# Unit 4B: Graphing Rational Functions

I CAN:

- Identify the important features of rational functions…
  - x- and y-intercepts
  - vertical, horizontal and/or slant asymptotes
  - holes
  - domain and range
- Graph rational functions


| Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|
| 22 DAY 1<br><br>Holes, Asymptotes, and Intercepts | 23 DAY 2<br><br>Graphing Rational Functions | 24 DAY 3<br><br>Graphing with Slant Asymptote | 25 DAY 4<br><br>More Graphing Practice<br><br>**DeltaMath Skills Check** | 26<br><br>Help Sessions |
| 29 DAY 5<br><br>Review | 30 DAY 6<br><br>Review<br><br>**Unit 4B Test** | 31<br><br>Help Sessions | 1<br><br>**Unit 4B Test Due 8 am** | 2 |

*THIS PLAN IS SUBJECT TO CHANGE. PLEASE REFER TO CTLS DIGITAL CLASSROOM FOR UPDATES.*

# Graphing Rational Functions

A rational function is a function of the form: [ ] where $p(x)$ and $q(x)$ are ______________ and $q(x) \neq$ ________

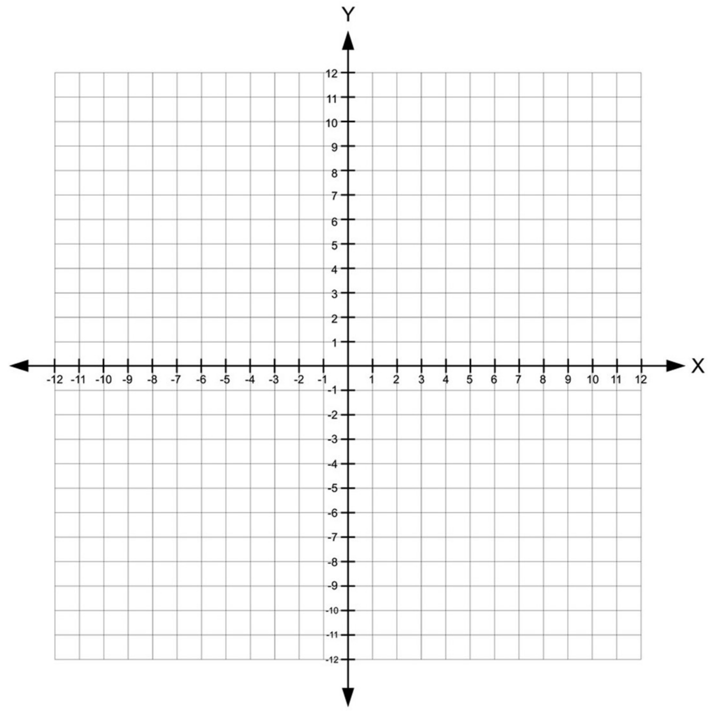
| Key Features of Rational Functions | |
|---|---|
| ① Simplify the function.<br>Factor the numerator and denominator, then eliminate common factors. | |
| **HOLES**<br>A hole is a point $(x, y)$ at which there is a ______________ in the graph.<br><br>A hole occurs when there is a ______________ between the numerator and denominator. | ② For each factor you eliminated in step 1, there is a hole! Locate each hole:<br>➢ To find the $x$-coordinate, set the factor = 0 and solve.<br>➢ To find the $y$-coordinate, substitute the x-coordinate into the simplified function. |
| **X-INTERCEPT(S)**<br>The points where the graph crosses the $x$-axis and where $y = 0$. | ③ Set the **numerator = 0** and solve. |
| **VERTICAL ASYMPTOTE(S)**<br>Vertical boundary lines which the graph will not cross, written in the form $x = a$.<br>These occur because the denominator of a fraction cannot = ____! | ④ Set the **denominator = 0** and solve. |
| **HORIZONTAL ASYMPTOTES**<br>Horizontal guidelines, written in the form $y = a$. | ⑤ Follow the rules below. |

| CASE | HORIZONTAL ASYMPTOTE |
|---|---|
| degree of $p <$ degree of $q$ | |
| degree of $p =$ degree of $q$ | |
| degree of $p >$ degree of $q$ | |

| | |
|---|---|
| **SLANT ASYMPTOTE**<br>If the degree of $p$ is greater than the degree of $q$ by 1, then the function has a slant asymptote in the form $y = mx + b$. | To find the slant asymptote, divide the numerator by the denominator using long or synthetic division. The **depressed polynomial** defines the asymptote – ignore the remainder. |
| **Y-INTERCEPT**<br>The point where the graph crosses the $y$-axis and where $x = 0$. | Substitute 0 for $x$ in the simplified equation and solve for $y$. |

Ex 1: Identify the key characteristics of the function and graph it.

$$f(x) = \frac{x - 4}{x + 4}$$




Hole(s):

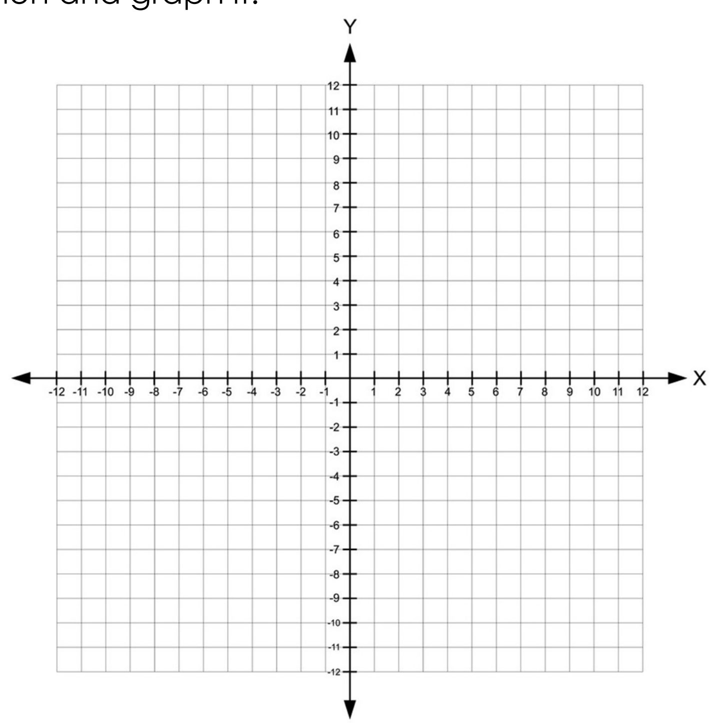
VA: x-int:

HA: y-int:

SA:

Ex 2: Identify the key characteristics of the function and graph it.

$$f(x) = \frac{5x - 10}{x^2 - 4}$$




Hole(s):

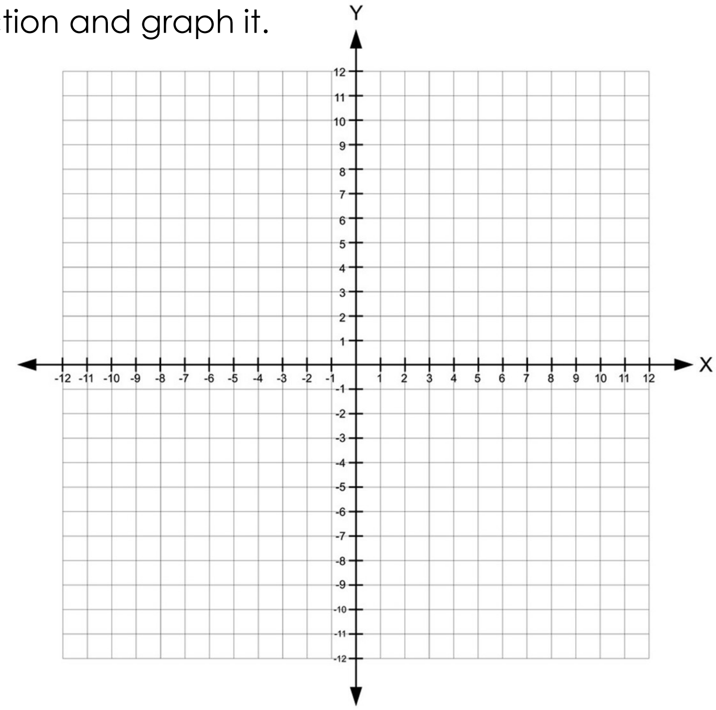
VA: x-int:

HA: y-int:

SA:

Ex 3: Identify the key characteristics of the function and graph it.

$$f(x) = \frac{2x^2 - 8}{x^2 - x - 6}$$




Hole(s):

VA: x-int:

HA: y-int:

SA:

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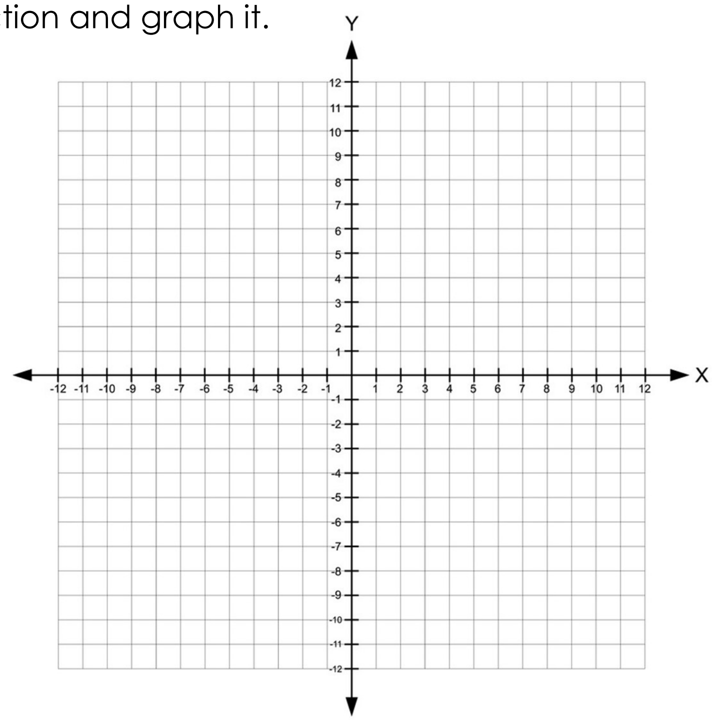
Ex 4: Identify the key characteristics of the function and graph it.

$$f(x) = \frac{3x}{x^2 - 9}$$




Hole(s):

VA: x-int:

HA: y-int:

SA:

Check your own understanding:

| |
|---|
| 1. Circle each function that has a horizontal asymptote at $y = 0$.<br>$f(x) = \frac{x+1}{x^2-9}$ $f(x) = \frac{2x+2}{x-3}$ $f(x) = \frac{x^2-4}{2x}$ $f(x) = \frac{3x}{2x^2-6x}$ $f(x) = \frac{x}{x^2-1}$ |
| 2. Circle each function that has a vertical asymptote at $x = 3$.<br>$f(x) = \frac{x+1}{x^2-9}$ $f(x) = \frac{2x+2}{x-3}$ $f(x) = \frac{x^2-4}{2x}$ $f(x) = \frac{3x}{2x^2-6x}$ $f(x) = \frac{x}{x^2-1}$ |
| 3. Circle the x-intercepts of the function $f(x) = \frac{x^2-4}{2x}$? -4 -2 0 2 4 |

Algebra 2 Unit 4B Name ______________________

## Graphing Rational Equations Practice

Identify the key characteristics of the function and graph it.

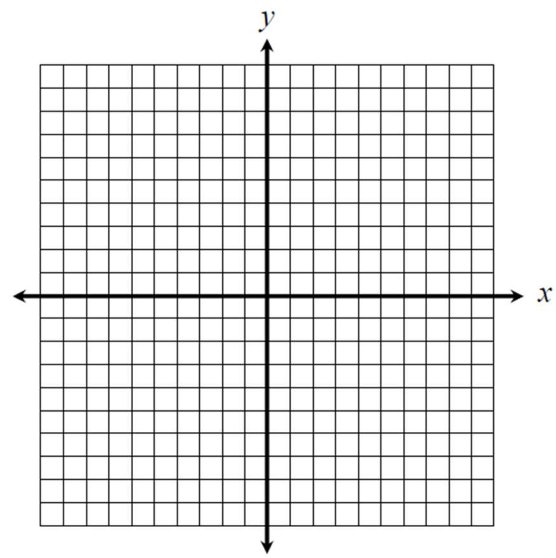
1. $f(x) = \frac{2x-6}{x-1}$

Hole(s):

VA: x-int:

HA: y-int:

SA:



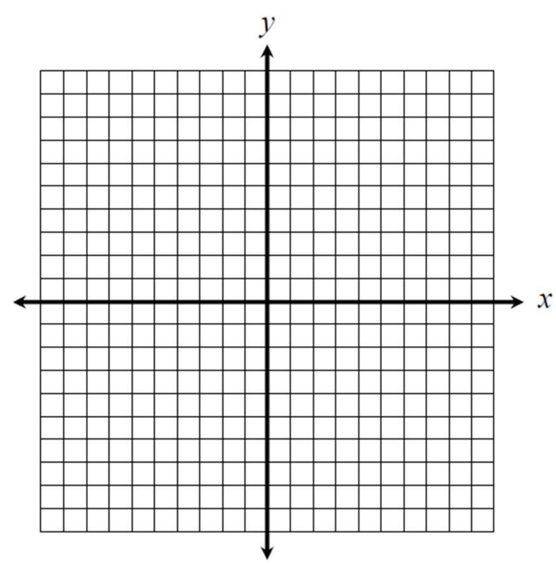

2. $f(x) = \frac{x+5}{x^2+9x+20}$

Hole(s):

VA: x-int:

HA: y-int:

SA:



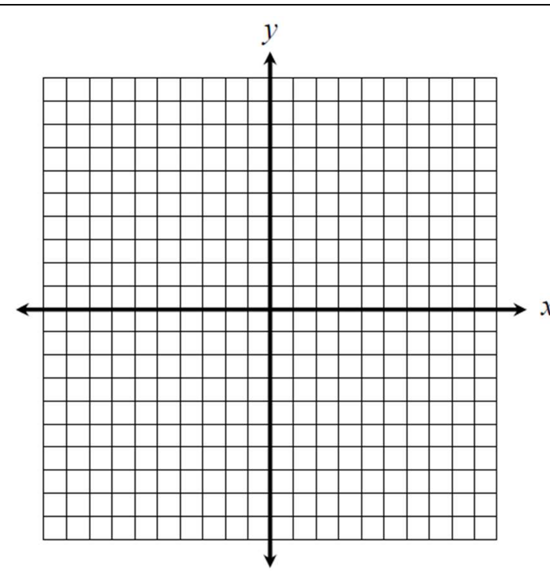

3. $f(x) = \frac{-x^2+7x-12}{x^2-3x}$

Hole(s):

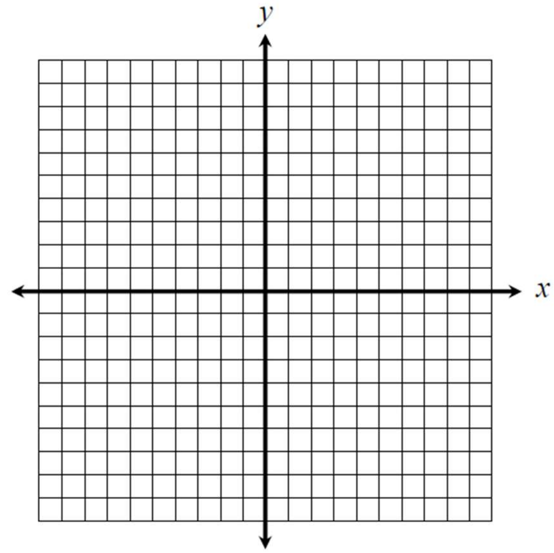
VA: x-int:

HA: y-int:

SA:



4. $f(x) = \frac{x-6}{x^2-6x}$



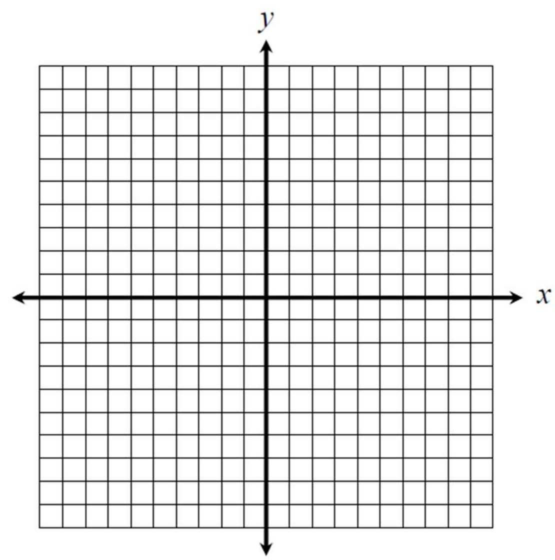

Hole(s):

VA: x-int:

HA: y-int:

SA:

5. $f(x) = \frac{x^2+3x-28}{x^2+12x+35}$




Hole(s):

VA: x-int:

HA: y-int:

SA:

Fill in the blank using words in the word bank.

| | | | | |
|---|---|---|---|---|
| DEGREES | DENOMINATOR | COEFFICIENTS | FACTOR | |
| ELIMINATE | MULTIPLY | NUMERATOR | X | Y |

6. The first step in graphing a rational function is to ______________ the numerator and denominator and simplify the equation.

7. When the equation is simplified, factors that _______________ create holes in the graph.

8. Determine the equation of the vertical asymptote by setting the _____________________ equal to zero.

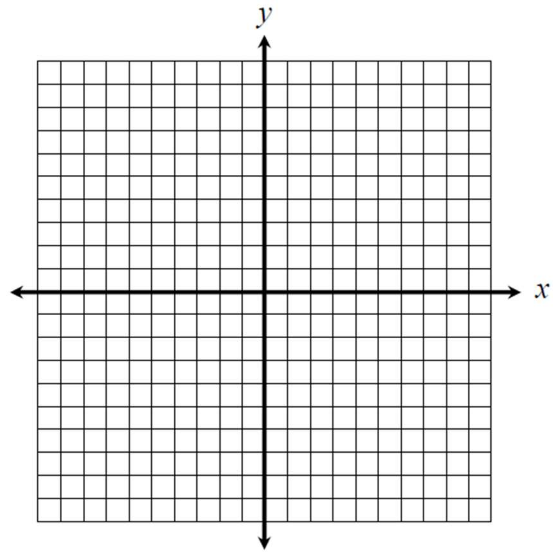
9. Determine the horizontal asymptote by comparing the ______________ of the numerator and denominator.

10. Find the x-intercepts of a rational function by setting the _____________________ equal to zero.

11. Determine the y-intercept of the function by substituting zero for _______.

## More Graphing Rational Functions

Ex 1: Graph $f(x) = \frac{x^3+3x^2}{x^2+2x-3}$




Hole(s):

VA: x-int:

HA: y-int:

SA:

Finding Slant Asymptote: $y = mx + b$

Divide the numerator by the denominator. The depressed polynomial is the "$mx + b$" of the slant asymptote – ignore the remainder!

Ex 2: Determine the equations of the vertical asymptote(s) and horizontal or slant asymptotes for each function.

| | VERTICAL ASYMPTOTE(S) | HORIZONTAL / SLANT |
|---|---|---|
| a. $f(x) = \frac{x+3}{2x}$ | | |
| b. $f(x) = \frac{x^2-5x+6}{x-1}$ | | |
| c. $f(x) = \frac{x-4}{x^2-x-6}$ | | |
| d. $f(x) = \frac{x^2-4}{x+3}$ | | |

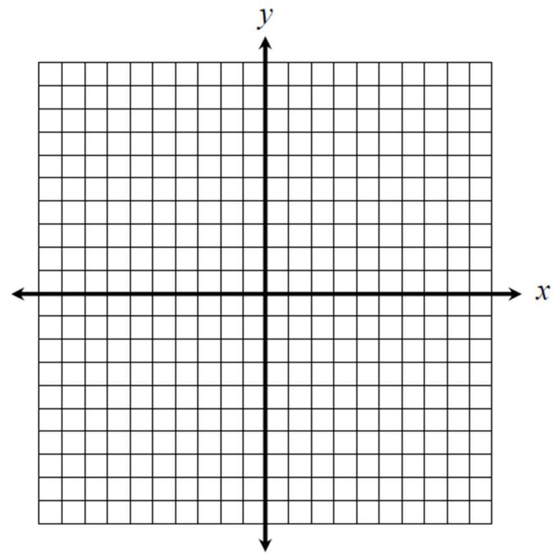
*Which of the functions in example 2 have holes? How do you know?

 Name ______________________________

Graphing Rational Equations Practice 2

Identify the key characteristics of the function and graph it.

1. $f(x) = \frac{x-1}{2x-4}$



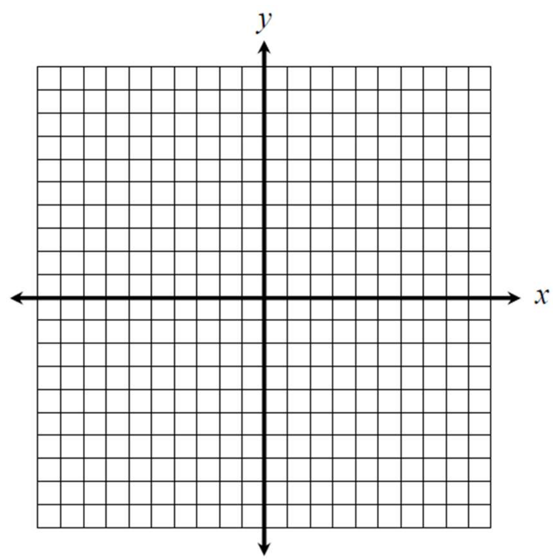

Hole(s):

VA: x-int:

HA: y-int:

SA:

2. $f(x) = \frac{x^3-9x}{x^2-4x}$



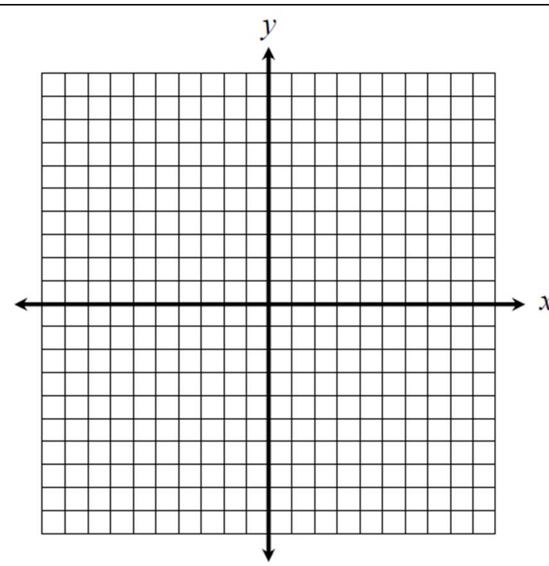

Hole(s):

VA: x-int:

HA: y-int:

SA:

3. $f(x) = \frac{4x-4}{x^2+x-6}$



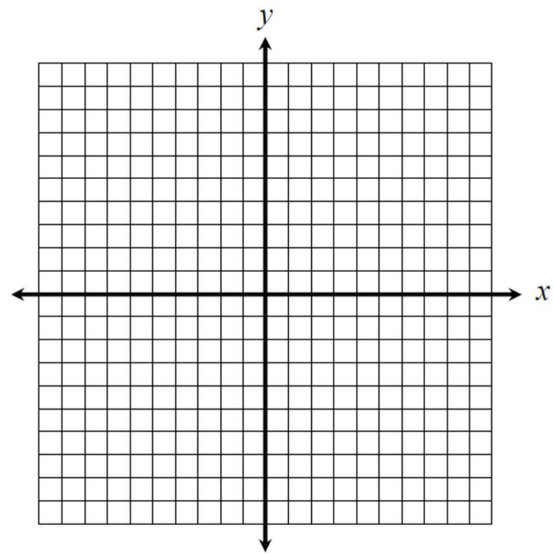

Hole(s):

VA: x-int:

HA: y-int:

SA:

4. $f(x) = \frac{x^2-4x+3}{x-4}$



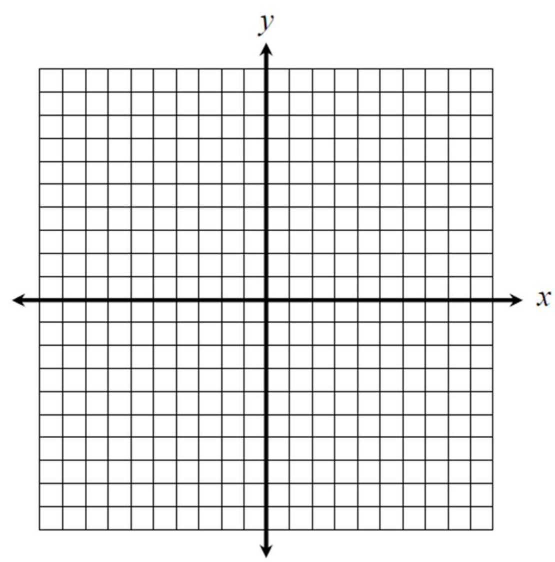

Hole(s):

VA: x-int:

HA: y-int:

SA:

5. $f(x) = \frac{x^2-7x+12}{x^2-5x+6}$




Hole(s):

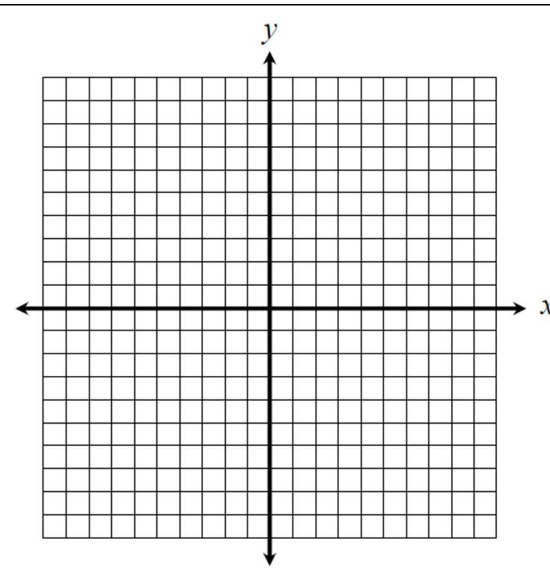
VA: x-int:

HA: y-int:

SA:

Domain: Range:

6. $f(x) = \frac{x^2-2x}{x-1}$




Hole(s):

VA: x-int:

HA: y-int:

SA:

# RATIONAL Functions

## EQUATION FORM:

## X-INTERCEPTS:

## VERTICAL ASYMPTOTES:

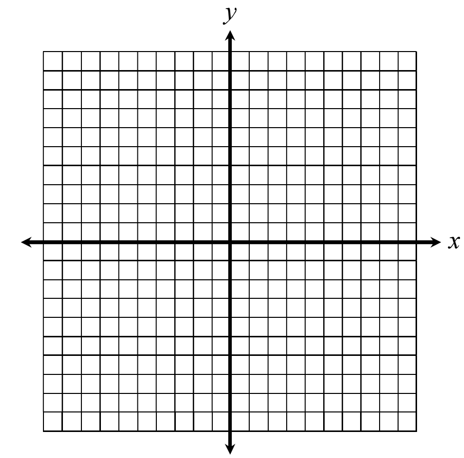
## HORIZONTAL ASYMPTOTES:

- If degree of $p$ > degree of $q$:
- If degree of $p$ < degree of $q$:
- If degree of $p$ = degree of $q$:

## HOLES:

## EXAMPLES

1. $f(x)=\dfrac{x^2-9}{x+2}$

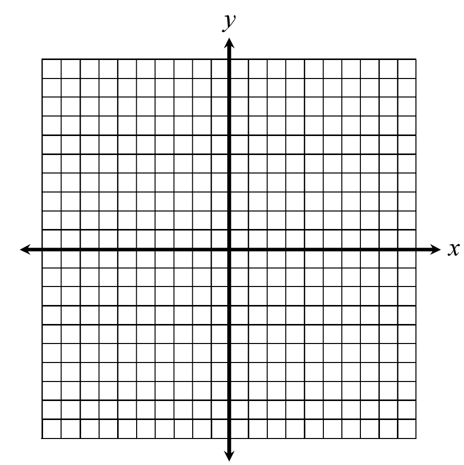



**x-int:** ____________________ **Holes:** ____________________

**Vertical Asymptote:** ___________ **D:** ____________________

**Horizontal Asymptote:** __________ **R:** ____________________

2. $f(x)=\dfrac{x^2-4x-5}{x^2-1}$




**x-int:** ____________________ **Holes:** ____________________

**Vertical Asymptote:** ___________ **D:** ____________________

**Horizontal Asymptote:** __________ **R:** ____________________

**Name:** ______________________________

**Date:** ____________________ **Bell:** ______

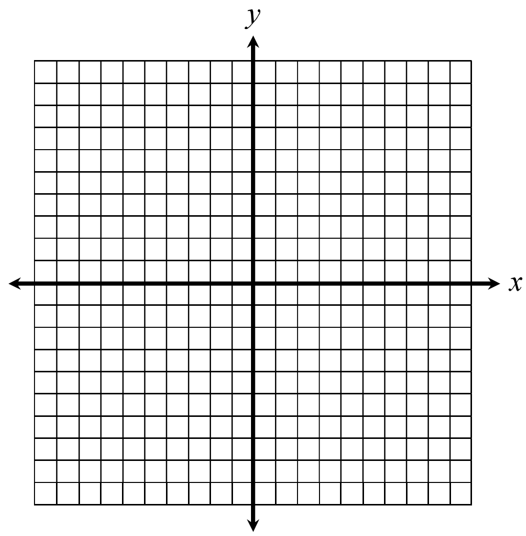
Algebra 2 Unit 4B

Graphing Rational Equations Practice

**** This is a 2-page document! ****

**Graph each function. Identify the domain, range, asymptotes, and holes.**

**1.** $f(x) = \dfrac{x+3}{x-2}$




***x*-int:** ____________________

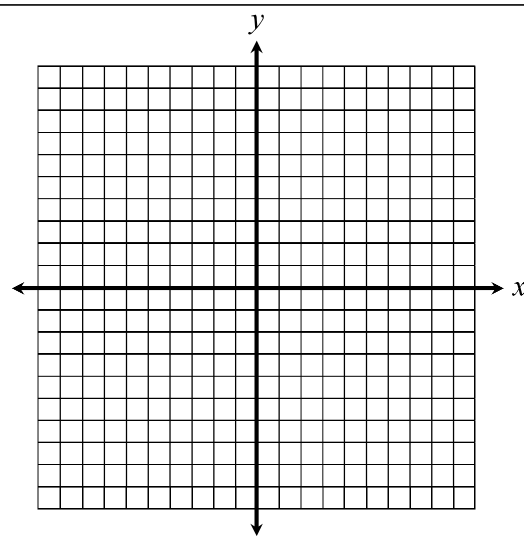
**VA:** ____________________

**HA:** ____________________

**Hole:** ____________________

**y-int:** ____________________

**2.** $f(x) = \dfrac{4x-24}{2x-4}$




***x*-int:** ____________________

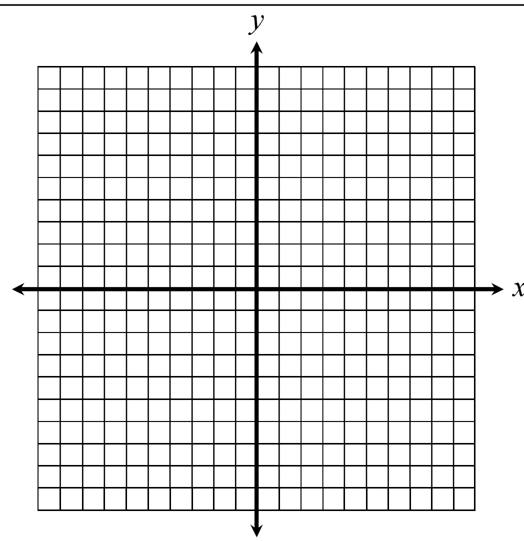
**VA:** ____________________

**HA:** ____________________

**Hole:** ____________________

**y-int:** ____________________

**3.** $f(x) = \dfrac{x^2+7x+10}{x+5}$




***x*-int:** ____________________

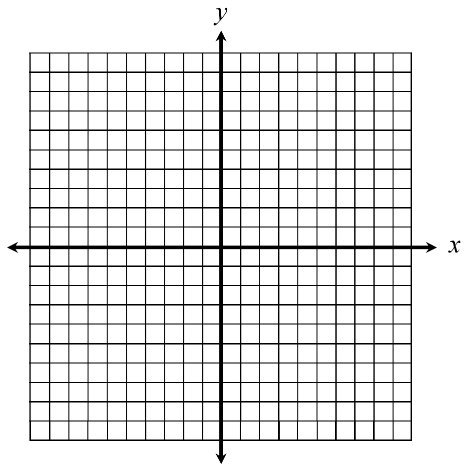
**VA:** ____________________

**HA:** ____________________

**Hole:** ____________________

**y-int:** ____________________

**4.** $f(x) = \dfrac{x+3}{x^2-9}$




***x*-int:** ______________________

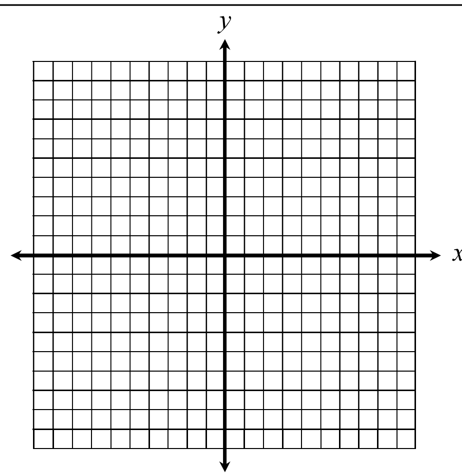
**VA:** ______________________

**HA:** ______________________

**Hole:** ______________________

**y-int:** ______________________

**5.** $f(x) = \dfrac{x^2+4x}{x^2+3x-4}$




***x*-int:** ______________________

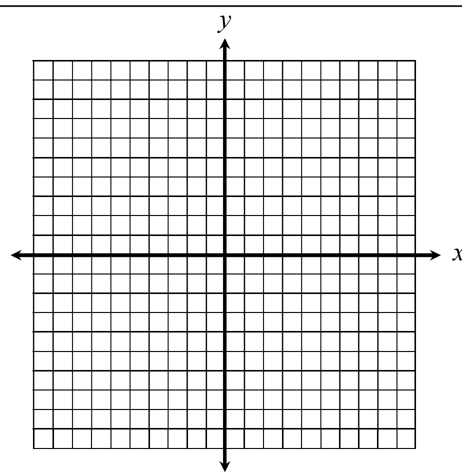
**VA:** ______________________

**HA:** ______________________

**Hole:** ______________________

**y-int:** ______________________

**6.** $f(x) = \dfrac{x^2-x-6}{x}$




***x*-int:** ______________________

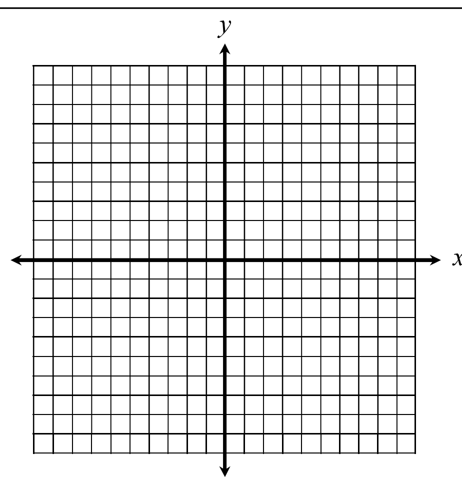
**VA:** ______________________

**HA:** ______________________

**Hole:** ______________________

**y-int:** ______________________

**7.** $f(x) = \dfrac{3x^2-5x-2}{x^2-3x+2}$




***x*-int:** ______________________

**VA:** ______________________

**HA:** ______________________

**Hole:** ______________________

**y-int:** ______________________

| Determine whether the statement is TRUE or FALSE.<br>If it is FALSE, choose the correct word to place in the blank to make it TRUE. | |
|---|---|
| Word Bank | |
| $x$  $y$  coefficient  factor  zero  one  two  less than  greater than | |
| 1. To find the y-intercept of a function, you must substitute zero for <u>y</u>. | TRUE / FALSE<br>________________________ |
| 2. The reason there are vertical asymptotes in the graph of a rational function is because the denominator of a fraction may not equal <u>zero</u>. | TRUE / FALSE<br>________________________ |
| 3. To determine whether a rational function has holes, you must factor both the numerator and denominator, to see if there are any <u>common factors</u>. | TRUE / FALSE<br>________________________ |
| 4. The equation of a horizontal line contains only one variable, and that variable is <u>x</u>. | TRUE / FALSE<br>________________________ |
| 5. A slant asymptote occurs when the degree of the numerator is <u>less than</u> the degree of the denominator by 1. | TRUE / FALSE<br>________________________ |

Multiple Choice: Write the letter of the correct response in the space provided.

______ 6. Which describes the horizontal asymptote of the function $f(x) = \frac{x+4}{4x^2-5}$ ?

A. $y = 0$  B. $y = 1$  C. $y = 2$  D. $y = \frac{1}{2}$

______ 7. Which describes the slant asymptote of the function $f(x) = \frac{x^2+7x+1}{x-1}$ ?

A. $y = x + 3$  B. $y = x + 8$  C. $y = x + 6$  D. $y = x + 1$

______ 8. Which describes one of the vertical asymptotes of the function $f(x) = \frac{6}{x^2-9}$ ?

A. $x = 0$  B. $x = 1$  C. $x = 2$  D. $x = 3$

9. Given the function $f(x) = \frac{x^2+4x+3}{x+1}$, what is the y-coordinate of the hole?

10. What values of $x$ are excluded from the domain of $f(x) = \frac{x^2-5x-6}{x^2-1}$ ?

11. Given the function $f(x) = \frac{x^2-5x-6}{x^2-1}$...

a. what is the x-intercept of the function?

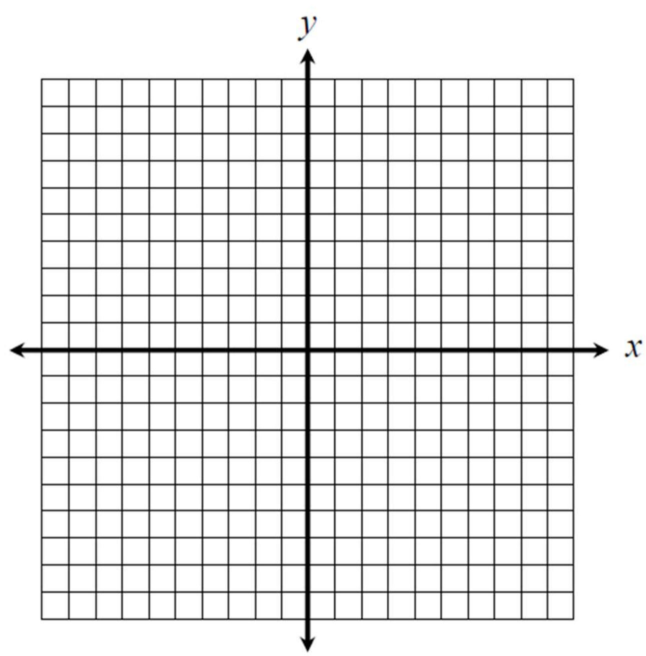
b. what is the y-intercept of the function?

12. Matching: Match the function and the correct asymptote.

| Function | Asymptote |
|---|---|
| $f(x) = \dfrac{2x-1}{x}$ | $y = 0$ |
| $f(x) = \dfrac{x}{2x^2-2}$ | $y = \dfrac{1}{2}$ |
| $f(x) = \dfrac{3x}{x-2}$ | $y = x + 2$ |
| $f(x) = \dfrac{x+1}{2x-1}$ | $x = 2$ |
| $f(x) = \dfrac{x^2-4x+3}{x+2}$ | $x = 0$ |

13. Identify the characteristics of the function and graph it.

$$f(x) = \frac{x^2-2x-15}{x^2-9}$$




Hole(s):

VA: x-int:

HA: y-int:

SA: