Unit 5

Introduction to Integration

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- ❖ Riemann Sums & Trapezoid Rule
- ❖ Indefinite Integrals (Antiderivatives)
- ❖ Definite Integrals (Fundamental Theorem of Calculus Part 2)
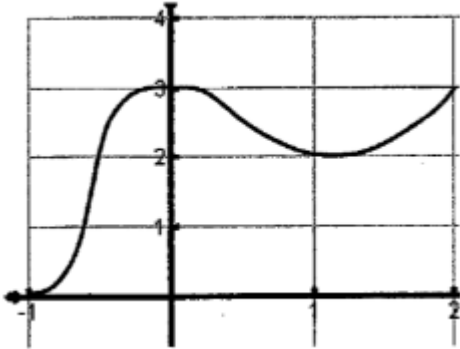- ❖ Fundamental Theorem of Calculus Part 1

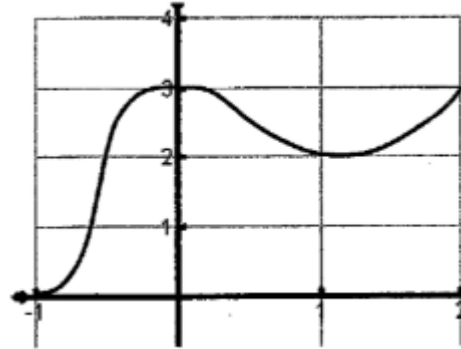# Quiz is _____

Test is _____

Name: _____

Riemann Sums Worksheet 1

Given the function estimate the area bounded by the curve and the x-axis using the specified method with 6 subintervals over the interval [-1,2]. Draw rectangles and use the graph to estimate y values.

1. Right Riemann Sums

2. Left Riemann Sums

3. Midpoint Method

4. Trapezoid Method

5. Inscribed Method

6. Circumscribed Method

Riemann Sums Worksheet 2

Given the function estimate the area bounded by the curve and the x-axis using the specified method with 8 subintervals over the interval [-1,1]. Draw rectangles and use the graph to estimate y values.
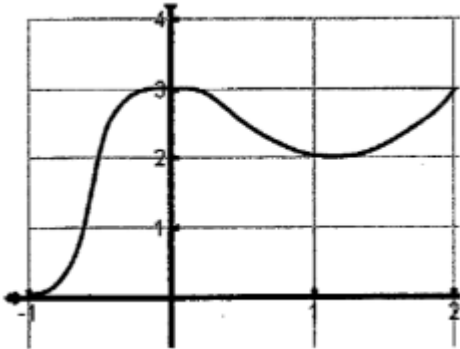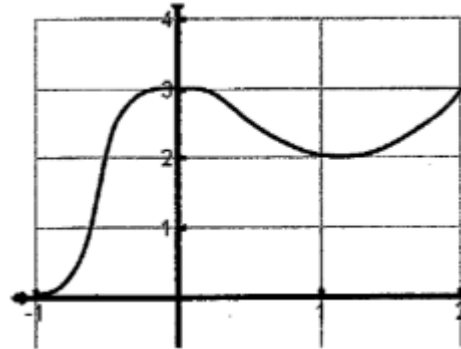
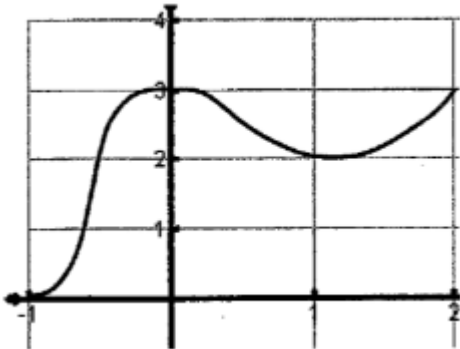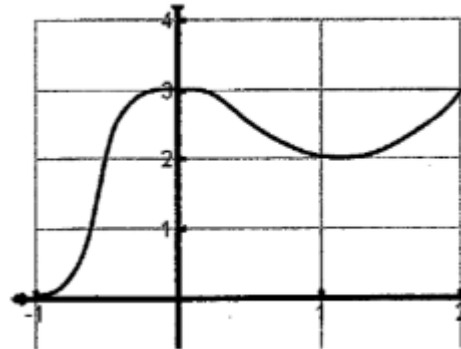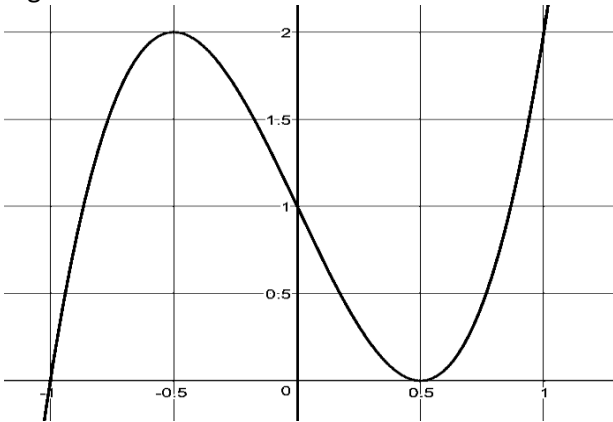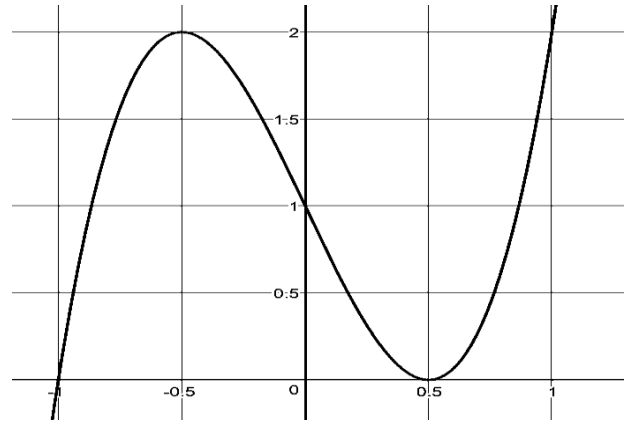1. Right Riemann Sums

2. Left Riemann Sums

3. Midpoint Method
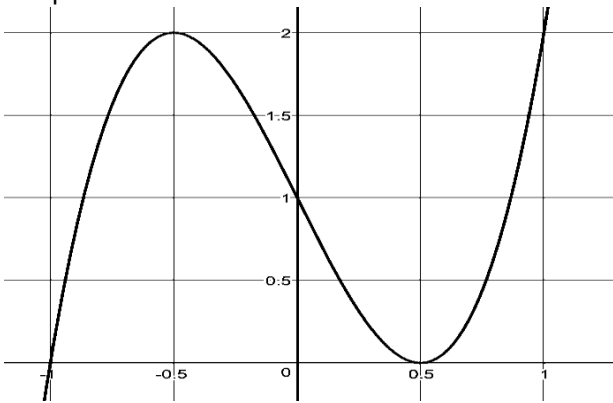
4. Trapezoid Method
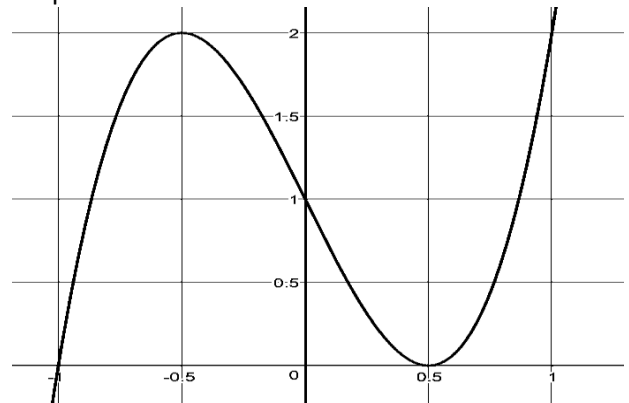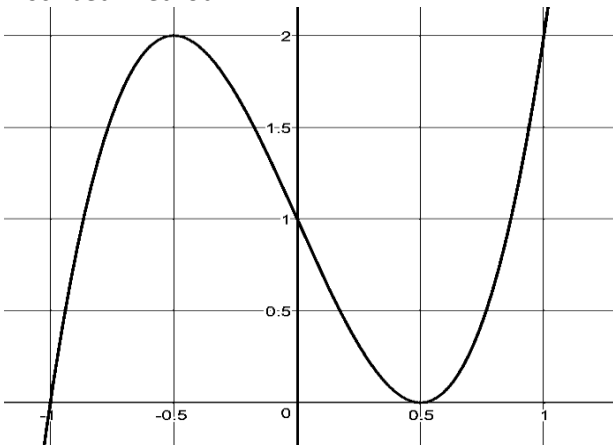
5. Inscribed Method

6. Circumscribed Method
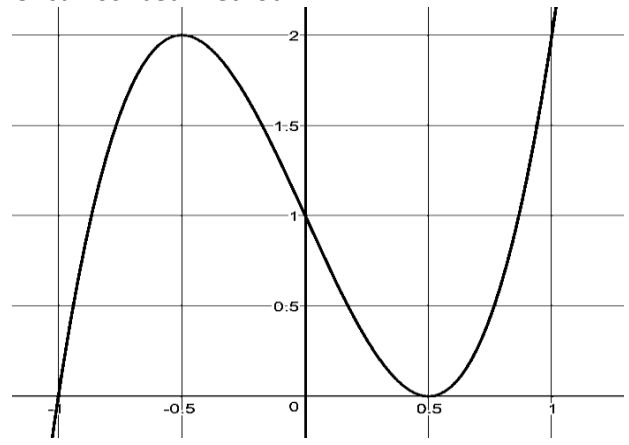
Integration Power Rule

Evaluate each indefinite integral

1. $\int -24x^5 dx$	2. $\int -3\, dx$
3. $\int -6x\, dx$	4. $\int 12x^2 dx$
5. $\int(-24x^5 - 10x)dx$	6. $\int(-9x^2 + 10x)\, dx$
7. $\int 4x^{-5} dx$	8. $\int -2x^{-3}\, dx$
9. $\int(-2x^{-3} + 20x^{-5})dx$	10. $\int(-4x^{-3} - 20x^{-5})dx$
11. $\int\left(-\frac{4}{x^3} - \frac{8}{x^5}\right)dx$	12. $\int\left(\frac{15}{x^4} + \frac{8}{x^5}\right)dx$

13. $\int -\frac{14x^{\frac{5}{2}}}{2} dx$	14. $\int -\frac{35x^{\frac{2}{5}}}{5} dx$
15. $\int -\frac{5\sqrt[3]{x^2}}{3} dx$	16. $\int -\frac{5\sqrt[4]{x}}{2} dx$
17. $\int \cos x \ dx$	18. $\int -5 \sin x \ dx$
19. $\int 3 \cdot \sec^2 x \ dx$	20. $\int -3 \csc x \cdot \cot x \ dx$
21. $\int \frac{2}{\sec x} dx$	22. $\int \frac{5}{\csc x} dx$

4

Indefinite Integration

	Original Integral	Rewrite	Integrate	Simplify
1.	$\int \frac{1}{x^3} dx$	$\int x^{-3} dx$	$\frac{x^{-2}}{-2} + c$	$-\frac{1}{2x^2} + c$
2.	$\int \sqrt{x}\, dx$			
3.	$\int \frac{1}{x^2} dx$			
4.	$\int \sqrt[3]{x}\, dx$			
5.	$\int 2\sin x\, dx$			
6.	$\int (x+2) dx$			
7.	$\int \frac{1}{x\sqrt{x}} dx$			
8.	$\int x(x^2+3) dx$			
9.	$\int \frac{1}{2x^3} dx$			
10.	$\int \frac{1}{(2x)^3} dx$			
11.	$\int x^2 \sqrt{x}\, dx$			
12.	$\int (x+3)(3x-2) dx$			
13.	$\int \frac{\sin x}{\cos^2 x} dx$			
14.	$\int \frac{x+1}{\sqrt{x}}\, dx$			
15.	$\int \frac{x^2+x+1}{\sqrt{x}}\, dx$			

5

The Fundamental Theorem of Calculus: Evaluating Definite Integrals

1. $\int_2^5 (x^3 - \pi x^2)dx$	2. $\int_2^3 2x\, dx$
3. $\int_1^2 3x^2 dx$	4. $\int_{12}^{20} dx$
5. $\int_1^3 (2x - 3)dx$	6. $\int_0^1 e^{2x}dx$
7. $\int_1^2 \frac{1}{x}dx$	8. $\int_1^{3.5} 2x^{-1}dx$

9. $\int_0^1 \sin\theta \; d\theta$	10. $\int_1^2 \frac{1+y^2}{y} dy$
11. $\int_0^2 \left(\frac{x^3}{3} + 2x\right) dx$	12. $\int_0^{\frac{\pi}{4}} (\sin t + \cos t) dt$
13. $\int_{-3}^{-1} \frac{2}{r^3} dr$	14. $\int_0^1 2e^x dx$
15. $\int_{-1}^1 2^x dx$	16. $\int_1^2 (2x^{-2} - 3) dx$
17. $\int_1^3 3\sqrt{x} \; dx$	18. $\int_1^2 (5 - 16x^{-3}) dx$

The Fundamental Theorem of Calculus Part 1

For each problem, find $F'(x)$.

1. $$F(x) = \int_{-4}^{x} (t-1)dt$$

2. $$F(x) = \int_{-3}^{x} (t^2 + 2t + 3)dt$$

3. $$F(x) = \int_{-1}^{x^2} (-2t + 2)dt$$

4. $$F(x) = \int_{4}^{3x} (-t^3 + 11t^2 - 39t + 44)dt$$

5. $$F(x) = \int_{2}^{x^3} \frac{1}{t^3}dt$$

6. $$F(x) = \int_{x}^{x^2} (-2t - 2)dt$$

7. $$F(x) = \int_{x}^{x^2} (t^2 - 8t + 11)dt$$

8. $$F(x) = \int_{x}^{2x} \left(\frac{2}{t}\right)dt$$

9. $\dfrac{d}{dx}\int_1^x (t^2-1)dt =$	10. $\dfrac{d}{dx}\int_{-1}^x \sqrt{t^3+1}\, dt =$
11. $\dfrac{d}{dx}\int_\pi^x \dfrac{1}{1+t^4}\, dt =$	12. $\dfrac{d}{dx}\int_x^2 \cos(t^2)\, dt =$
13. $\dfrac{d}{dx}\int_3^{x^2}(\sin^4 t)dt =$	14. $\dfrac{d}{dx}\int_1^{\sqrt{x}}\left(\dfrac{s^2}{s^2+1}\right)ds =$
16. $\dfrac{d}{dx}\int_{2x}^{3x}\left(\dfrac{u-1}{u+1}\right)du =$	16. $\dfrac{d}{dx}\int_{\tan x}^{x^2}\left(\dfrac{1}{\sqrt{2+t^2}}\right)dt =$

Antiderivative Practice

Find the antiderivative of each.

1. $\int x^7 dx$

2. $\int 5x^3 dx$

3. $\int (-2x^{-3})dx$

4. $\int (x^2 - x + 1)dx$

5. $\int (7x^3 + 2x^2 - 5)dx$

6. $\int x^{\frac{1}{2}} dx$

7. $\int x^{\frac{3}{5}} dx$

8. $\int \left(x^{-5} + x^{\frac{1}{4}} \right) dx$

9. $\int \sec^2 x \; dx$

10. $\int \cos x \; dx$

11. $\int \sec x \cdot \tan x \; dx$

12. $\int \csc^2 x \; dx$

13. $\int \sin x \; dx$

14. $\int \csc x \cdot \cot x \; dx$

The Fundamental Theorem of Calculus Practice

For each of the following problems, find the value of the integral.

1. $\int_{-2}^{1} 5\, dx$

2. $\int_{1}^{4} (3x^2)\, dx$

3. $\int_{-1}^{1} (x^3)\, dx$

4. $\int_{4}^{9} (5\sqrt{x})\, dx$

5. $\int_{2}^{3} \left(\frac{4}{x^3}\right) dx$

6. $\int_{1}^{8} (\sqrt[3]{x^2})\, dx$

7. $\int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$

8. $\int_{0}^{\pi/2} (\sin x)\, dx$

For each of the following problems, find the derivative.

9. $\int_{-4}^{x} \left(\frac{5}{t^3}\right) dt$

10. $\int_{1}^{x^2} (t^3 - 4t^2 + 3)\, dt$

11. $\int_{-\frac{\pi}{4}}^{3x} (-\csc t\ \cot t)\, dt$

12. $\int_{x}^{x^2} (-t - 1)\, dt$

Meaning of Integration Unit Review

Section 1: Indefinite Integrals

1. $\int \sec^2 x\,dx$

2. $\int dx$

3. $\int 4\sin x\,dx$

4. $\int \dfrac{5}{x^3}\,dx$

5. $\int \sqrt[2]{x^5}\,dx$

6. $\int \csc x \cot x\,dx$

7. $\int \dfrac{-5}{(2x)^3}\,dx$

8. $\int (5x+1)^2\,dx$

9. $\int (x^{-4}+3x^3)\,dx$

10. $\int 3\sec x \tan x\,dx$

11. $\int (\dfrac{1}{\sqrt[3]{x^2}} - \sec^2 x)\,dx$

12. $\int (\pi e^x + 7\cos x)\,dx$

13. $\int (2x-5)^2\,dx$

14. $\int \csc x \cot x\,dx$

15. $\int 3x^{-1}\,dx$

16. $\int (3x-4)^2\,dx$

17. $\int e x^{-1}\,dx$

18. $\int \dfrac{dx}{\cos^2 x}$

Section 2: Definite Integrals – Fundamental Theorem of Calculus

19. $\displaystyle\int_1^8 (x^{-\frac{1}{3}} + \dfrac{1}{x})\,dx$

20. $\displaystyle\int_0^\pi (\sin t + 1)\,dt$

21. $\dfrac{d}{dx}\displaystyle\int_2^x \sqrt{1+4t^2}\,dt$

22. $\dfrac{d}{dx}\displaystyle\int_3^{x^2} \dfrac{1}{\sqrt{2-t^5}}\,dt$

23. $\dfrac{d}{dx}\displaystyle\int_{\tan x}^{x^2} \dfrac{1}{4-t^3}\,dt$

24. $\dfrac{d}{dx}\displaystyle\int_2^x \sec(t)\,dt$

25. $\dfrac{d}{dx}\displaystyle\int_{-5}^x \sec(t)\,dt$

26. $\dfrac{d}{dx}\displaystyle\int_x^2 \sec(t)\,dt$

27. $\dfrac{d}{dx}\displaystyle\int_2^{2x^5} \sec(t)\,dt$

12

28. $f(x) = \int_{-1}^{x} \ln(t)\,dt \quad f'(x) =$

29. $f(x) = \int_{x}^{-4} \ln(t)\,dt \quad f'(x) =$

30. $f(x) = \int_{0}^{\sin x} \ln(t)\,dt \quad f'(x) =$

31. $f(x) = \int_{3+x^2}^{x} \ln(t)\,dt \quad f'(x) =$

32. $\dfrac{d}{dx} \int_{-1}^{x} t^2\,dt$

33. $\int_{-1}^{x} t^2\,dt$

34. $\dfrac{d}{dx} \int_{x^3}^{5} \left(\dfrac{t^2 - 3}{2t} \right) dt$

35. $\int_{1}^{25} x^{1/2}\,dx$

36. $\int_{0}^{3\pi/2} (\sin t + 1)\,dt$

37. $\dfrac{d}{dx} \int_{2}^{x} \sec^3 t\,dt$

38. $\dfrac{d}{dx} \int_{e^x}^{x^3} \dfrac{1}{1+t^2}\,dt$

39. $\dfrac{d}{dx} \int_{3}^{x^2} \sqrt{5t - t^2}\,dt$

40.
If $\displaystyle\int_{2}^{7} f(x)\,dx = 8$ and $\displaystyle\int_{2}^{7} g(x)\,dx = -3$ and $\displaystyle\int_{5}^{7} f(x)\,dx = -1$, find the following:

a. $\displaystyle\int_{2}^{7} [3f(x) + 2g(x)]\,dx$

b. $\displaystyle\int_{7}^{2} 3g(x)\,dx$

c. $\displaystyle\int_{2}^{5} f(x)\,dx$

d. $\displaystyle\int_{3}^{3} g(x)\,dx$

Section 3: Area

Given the graph below, if $g(x) = \displaystyle\int_{-2}^{x} f(t)\,dt$, find the following
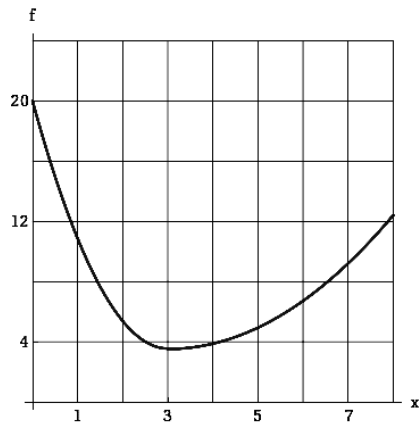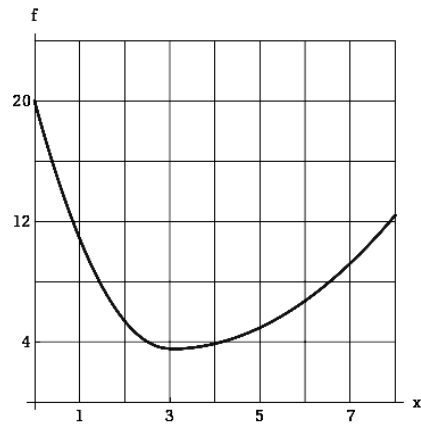
41. g(2)

42. g(0)

43. g(4)

44. g(-3)

13
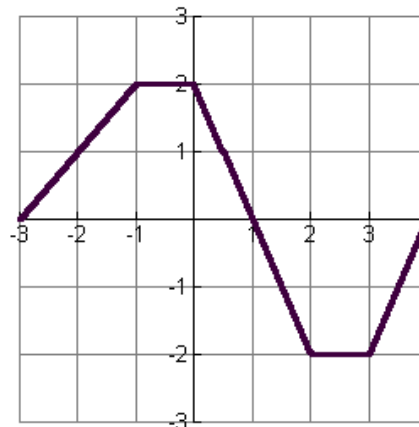
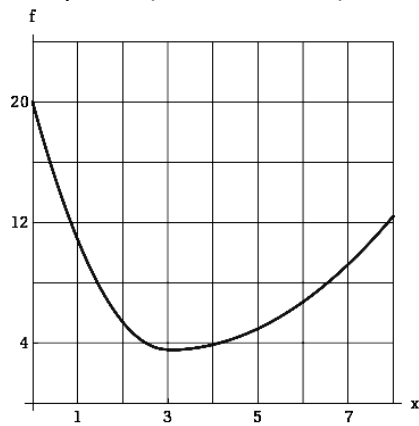**Draw the picture only for #46, 48-50. Draw the picture and find the area for #45 & 47.**
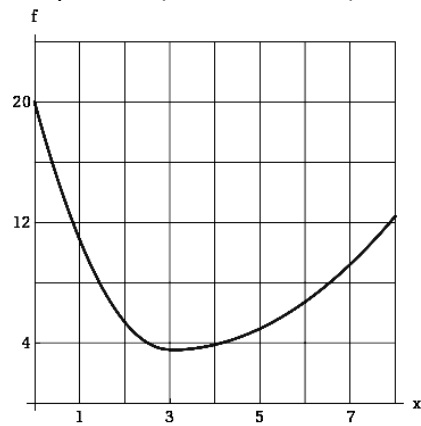
45. Right Riemann Sum (8 subintervals)

f

20

12

4

1   3   5   7   x
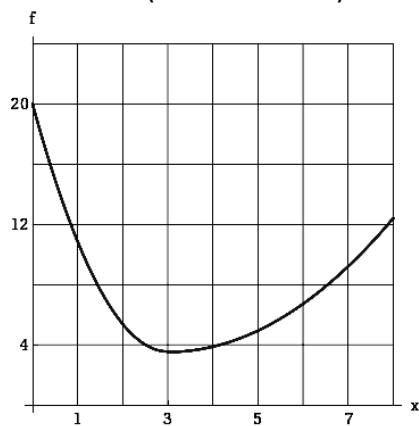
46. Left Riemann Sum (16 subintervals)

f

20

12

4

1   3   5   7   x

47. Midpoint (4 subintervals)

f

20

12

4

1   3   5   7   x

48. Trapezoid (8 subintervals)

f

20

12

4

1   3   5   7   x

49. Inscribed (8 subintervals)

f

20

12

4

1   3   5   7   x

50. Circumscribed (4 subintervals)

f

20

12

4

1   3   5   7   x