Riemann Sums & Trapezoid Rule
- Indefinite Integrals (Antiderivatives)
- Definite Integrals (Fundamental Theorem of Calculus Part 2)
- Fundamental Theorem of Calculus Part 1

Quiz is ________________

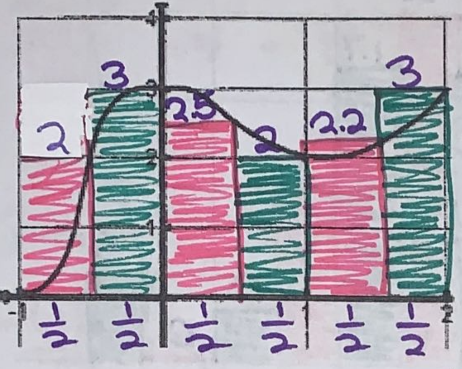
Test is ________________

Name: Bonanni

Riemann Sums Worksheet 1

Given the function estimate the area bounded by the curve and the x-axis using the specified method with 6 subintervals over the interval [-1,2]. Draw rectangles and use the graph to estimate y values.

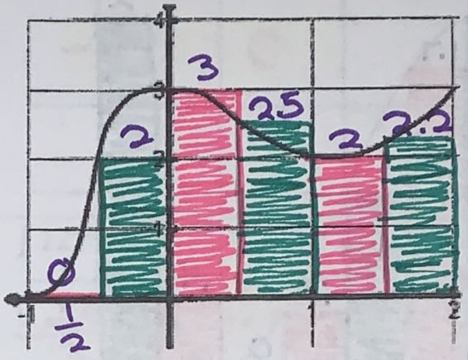
1. Right Riemann Sums



$A=\frac{1}{2}(2+3+2.5+2+2.2+3)$

$A\approx 7.35\,u^2$

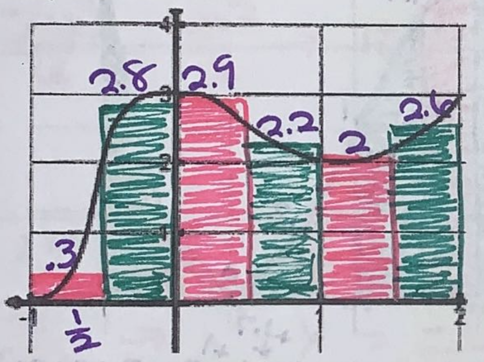
2. Left Riemann Sums



$A=\frac{1}{2}(0+2+3+2.5+2+2.2)$

$A\approx 5.85\,u^2$

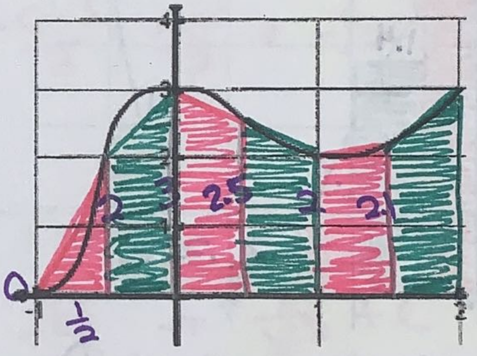
3. Midpoint Method



$A\approx\frac{1}{2}(.3+2.8+2.9+2.2+2+2.6)$

$A\approx 6.4\,u^2$

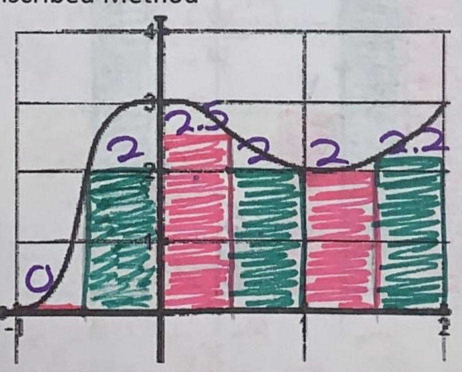
4. Trapezoid Method



$A=\frac{1}{2}\cdot\frac{1}{2}(0+2+2+3+3+2.5+2.5+2+2+2.1+2.1+3)$

$A\approx 6.55\,u^2$

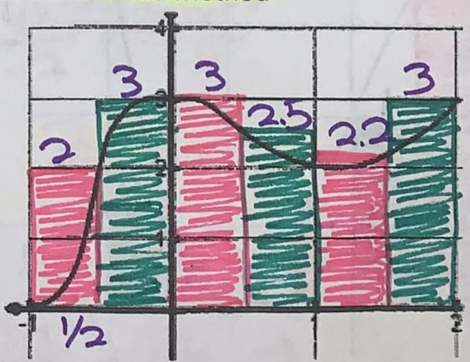
5. Inscribed Method



$A\approx\frac{1}{2}(0+2+2.5+2+2+2.2)$

$A\approx 5.35\,u^2$

6. Circumscribed Method



$A=\frac{1}{2}(2+3+3+2.5+2.2+3)$

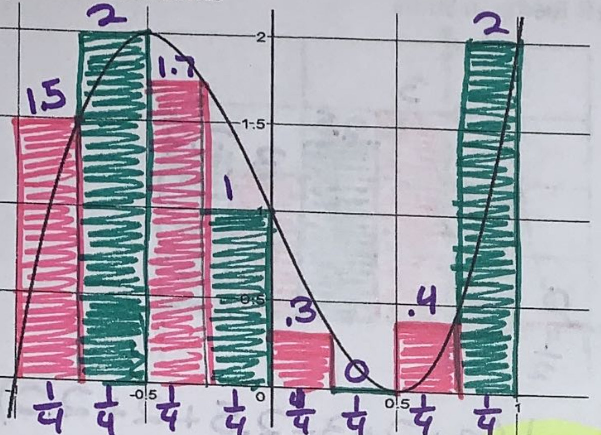
$A\approx 7.85\,u^2$

Riemann Sums Worksheet 2

Given the function estimate the area bounded by the curve and the x-axis using the specified method with 8 subintervals over the interval [-1,1]. Draw rectangles and use the graph to estimate y values.

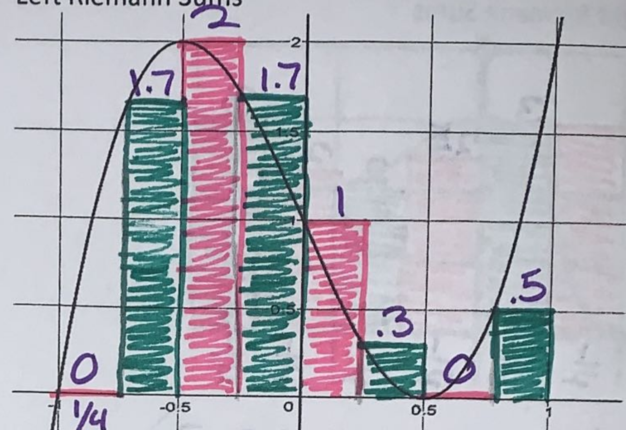
1. Right Riemann Sums

$\frac{1--1}{8} = \frac{1}{4}$



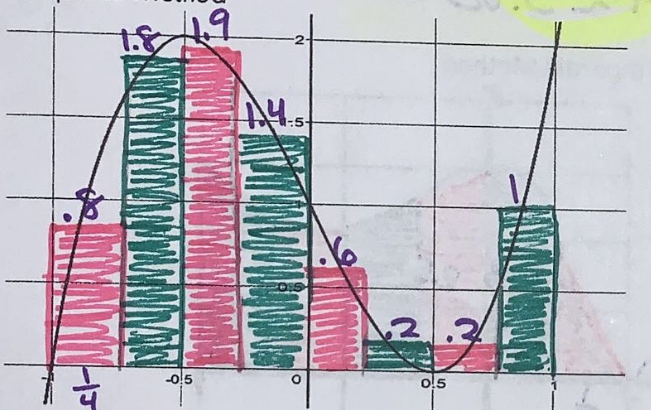
$A \approx \frac{1}{4}(1.5+2+1.7+1+.3+0+.4+2) \approx 2.225u^2$

2. Left Riemann Sums



$A \approx \frac{1}{4}(0+1.7+2+1.7+1+.3+0+.5) \approx 1.8u^2$

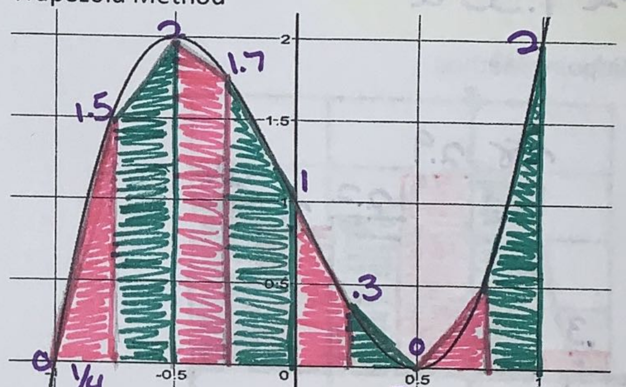
3. Midpoint Method



$A \approx \frac{1}{4}(.8+1.8+1.9+1.4+.6+.2+.2+1)$

$\approx 1.975u^2$

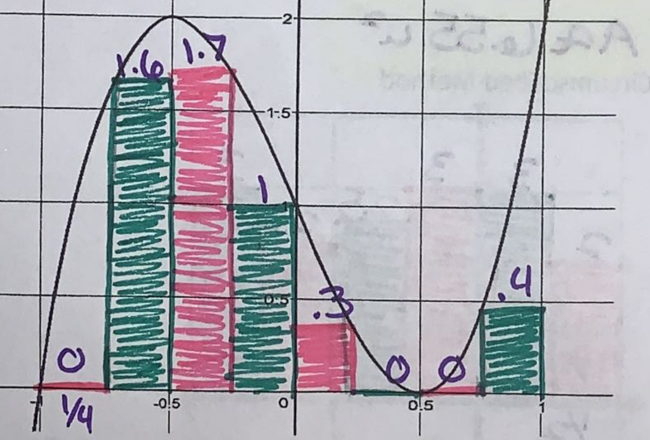
4. Trapezoid Method



$A = \frac{1}{2} \cdot \frac{1}{4}(0+1.5+2+2+1.7+1.7+1+1+.3+.3+0+0+1+1+2)$

$\approx 1.9375u^2$

5. Inscribed Method



$A = \frac{1}{4}(0+1.6+1.7+1+.3+0+0+.4)$

$A \approx 1.25u^2$

6. Circumscribed Method

$A = \frac{1}{4}(1.7+2+2+1.7+1+.25+.6+2)$

$A \approx 2.8125u^2$

Integration Power Rule

valuate each indefinite integral

1. $\int -24x^5 dx$

$\frac{-24x^6}{6} \rightarrow -4x^6 + C$

2. $\int -3\, dx$

$-3x + C$

3. $\int -6x\, dx$

$-\frac{6x^2}{2} \rightarrow -3x^2 + C$

4. $\int 12x^2 dx$

$\frac{12x^3}{3} \rightarrow 4x^3 + C$

5. $\int (-24x^5 - 10x) dx$

$-\frac{24x^6}{6} - \frac{10x^2}{2} \rightarrow -4x^6 - 5x^2 + C$

6. $\int (-9x^2 + 10x)\, dx$

$-\frac{9x^3}{3} + \frac{10x^2}{2} \rightarrow -3x^3 + 5x^2 + C$

7. $\int 4x^{-5} dx$

$\frac{4x^{-4}}{-4} \rightarrow -x^{-4} = \frac{-1}{x^4} + C$

8. $\int -2x^{-3}\, dx$

$\frac{-2x^{-2}}{-2} \rightarrow \frac{1}{x^2} + C$

9. $\int (-2x^{-3} + 20x^{-5}) dx$

$-\frac{2x^{-2}}{-2} + \frac{20x^{-4}}{-4}$

$\frac{1}{x^2} - \frac{5}{x^4} + C$

10. $\int (-4x^{-3} - 20x^{-5}) dx$

$\frac{-4x^{-2}}{-2} - \frac{20x^{-4}}{-4}$

$\frac{2}{x^2} + \frac{5}{x^4} + C$

11. $\int \left(-\frac{4}{x^3} - \frac{8}{x^5}\right) dx = -4x^{-3} - 8x^{-5}$

$\frac{-4x^{-2}}{-2} - \frac{8x^{-4}}{-4} \rightarrow \frac{2}{x^2} + \frac{2}{x^4} + C$

12. $\int \left(\frac{15}{x^4} + \frac{8}{x^5}\right) dx \quad 15x^{-4} + 8x^{-5}$

$\frac{15x^{-3}}{-3} + \frac{8x^{-4}}{-4} \rightarrow \frac{-5}{x^3} - \frac{2}{x^4} + C$

13. $\int -\frac{14x^{\frac{5}{2}}}{2}\,dx$ $\frac{-7x^{7/2}}{\frac{7}{2}} = -7x^{7/2}\cdot\frac{2}{7}$ $-2x^{7/2}+C$	14. $\int -\frac{35x^{\frac{2}{5}}}{5}\,dx$ $\frac{-7x^{7/5}}{\frac{7}{5}} = -7x^{7/5}\cdot\frac{5}{7}$ $-5x^{7/5}+C$
15. $\int -\frac{5\sqrt[3]{x^2}}{3}\,dx = \frac{-5x^{2/3}}{3}$ $\frac{-\frac{5}{3}x^{5/3}}{\frac{5}{3}} = -x^{5/3}+C$	16. $\int -\frac{5\sqrt[4]{x}}{2}\,dx$ $\int -\frac{5}{2}x^{1/4}dx \to \frac{-\frac{5}{2}x^{5/4}}{\frac{5}{4}} = -\frac{5}{2}x^{5/4}\cdot\frac{4}{5}$ $-2x^{5/4}+C$
17. $\int \cos x\,dx$ $\sin x + C$	18. $\int -5\sin x\,dx$ $-5(-\cos x)$ $5\cos x + C$
19. $\int 3\cdot\sec^2 x\,dx$ $3\tan x + C$	20. $\int -3\csc x\cdot\cot x\,dx$ $-3(-\csc x)$ $3\csc x + C$
21. $\int \frac{2}{\sec x}\,dx = \int 2\cdot\frac{1}{\sec x}dx = \int 2\cos x\,dx$ $2\sin x + C$	22. $\int \frac{5}{\csc x}\,dx$ $\quad 5\int\frac{1}{\csc x}dx = 5\int \sin x\,dx$ $5(-\cos x)$ $-5\cos x + C$

Indefinite Integration

	Original Integral	Rewrite	Integrate	Simplify
1.	$\int \frac{1}{x^3} dx$	$\int x^{-3} dx$	$\frac{x^{-2}}{-2} + c$	$-\frac{1}{2x^2} + c$
2.	$\int \sqrt{x}\, dx$	$\int x^{1/2} dx$	$\frac{x^{3/2}}{\frac{3}{2}} + C$	$\frac{2}{3}x^{3/2} + C$
3.	$\int \frac{1}{x^2} dx$	$\int x^{-2} dx$	$\frac{x^{-1}}{-1} + C$	$\frac{-1}{x} + C$
4.	$\int \sqrt[3]{x}\, dx$	$\int x^{1/3} dx$	$\frac{x^{4/3}}{4/3} + C$	$\frac{3}{4}x^{4/3} + C$
5.	$\int 2 \sin x\, dx$	$2\int \sin x\, dx$	$2(-\cos x) + C$	$-2\cos x + C$
6.	$\int (x+2) dx$	not needed or... $\int x\, dx + \int 2 dx$	$\frac{x^2}{2} + \frac{2x^1}{1} + C$	$\frac{x^2}{2} + 2x + C$
7.	$\int \frac{1}{x\sqrt{x}} dx$ $x \cdot x^{1/2}$	$\int x^{-3/2} dx$	$\frac{x^{-1/2}}{-1/2} + C$	$\frac{-2}{x^{1/2}} + C$
	$\int x(x^2+3) dx$	$\int (x^3 + 3x) dx$	$\frac{x^4}{4} + \frac{3x^2}{2} + c$	$\frac{x^4}{4} + \frac{3}{2}x^2 + C$
9.	$\int \frac{1}{2x^3} dx$	$\frac{1}{2}\int x^{-3} dx$	$\frac{1}{2}\left(\frac{x^{-2}}{-2}\right) + C$	$\frac{-1}{4x^2} + C$
10.	$\int \frac{1}{(2x)^3} dx = \frac{1}{8x^3}$	$\frac{1}{8}\int x^{-3} dx$	$\frac{1}{8}\left(\frac{x^{-2}}{-2}\right) + C$	$\frac{-1}{16x^2} + C$
11.	$\int x^2\sqrt{x}\, dx$ $x^2 \cdot x^{1/2}$	$\int x^{5/2} dx$	$\frac{x^{7/2}}{7/2} + C$	$\frac{2}{7}x^{7/2} + C$
12.	$\int (x+3)(3x-2) dx$	$\int (3x^2 + 7x - 6) dx$	$\frac{3x^3}{3} + \frac{7x^2}{2} - \frac{6x^1}{1} + C$	$x^3 + \frac{7}{2}x^2 - 6x + C$
13.	$\int \frac{\sin x}{\cos^2 x} dx$	$\int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx$ $\int \tan x \sec x\, dx$		$\sec x + C$
14.	$\int \frac{x+1}{\sqrt{x}} dx$ $\frac{x^1}{x^{1/2}} + \frac{1}{x^{1/2}}$	$\int (x^{1/2} + x^{-1/2}) dx$	$\frac{x^{3/2}}{\frac{3}{2}} + \frac{x^{1/2}}{\frac{1}{2}} + C$	$\frac{2}{3}x^{3/2} + 2x^{1/2} + C$ or $2\sqrt{x}$
15.	$\int \frac{x^2+x+1}{\sqrt{x}} dx$ $\frac{x^2}{x^{1/2}} + \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}}$	$\int (x^{3/2} + x^{1/2} + x^{-1/2}) dx$	$\frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C$	$\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + 2x^{1/2} + C$

The Fundamental Theorem of Calculus: Evaluating Definite Integrals

1. $\int_2^5 (x^3 - \pi x^2)dx$

$\frac{x^4}{4} - \frac{\pi x^3}{3}\Big|_2^5$

$\left(\frac{5^4}{4} - \frac{\pi 5^3}{3}\right) - \left(\frac{2^4}{4} - \frac{\pi \cdot 2^3}{3}\right)$

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$\left(\frac{625}{4} - \frac{125\pi}{3}\right) - \left(\frac{16}{4} - \frac{8\pi}{3}\right) = \frac{609}{4} - 39\pi$

2. $\int_2^3 2x\,dx$

$\frac{2x^2}{2}\Big|_2^3 = x^2\Big|_2^3$

$3^2 - 2^2 = 5$

3. $\int_1^2 3x^2 dx$

$\frac{3x^3}{3}\Big|_1^2 = x^3\Big|_1^2$

$2^3 - 1^3 = 7$

4. $\int_{12}^{20} dx$

$x\Big|_{12}^{20}$

$20 - 12 = 8$

5. $\int_1^3 (2x - 3)dx$

$\left(\frac{2x^2}{2} - 3x\right)\Big|_1^3$

$(3^2 - 3(3)) - (1^2 - 3(1))$

$0 - -2 = 2$

6. $\int_0^1 e^{2x} dx$

$\frac{e^{2x}}{2}\Big|_0^1$

$\frac{e^{2(1)}}{2} - \frac{e^{2(0)}}{2} = \frac{e^2}{2} - \frac{1}{2}$ or

$\frac{e^2 - 1}{2}$

7. $\int_1^2 \frac{1}{x} dx$

$\int_1^2 x^{-1} dx$ $\quad \frac{x^{-1+1}}{0}$ so you have to memorize this one!

$\ln|x|\Big|_1^2$

$\ln 2 - \ln 1 = \ln 2 - 0 =$

$\ln 2$

8. $\int_1^{3.5} 2x^{-1} dx$

$2\ln|x|\Big|_1^{3.5}$

$2\ln 3.5 - 2\ln 1$

$2\ln 3.5 - 2(0)$

$= 2\ln 3.5$

rals

$\int_0^1 \sin\theta \, d\theta$

$-\cos\theta \Big|_0^1$

$-\cos 1 - (-\cos 0)$

$-\cos(1) + 1$

10. $\int_1^2 \frac{1+y^2}{y} dy = \int_1^2 \left(\frac{1}{y} + y\right) dx$

$\left(\ln|y| + \frac{y^2}{2}\right)\Big|_1^2$

$(\ln(2) + 2) - (\ln(1) + \frac{1}{2})$, with $\ln(1) = 0$

$\ln 2 + \frac{3}{2}$

11. $\int_0^2 \left(\frac{x^3}{3} + 2x\right) dx$

$\frac{x^4}{3 \cdot 4} + \frac{2x^2}{2} = \frac{x^4}{12} + x^2 \Big|_0^2$

$\left(\frac{2^4}{12} + 2^2\right) - \left(\frac{0^4}{12} + 0^2\right)$

$\frac{16}{12} + 4 = 16/3$

12. $\int_0^{\frac{\pi}{4}} (\sin t + \cos t) dt$

$(-\cos t + \sin t)\Big|_0^{\pi/4}$

$(-\cos\frac{\pi}{4} + \sin\frac{\pi}{4}) - (-\cos 0 + \sin 0)$

$\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (-1 + 0)$

$0 + 1 = 1$

13. $\int_{-3}^{-1} \frac{2}{r^3} dr = \int_{-3}^{-1} 2r^{-3} dr$

$\frac{2r^{-2}}{-2}\Big|_{-3}^{-1} = \frac{-1}{r^2}\Big|_{-3}^{-1}$

$\frac{-1}{(-1)^2} - \frac{-1}{(-3)^2} = -1 + \frac{1}{9} = -8/9$

14. $\int_0^1 2e^x dx$

$2e^x\Big|_0^1$

$2e^1 - 2e^0 = 2e - 2$

15. $\int_{-1}^1 2^x dx$

$\frac{2^x}{\ln 2}\Big|_{-1}^1$

$\frac{2}{\ln 2} - \frac{2^{-1}}{\ln 2} = \frac{2}{\ln 2} - \frac{1}{2\ln 2}$

or $\frac{3}{2\ln 2}$

16. $\int_1^2 (2x^{-2} - 3) dx$

$\frac{2x^{-1}}{-1} - 3x\Big|_1^2$ or $\left(\frac{-2}{x} - 3x\right)\Big|_1^2$

$\left(\frac{-2}{2} - 3 \cdot 2\right) - \left(\frac{-2}{1} - 3 \cdot 1\right)$

$-1 - 6 + 2 + 3 = -2$

17. $\int_1^3 3\sqrt{x} \, dx$

$\int_1^3 3x^{1/2} dx \quad \frac{3x^{3/2}}{3/2}\Big|_1^3 \quad 3 \cdot \frac{2}{3}$

$2x^{3/2}\Big|_1^3$

$2(3)^{3/2} - 2(1)^{3/2} = 2\sqrt{27} - 2$ ($9 \cdot 3$)

$6\sqrt{3} - 2$

18. $\int_1^2 (5 - 16x^{-3}) dx$

$5x - \frac{16x^{-2}}{-2}\Big|_1^2 = \left(5x + \frac{8}{x^2}\right)\Big|_1^2$

$\left(5 \cdot 2 + \frac{8}{2^2}\right) - \left(5 \cdot 1 + \frac{8}{1^2}\right)$

$(10 + 2) - (5 + 8)$

$12 - 13 = -1$

$\frac{d}{dx}\int_1^x (t^2 -$

The Fundamental Theorem of Calculus Part 1

For each problem, find $F'(x)$.

1. $F(x) = \int_{-4}^{x} (t-1)dt$

$x - 1$

2. $F(x) = \int_{-3}^{x} (t^2 + 2t + 3)dt$

$x^2 + 2x + 3$

3. $F(x) = \int_{-1}^{x^2} (-2t + 2)dt$

$(-2 \cdot x^2 + 2)(2x)$

$-4x^3 + 4x$

4. $F(x) = \int_{4}^{3x} (-t^3 + 11t^2 - 39t + 44)dt$

$(-(3x)^3 + 11(3x)^2 - 39(3x) + 44)(3)$

$3(-27x^3 + 99x^2 - 117x + 44)$

$-81x^3 + 297x^2 - 351x + 132$

5. $F(x) = \int_{2}^{x^3} \frac{1}{t^3} dt$

$\frac{1}{(x^3)^3} \cdot 3x^2 = \frac{1}{x^9} \cdot 3x^2$

$\frac{3}{x^7}$

6. $F(x) = \int_{x}^{x^2} (-2t - 2)dt$

$(-2(x^2) - 2)(2x) - (-2x - 2)(1)$

$-4x^3 - 4x + 2x + 2$

$-4x^3 - 2x + 2$

7. $F(x) = \int_{x}^{x^2} (t^2 - 8t + 11)dt$

$((x^2)^2 - 8(x^2) + 11)(2x) - (x^2 - 8x + 11)(1)$

$2x^5 - 16x^3 + 22x - x^2 + 8x - 11$

$2x^5 - 16x^3 - x^2 + 30x - 11$

8. $F(x) = \int_{x}^{2x} \left(\frac{2}{t}\right) dt$

$\left(\frac{2}{2x}\right)(2) - \left(\frac{2}{x}\right)(1)$

$\frac{2}{x} - \frac{2}{x} = 0$

$\frac{d}{dx}\int_1^x (t^2-1)dt =$ x^2-1	10. $\frac{d}{dx}\int_{-1}^x \sqrt{t^3+1}\,dt =$ $\sqrt{x^3+1}$
11. $\frac{d}{dx}\int_\pi^x \frac{1}{1+t^4}\,dt =$ $\frac{1}{1+x^4}$	12. $\frac{d}{dx}\int_x^2 \cos(t^2)\,dt =$ $-\frac{d}{dx}\int_2^x \cos(t^2)\,dt$ $-\cos(x^2)$
13. $\frac{d}{dx}\int_3^{x^2} (\sin^4 t)dt =$ $(\sin^4 x^2)\cdot 2x$ $2x\sin^4 x^2$	14. $\frac{d}{dx}\int_1^{\sqrt{x}} \left(\frac{s^2}{s^2+1}\right) ds =$ $x^{1/2}$ $\frac{d}{dx}x^{1/2}=\frac{1}{2x^{1/2}}$ $\frac{(\sqrt{x})^2}{(\sqrt{x})^2+1}\cdot\frac{1}{2\sqrt{x}} =$ $\frac{x}{x+1}\cdot\frac{1}{2\sqrt{x}} = \frac{x}{2\sqrt{x}(x+1)}$ or $\frac{\sqrt{x}}{2(x+1)}$
16. $\frac{d}{dx}\int_{2x}^{3x} \left(\frac{u-1}{u+1}\right) du =$ $\left(\frac{3x-1}{3x+1}\right)\cdot 3 - \left(\frac{2x-1}{2x+1}\right)\cdot 2$ $\frac{9x-3}{3x+1} - \frac{4x-2}{2x+1}$	16. $\frac{d}{dx}\int_{\tan x}^{x^2} \left(\frac{1}{\sqrt{2+t^2}}\right) dt =$ $\frac{1}{\sqrt{2+x^4}}\cdot 2x - \frac{1}{\sqrt{2+\tan^2 x}}\cdot\sec^2 x$ $\frac{2x}{\sqrt{2+x^4}} - \frac{\sec^2 x}{\sqrt{2+\tan^2 x}}$

Antiderivative Practice

Find the antiderivative of each.

1. $\int x^7 dx$

$\frac{x^8}{8} + C$

2. $\int 5x^3 dx$

$\frac{5x^4}{4} + C$

3. $\int (-2x^{-3}) dx$

$\frac{-2x^{-2}}{-2} + C$

$\frac{1}{x^2} + C$

4. $\int (x^2 - x + 1) dx$

$\frac{x^3}{3} - \frac{x^2}{2} + x + C$

5. $\int (7x^3 + 2x^2 - 5) dx$

$\frac{7x^4}{4} + \frac{2x^3}{3} - 5x + C$

6. $\int x^{\frac{1}{2}} dx$

$\frac{x^{3/2}}{3/2} + C$

$\frac{2}{3}x^{3/2} + C$

7. $\int x^{\frac{3}{5}} dx$

$\frac{x^{8/5}}{8/5} + C$

$\frac{5}{8}x^{8/5} + C$

8. $\int \left(x^{-5} + x^{\frac{1}{4}}\right) dx$

$\frac{x^{-4}}{-4} + \frac{x^{5/4}}{5/4} + C$

$\frac{-1}{4x^4} + \frac{4}{5}x^{5/4} + C$

9. $\int \sec^2 x \, dx$

$\tan x + C$

10. $\int \cos x \, dx$

$\sin x + C$

11. $\int \sec x \cdot \tan x \, dx$

$\sec x + C$

12. $\int \csc^2 x \, dx$

$-\cot x + C$

13. $\int \sin x \, dx$

$-\cos x + C$

14. $\int \csc x \cdot \cot x \, dx$

$-\csc x + C$

10

The Fundamental Theorem of Calculus Practice

r each of the following problems, find the value of the integral.

1. $\int_{-2}^{1} 5\,dx$

$5x\Big|_{-2}^{1} = 5(1) - 5(-2)$

$5 + 10$

15

2. $\int_{1}^{4} (3x^2)\,dx$

$\frac{3x^3}{3}\Big|_1^4 = x^3\Big|_1^4 = 4^3 - 1^3$

$= 64 - 1$

$= 63$

3. $\int_{-1}^{1} (x^3)dx$

$\frac{x^4}{4}\Big|_{-1}^{1} = \frac{(1)^4}{4} - \frac{(-1)^4}{4}$

$= \frac{1}{4} - \frac{1}{4}$

0

4. $\int_{4}^{9} (5\sqrt{x})dx$ $5x^{1/2}$

$\frac{5x^{3/2}}{3/2}\Big|_4^9 = \frac{10x^{3/2}}{3}\Big|_4^9$

$\frac{10\sqrt{9}^3}{3} - \frac{10\sqrt{4}^3}{3} = \frac{270}{3} - \frac{80}{3} = \frac{190}{3}$

5. $\int_{2}^{3} \left(\frac{4}{x^3}\right) dx$ $4x^{-3}$

$\frac{4x^{-2}}{-2}\Big|_2^3 = \frac{-2}{x^2}\Big|_2^3$

$\frac{-2}{9} - \frac{-2}{4} = 5/18$

6. $\int_{1}^{8} \left(\sqrt[3]{x^2}\right)dx$ $x^{2/3}$

$\frac{3x^{5/3}}{5}\Big|_1^8 = \frac{3\sqrt[3]{8}^5}{5} - \frac{3\sqrt[3]{1}^5}{5}$

$\frac{96}{5} - \frac{3}{5} = \frac{93}{5}$

7. $\int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$

$\frac{1}{2}\frac{x^2}{2} + 3x\Big|_{-2}^{4} = \frac{x^2}{4} + 3x\Big|_{-2}^{4}$

$\left(\frac{4^2}{4} + 3\cdot 4\right) - \left(\frac{(-2)^2}{4} + 3\cdot -2\right) = 16 - (-5) = 21$

8. $\int_{0}^{\pi/2} (\sin x)dx$

$-\cos x\Big|_0^{\pi/2}$

$\left(-\cos\frac{\pi}{2}\right) - (-\cos 0)$

$0 - (-1) = 1$

Find the derivative of each of the following.

9. $\int_{-4}^{x} \left(\frac{5}{t^3}\right) dt$

$\frac{5}{x^3}$

10. $\int_{1}^{x^2} (t^3 - 4t^2 + 3)dt$

$[(x^2)^3 - 4(x^2)^2 + 3](2x)$

$2x(x^6 - 4x^4 + 3)$

$2x^7 - 8x^5 + 6x$

11. $\int_{\frac{\pi}{4}}^{3x} (-\csc t\ \cot t)dt$

$(-\csc 3x \cot 3x) \cdot 3$

$-3\csc(3x)\cot(3x)$

12. $\int_{x}^{x^2} (-t - 1)dt$

$(-x^2 - 1)(2x) - (-x - 1)(1)$

$-2x^3 - 2x + x + 1$

$-2x^3 - x + 1$

Meaning of Integration Unit Review

Section 1: Indefinite Integrals

1. $\int \sec^2 x dx$ $\quad \tan x + C$

2. $\int dx$ $\quad X + C$

3. $\int 4\sin x dx$

$-4\cos x + C$

4. $\int \frac{5}{x^3} dx$ $\quad 5x^{-3} \quad \frac{5x^{-2}}{-2}$

$\frac{-5}{2x^2} + C$

5. $\int \sqrt[2]{x^5} dx \rightarrow x^{5/2} \quad \frac{x^{7/2}}{7/2}$

$\frac{2x^{7/2}}{7} + C$

6. $\int \csc x \cot x dx$

$-\csc x + C$

7. $\int \frac{-5}{(2x)^3} dx$ $\quad -\frac{5}{8}x^{-3} \rightarrow \frac{-5x^{-2}}{8 \cdot -2}$

$\frac{5}{16x^2} + C$

8. $\int (5x+1)^2 dx$ $\quad (5x+1)(5x+1$

$\int (25x^2 + 10x + 1) dx$

$\frac{25x^3}{3} + \frac{10x^2}{2} + x + C$

$\frac{25x^3}{3} + 5x^2 + x + C$

9. $\int (x^{-4} + 3x^3) dx$

$\frac{x^{-3}}{-3} + \frac{3x^4}{4} + C$

$\frac{-1}{3x^3} + \frac{3}{4}x^4 + C$

10. $\int 3\sec x \tan x dx$

$3\sec x + C$

11. $\int (\frac{1}{\sqrt[3]{x^2}} - \sec^2 x) dx$

$x^{-2/3} - \sec^2 x$

$\frac{x^{1/3}}{1/3} - \tan x + C =$

$3x^{1/3} - \tan x + C$

12. $\int (\pi e^x + 7\cos x) dx$

$\pi e^x + 7\sin x + C$

13. $\int (2x-5)^2 dx$ $\quad 4x^2 - 10x + 25$

$\frac{4x^3}{3} - \frac{10x^2}{2} + 25x + C$

$\frac{4x^3}{3} - 5x^2 + 25x + C$

14. $\int \csc x \cot x dx$

$-\csc x + C$

15. $\int 3x^{-1} dx$

$3\ln|x| + C$

16. $\int (3x-4)^2 dx$ $\quad 9x^2 - 24x + 16$

$\frac{9x^3}{3} - \frac{24x^2}{2} + 16x + C$

$3x^3 - 12x^2 + 16x + C$

17. $\int e x^{-1} dx$

$e\ln|x| + C$

18. $\int \frac{dx}{\cos^2 x}$ $\quad = \int \frac{1}{\cos^2 x} = \int \sec^2 x dx$

$\tan x + C$

Section 2: Definite Integrals – Fundamental Theorem of Calculus

19. $\int_1^8 (x^{-1/3} + \frac{1}{x}) dx$

$\frac{3x^{2/3}}{2} + \ln|x| \Big|_1^8$

$(\frac{3\sqrt[3]{8}^2}{2} + \ln 8) - (\frac{3\sqrt[3]{1}^2}{2} + \ln 1)$

$6 + \ln 8 - \frac{3}{2} + 0$

$\frac{9}{2} + \ln 8$

20. $\int_0^\pi (\sin t + 1) dt$

$-\cos t + t \Big|_0^\pi$

$(-\cos(\pi) + \pi) - (-\cos(0) + 0)$

$1 + \pi + 1 - 0 =$

$2 + \pi$

21. $\frac{d}{dx} \int_2^x \sqrt{1+4t^2} dt$

$\sqrt{1+4x^2}$

22. $\frac{d}{dx} \int_3^{x^2} \frac{1}{\sqrt{2-t^5}} dt$

$\frac{1}{\sqrt{2-(x^2)^5}} \cdot 2x = \frac{2x}{\sqrt{2-x^{10}}}$

23. $\frac{d}{dx} \int_{\tan x}^{x^2} \frac{1}{4-t^3} dt$

$(\frac{1}{4-x^6}) \cdot 2x - (\frac{1}{4-\tan^3 x}) \sec^2 x$

$\frac{2x}{4-x^6} - \frac{\sec^2 x}{4-\tan^3 x}$

24. $\frac{d}{dx} \int_2^x \sec(t) dt$

$\sec x$

25. $\frac{d}{dx} \int_{-5}^x \sec(t) dt$

$\sec x$

26. $\frac{d}{dx} \int_x^2 \sec(t) dt$ $\quad -1 \frac{d}{dx} \int_2^x \sec(t) dt$

$-\sec x$

27. $\frac{d}{dx} \int_2^{2x^5} \sec(t) dt$

$\sec(2x^5) \cdot 10x^4$

$10x^4 \sec(2x^5)$

$f(x)=\int_{-1}^{x}\ln(t)dt \quad f'(x)=$

$\ln|x|$

29. $f(x)=\int_{x}^{-4}\ln(t)dt \quad f'(x)=$

$-\ln|x|$

30. $f(x)=\int_{0}^{\sin x}\ln(t)dt \quad f'(x)=$

$\ln|\sin x|\cdot\cos x$

$f(x)=\int_{3+x^2}^{x}\ln(t)dt \quad f'(x)=$

$\ln|x|\cdot 1-\ln|3+x^2|\cdot 2x$

$\ln|x|-2x\ln|3+x^2|$

32. $\frac{d}{dx}\int_{-1}^{x}t^2dt$

x^2

33. $\int_{-1}^{x}t^2dt \quad \frac{t^3}{3}\Big|_{-1}^{x}$

$\frac{x^3}{3}-\frac{(-1)^3}{3}=\frac{x^3}{3}+\frac{1}{3}$

4. $\frac{d}{dx}\int_{x^3}^{5}\left(\frac{t^2-3}{2t}\right)dt$

$-\frac{(x^6-3)\cdot 3x^2}{2x^3}=\frac{-3x^6+9}{2x}$

35. $\int_{1}^{25}x^{1/2}dx \quad \frac{2x^{3/2}}{3}\Big|_{1}^{25}$

$\frac{2\sqrt{25}^3}{3}-\frac{2\sqrt{1}^3}{3}=$

$\frac{250}{3}-\frac{2}{3}=\frac{248}{3}$

36. $\int_{0}^{3\pi/2}(\sin t+1)dt \quad -\cos t+t\Big|_{0}^{3\pi/2}$

$(-\cos 3\pi/2+3\pi/2)-(-\cos 0+0)$

$0+\frac{3\pi}{2}+1+0=$

$\frac{3\pi}{2}+1$

37. $\frac{d}{dx}\int_{2}^{x}\sec^3 t\,dt$

$\sec^3 x$

38. $\frac{d}{dx}\int_{e^x}^{x^3}\frac{1}{1+t^2}dt$

$\left(\frac{1}{1+x^6}\cdot 3x^2\right)-\left(\frac{1}{1+e^{2x}}\right)e^x=\frac{3x^2}{1+x^6}-\frac{e^x}{1+e^{2x}}$

39. $\frac{d}{dx}\int_{3}^{x^2}\sqrt{5t-t^2}\,dt$

$\sqrt{5x^2-(x^2)^2}\cdot 2x$

$2x\sqrt{5x^2-x^4}$

40 If $\int_{2}^{7}f(x)dx=8$ and $\int_{2}^{7}g(x)dx=-3$ and $\int_{5}^{7}f(x)dx=-1$, find the following:

a. $\int_{2}^{7}[3f(x)+2g(x)]dx$

$3(8)+2(-3)=18$

b. $\int_{7}^{2}3g(x)dx$

$-1\cdot 3\cdot -3=9$

c. $\int_{2}^{5}f(x)dx \quad \int_{2}^{7}f(x)dx-\int_{5}^{7}f(x)dx$

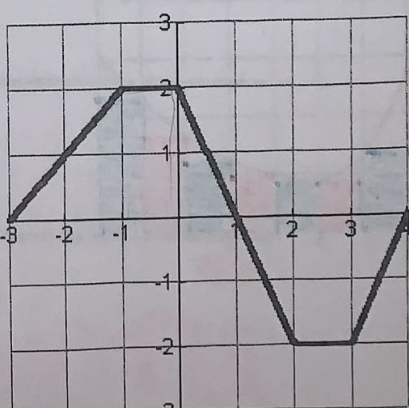
$8--1=9$

d. $\int_{3}^{3}g(x)dx$

0

Section 3: Area

Given the graph below, if $g(x)=\int_{-2}^{x}f(t)dt$, find the following



41. g(2) $\int_{-2}^{2}=1.5+2+\frac{1}{2}(2)-\frac{1}{2}(2)=3.5$

$1+\frac{1}{2}$

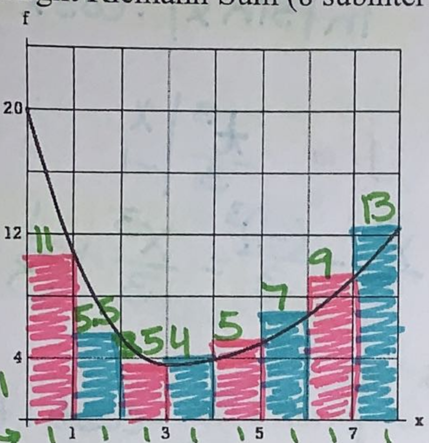
42. g(0) $\int_{-2}^{0}=1.5+2=3.5$

g(4) $\int_{-2}^{4}1.5+2+1-1-2-1=0.5$

44. g(-3) $\int_{-2}^{-3}=-1\int_{-3}^{-2}=-1(.5)=-0.5$

Draw the picture only for #46, 48-50. Draw the picture and find the area for #45 & 47.

45. Right Riemann Sum (8 subintervals)

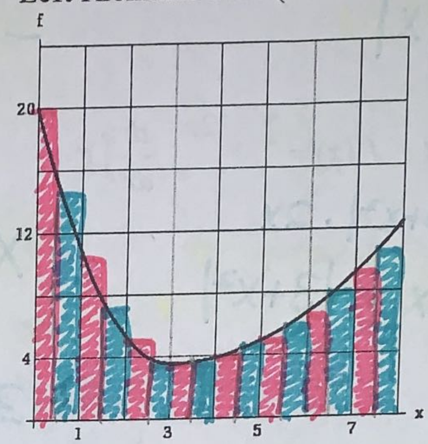


$\frac{8-0}{8}=1$

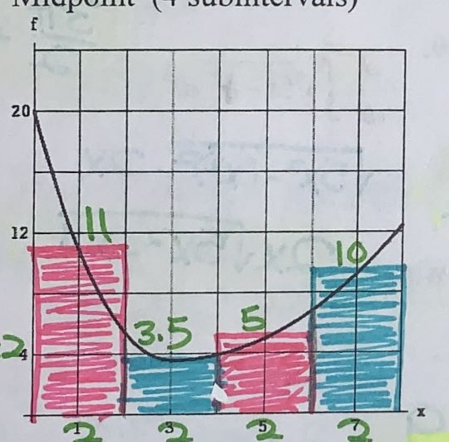
$A \approx 1(11+5.5+3.5+4+5+7+9+13)$

$A \approx 58$ units^2

46. Left Riemann Sum (16 subintervals)



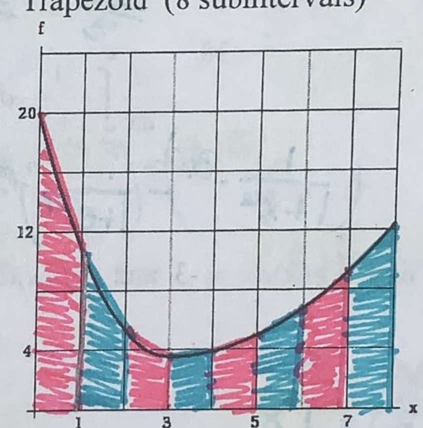
47. Midpoint (4 subintervals)



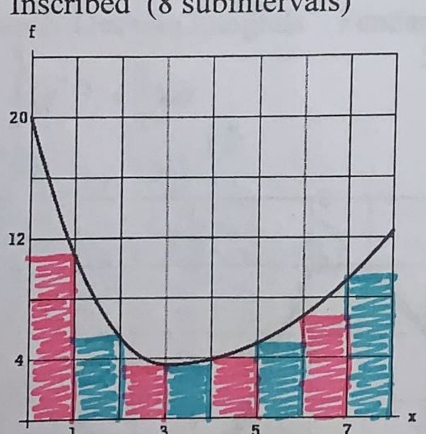
$\frac{8-0}{4}=2$

$A \approx 2(11+3.5+5+10) \approx 59u^2$

48. Trapezoid (8 subintervals)



49. Inscribed (8 subintervals)



50. Circumscribed (4 subintervals)