Unit 4A – Curve Sketching

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- Graphing f’ from f
- Graphing f’’ from f and f’
- Graphing f from f’ and f’’
- Interpreting Graphs using characteristics of the first and second derivative

Quiz is ____________________

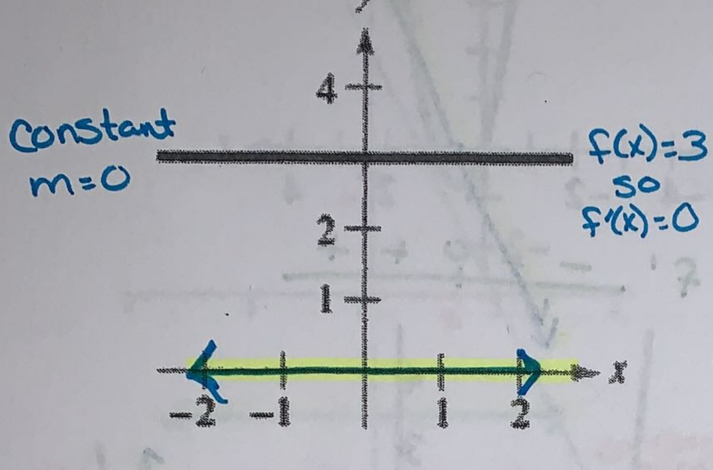
Name: Bonanni

F19

Curve Sketching - Graphing f' from f

The graph of f is given below. Sketch a possible graph of f'

1.



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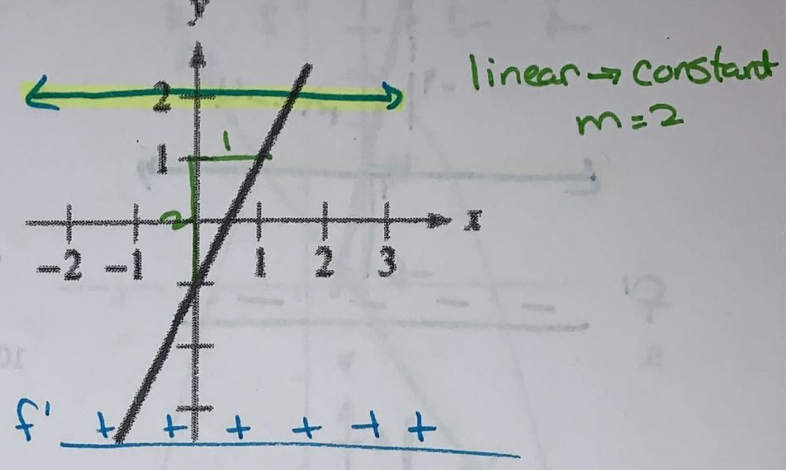

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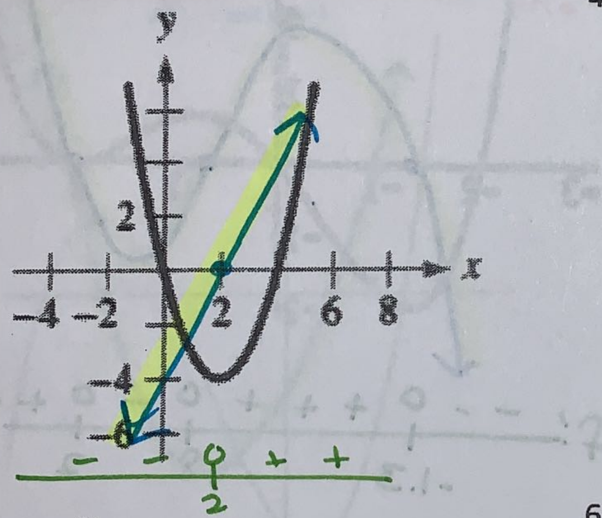

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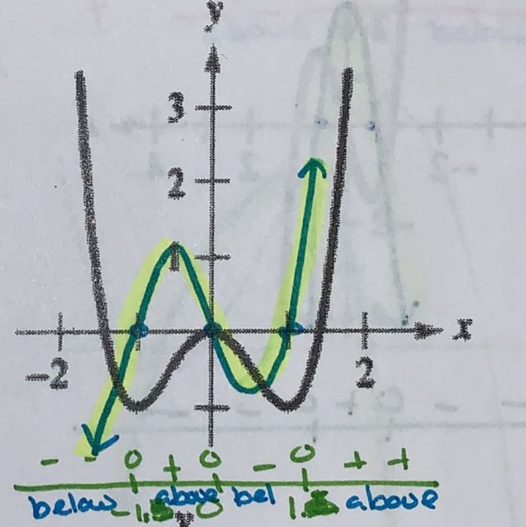

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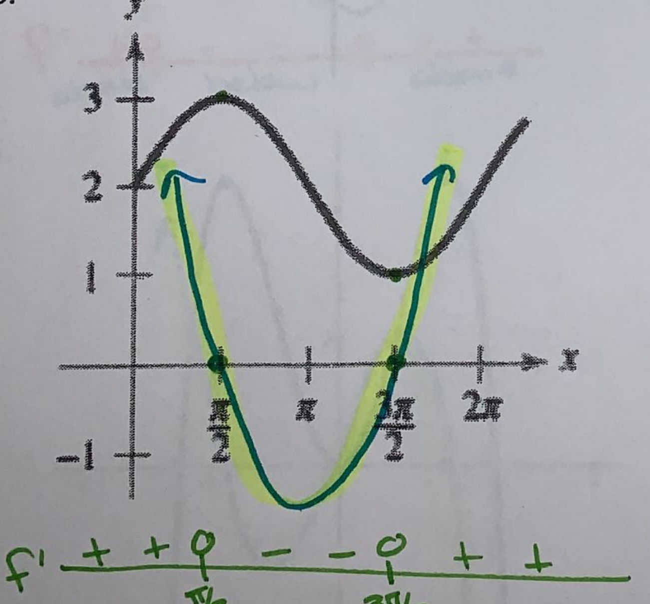

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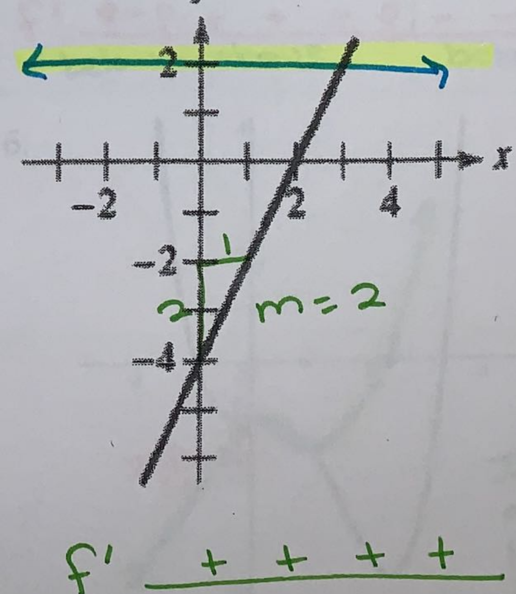

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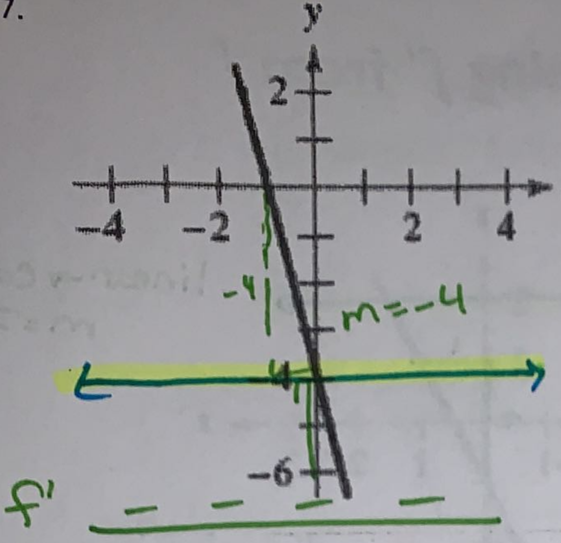
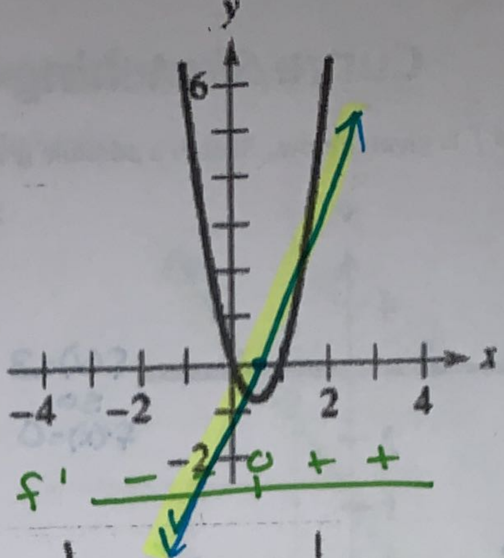


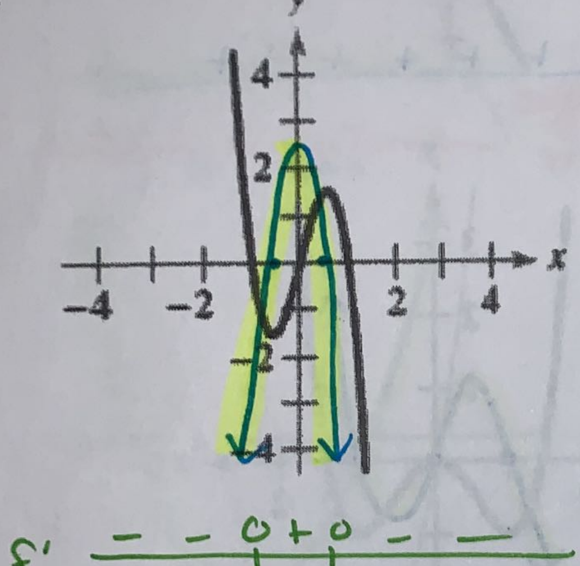
y
x
2
-4
-2
2
4
-4
-6
m=-4
-4
f'
- - - -

8.



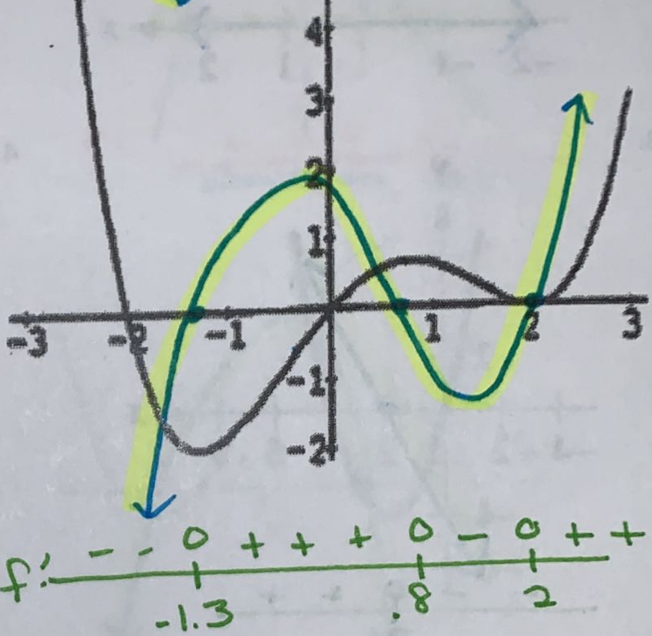
y
x
6
-4
-2
2
4
-2
f'
- 0 + +

9.



y
x
4
2
-4
-2
2
4
-2
-4
f'
- - 0 + 0 - -

10.

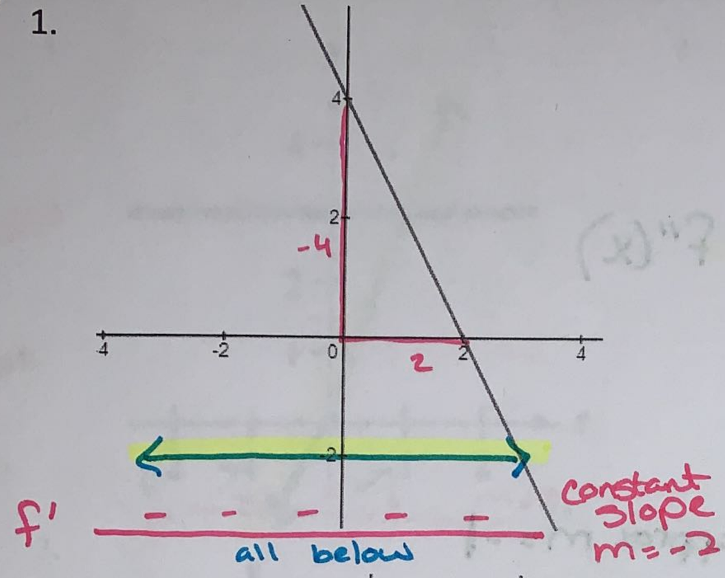


4
3
2
1
-1
-2
-3
-2
-1
1
2
3
f'
- - 0 + + + 0 - 0 + +
-1.3
.8
2

Graphs of Derivatives

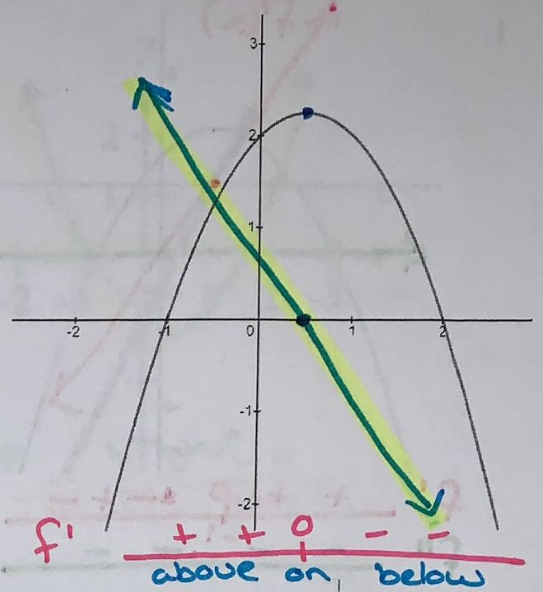
Sketch a graph of the derivative function of each function.

1.

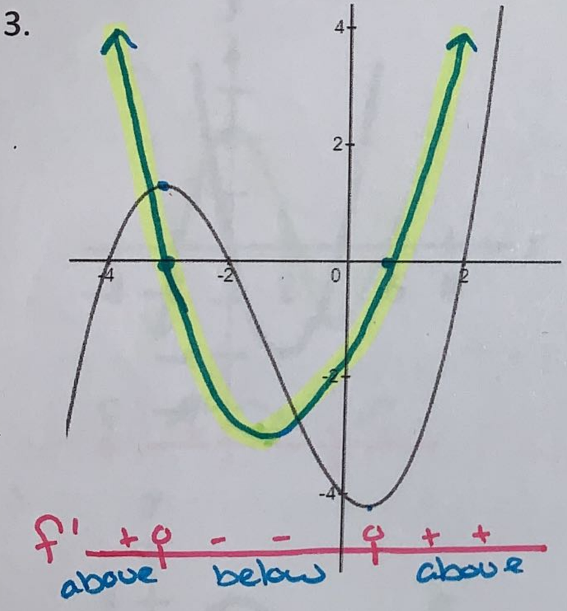


Constant slope m=-2

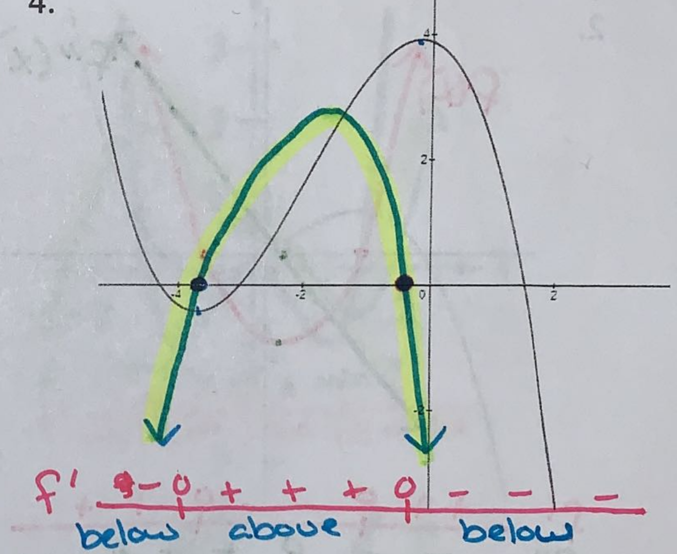
2.



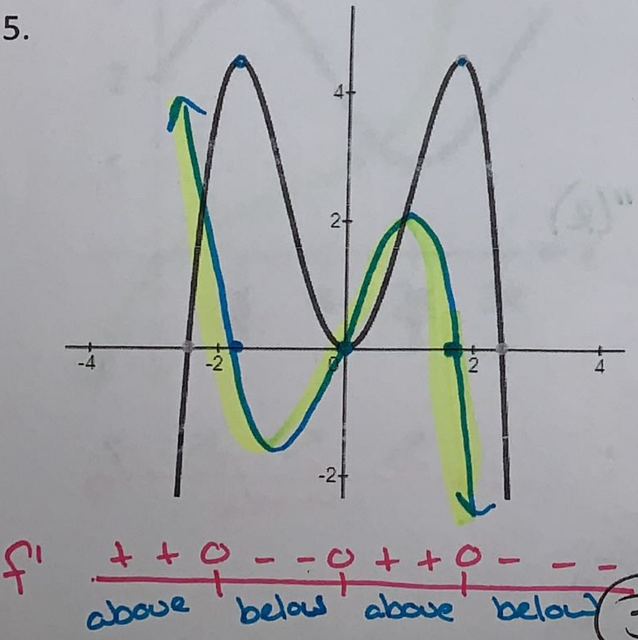
3.



4.



5.



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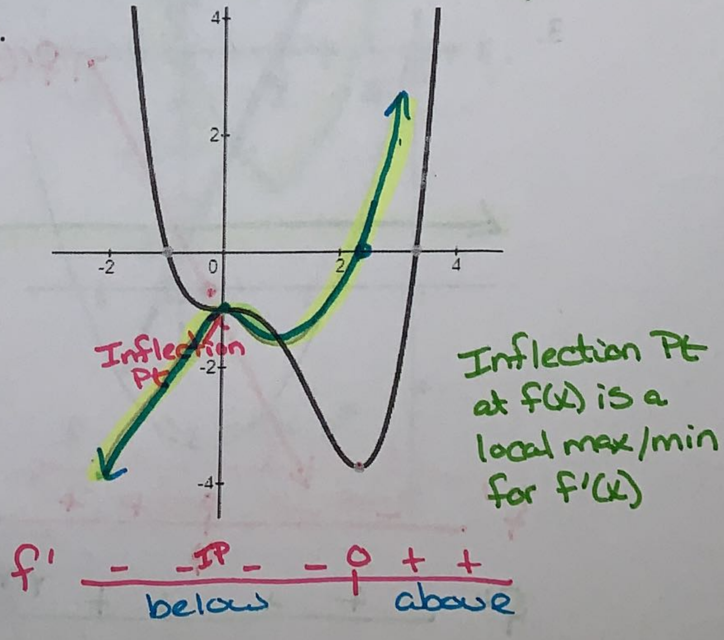
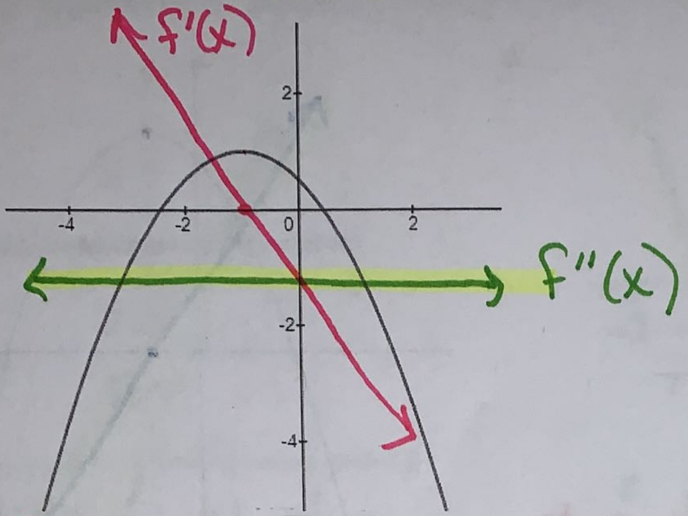


Inflection Pt at f(x) is a local max/min for f'(x)

Graph f" from f(x)

Sketch a graph of the second derivative function given each function.

1.

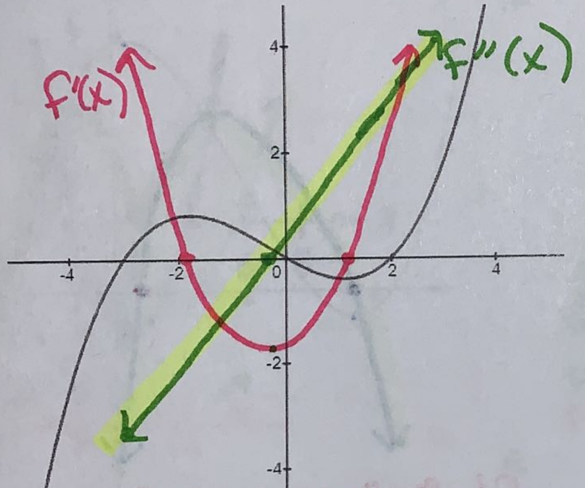


f' + + 0 − − −

f'' − − −1 − −

approx m= -1

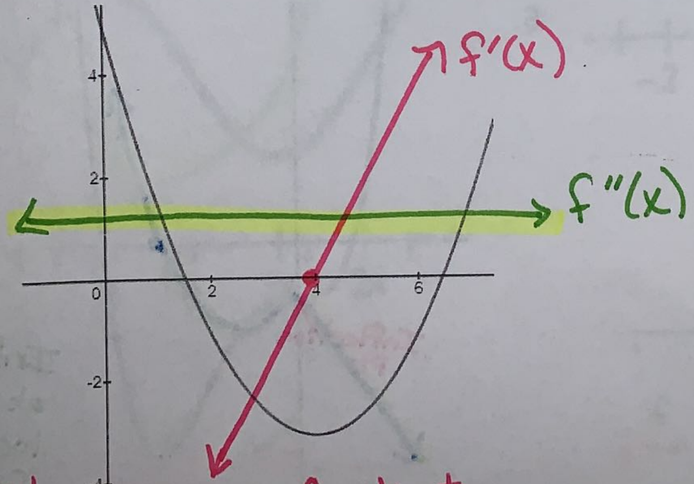
2.



f' + + 0 − − 0 + +

f'' − − 0 + + +

3.



f' − − 0 + +
4

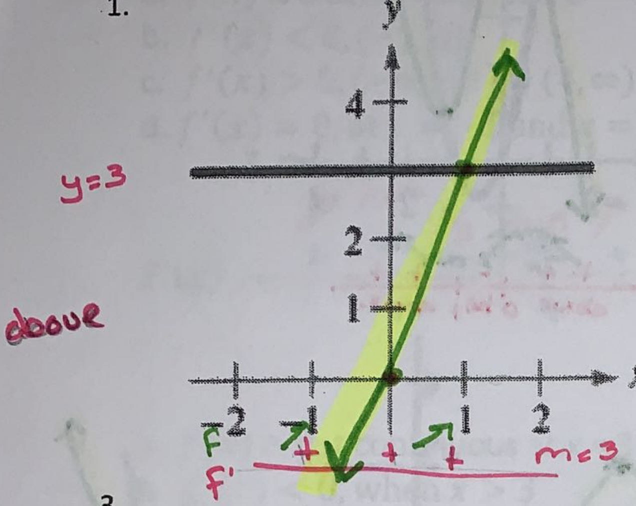
f'' + + + + m≈1

④

Curve Sketching - Graphing f from f'

The graph of f' is given below. Sketch a possible graph of f

1.



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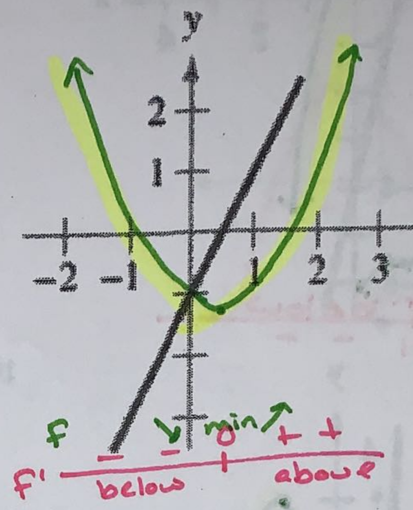
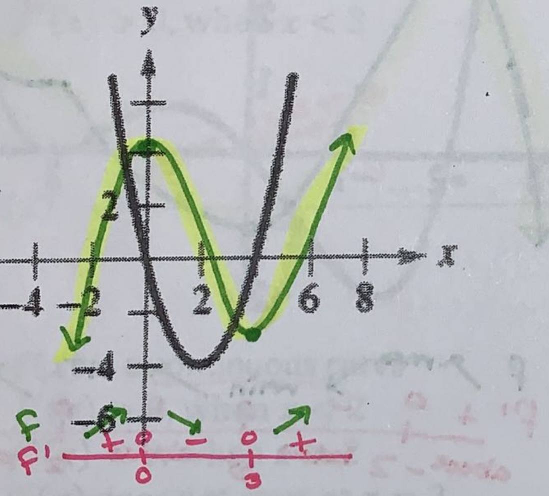


3.



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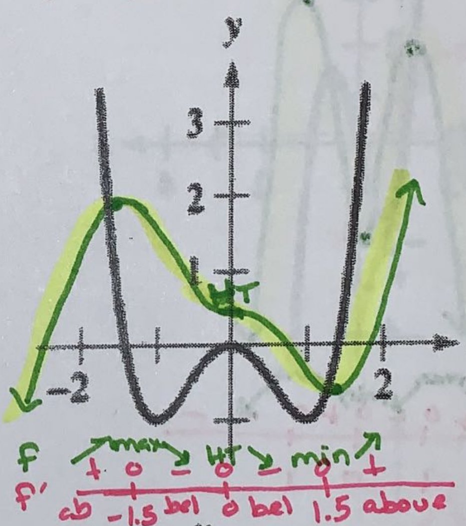

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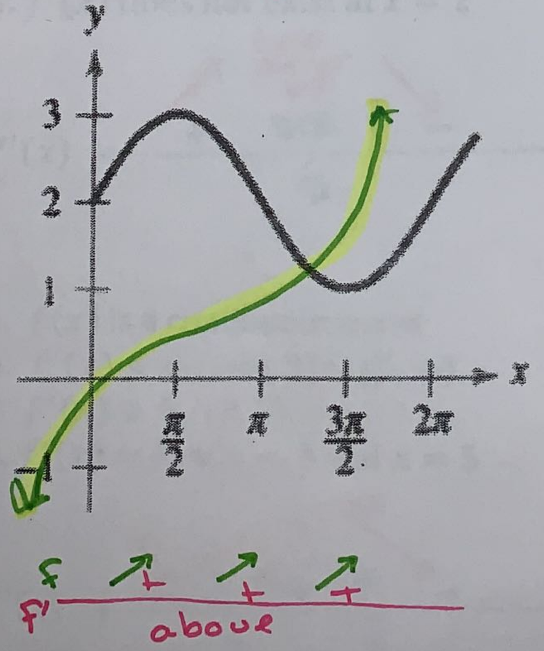

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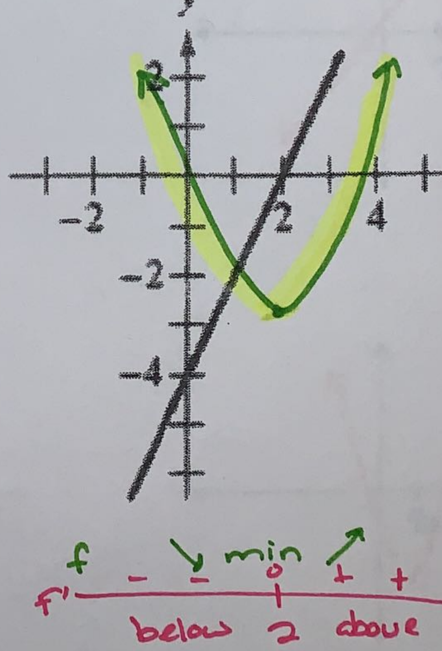

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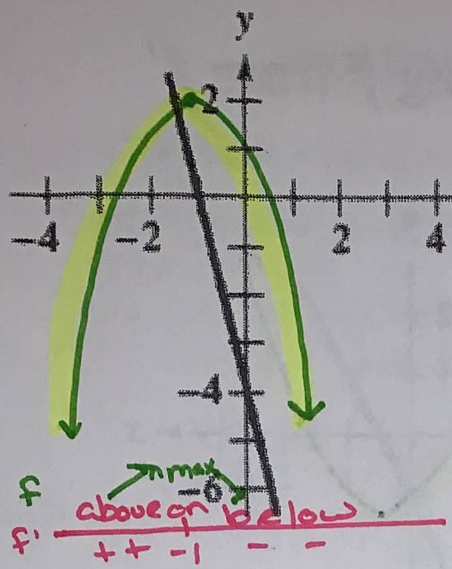
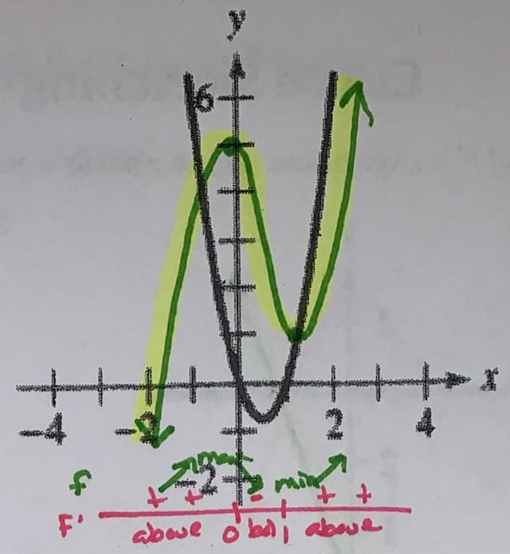
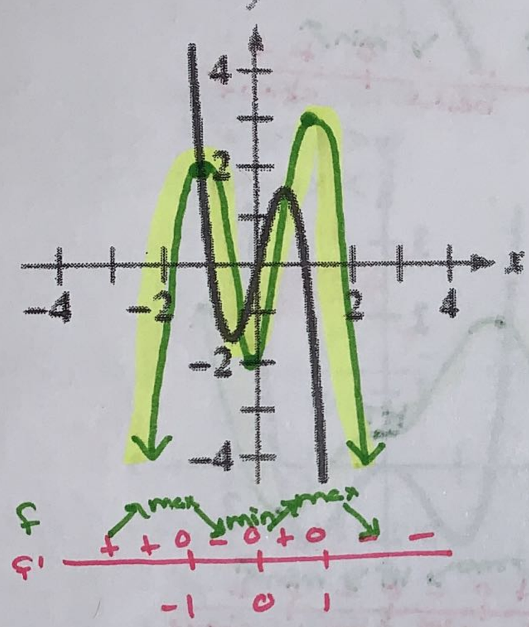
7.


y
x
2
-2
-4
2
4
-4
-6
f
max
f'
above or below
+ + -1 - -

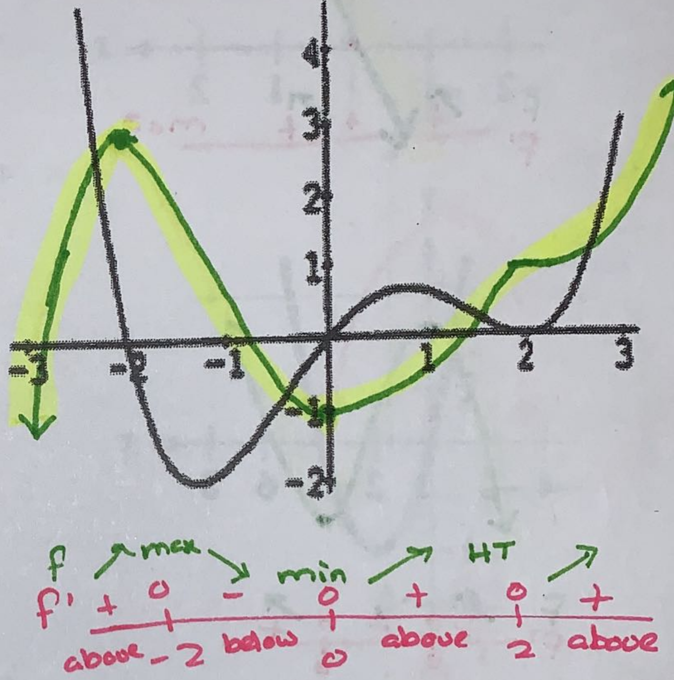
8.


y
x
6
-4
-2
2
4
f
max
min
+ + - + +
f'
above 0 bel 1 above

9.


y
x
4
2
-2
-4
-4
-2
2
4
f
max
min
max
f'
+ + 0 - 0 + 0 - -
-1 0 1

10.


4
3
2
1
-2
-3
-2
-1
1
2
3
f
max
min
HT
f'
+ 0 - 0 + 0 +
above -2 below 0 above 2 above

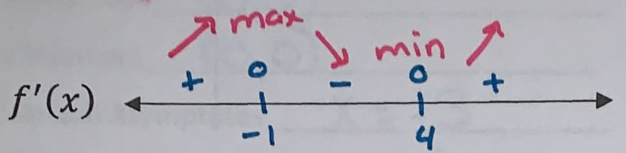
3

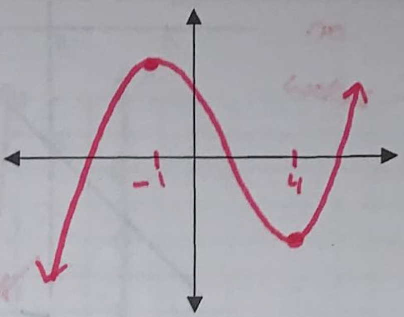
⑥

First Derivative Test & Critical Points

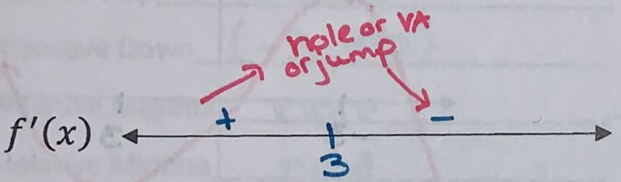
Draw a possible graph of f(x) given the information below.

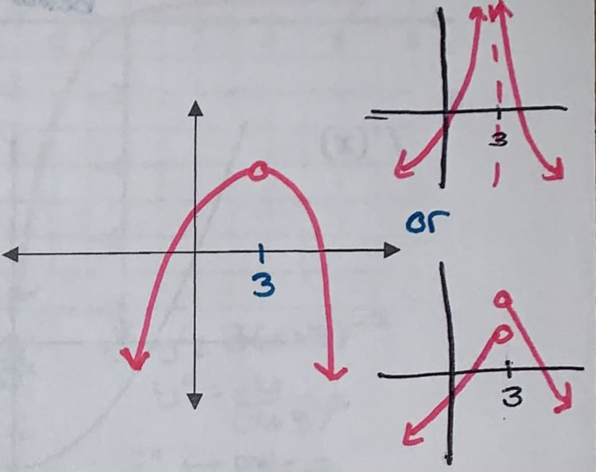
1.
   a. $f(x)$ is a continuous curve
   b. $f'(x) < 0, (-1, 4)$
   c. $f'(x) > 0, (-\infty, -1) \cup (4, \infty)$
   d. $f'(x) = 0$, at $x = -1$ and $x = 4$



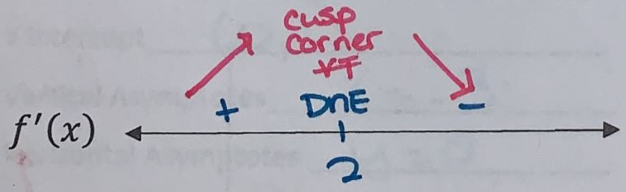


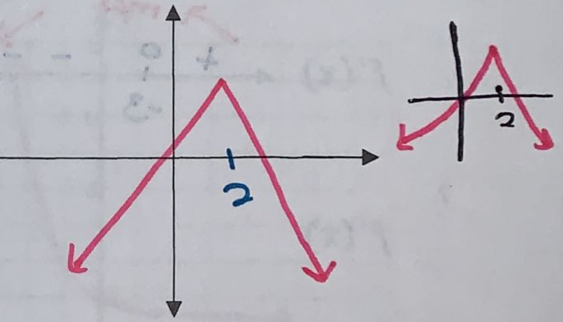
2.
   a. $f(x)$ is not continuous at x = 3
   b. $f'(x) < 0$, when $x > 3$
   c. $f'(x) > 0$, when $x < 3$



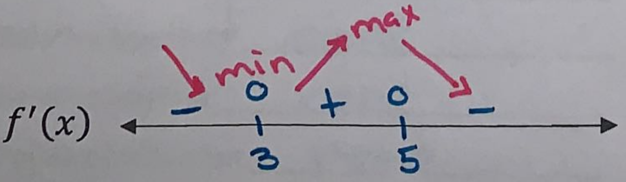


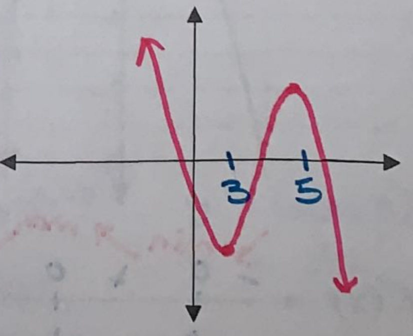
3.
   a. $f(x)$ is a continuous curve
   b. $f'(x) > 0$, when $x < 2$
   c. $f'(x) < 0$, when $x > 2$
   d. $f'(x)$ does not exist at $x = 2$



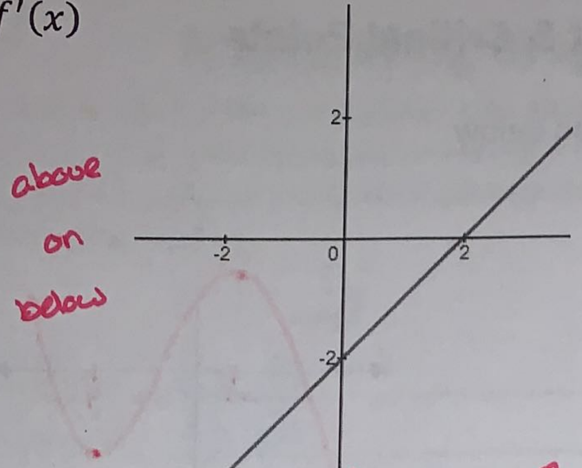


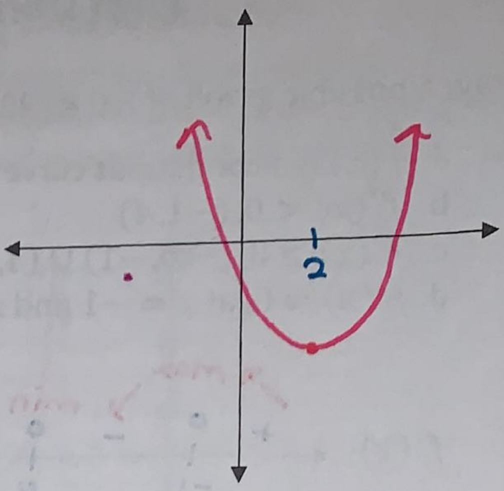
4.
   a. $f(x)$ is a continuous curve
   b. $f'(x) < 0, (-\infty, 3) \cup (5, \infty)$
   c. $f'(x) > 0$, $(3, 5)$
   d. $f'(x) = 0$, at $x = 3$ and $x = 5$



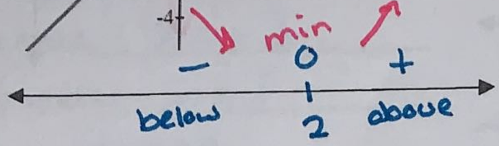


5. $f'(x)$

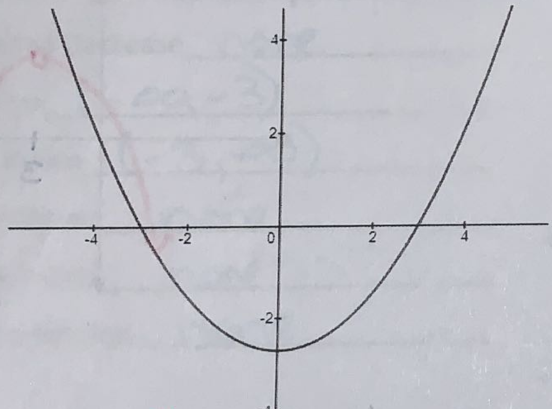


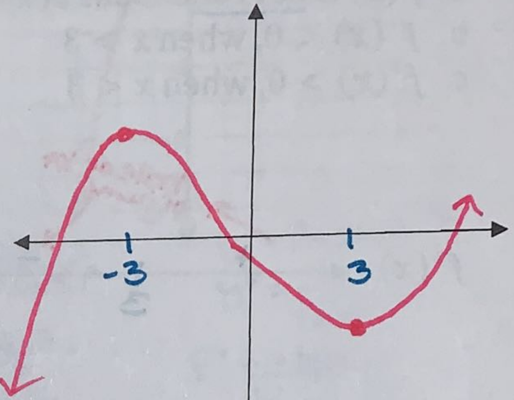


$f'(x)$

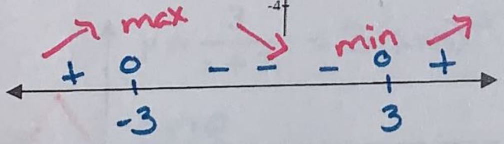


6. $f'(x)$

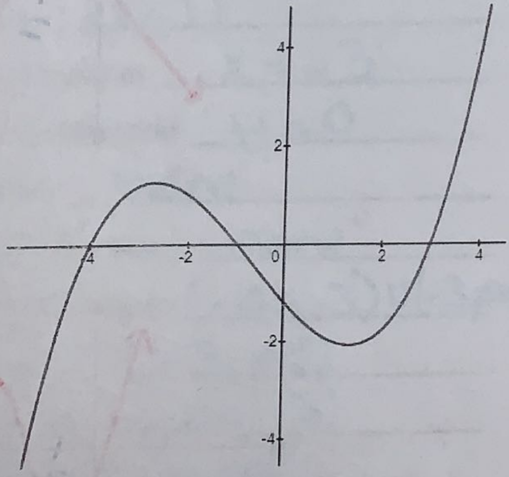


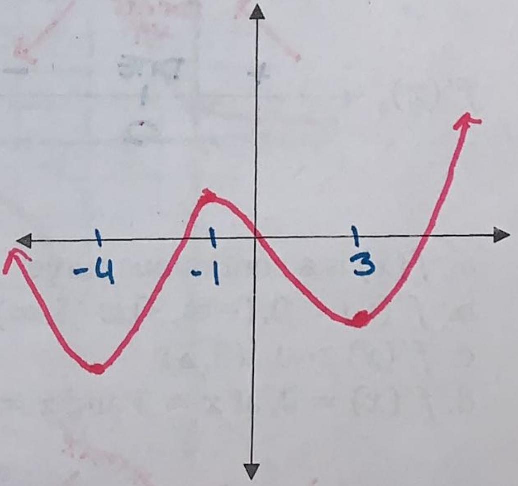


$f'(x)$

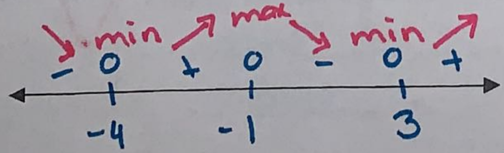


7. $f'(x)$





$f'(x)$



(8)

Interpreting Graphs Using Derivatives

For each problem, find the: x and y intercepts, asymptotes, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

1. $y = \frac{x}{x+3}$

$\frac{0}{1} = \frac{x}{x+3}$

$x = 0$

$y = \frac{0}{0+3}$

$y = 0$

X intercept (0,0)

Y intercept (0,0)

Vertical Asymptotes X = -3

Horizontal Asymptotes y = 1

Critical Points none

Interval(s) of Increase $(-\infty,-3) \cup (-3,\infty)$

Interval(s) of Decrease none

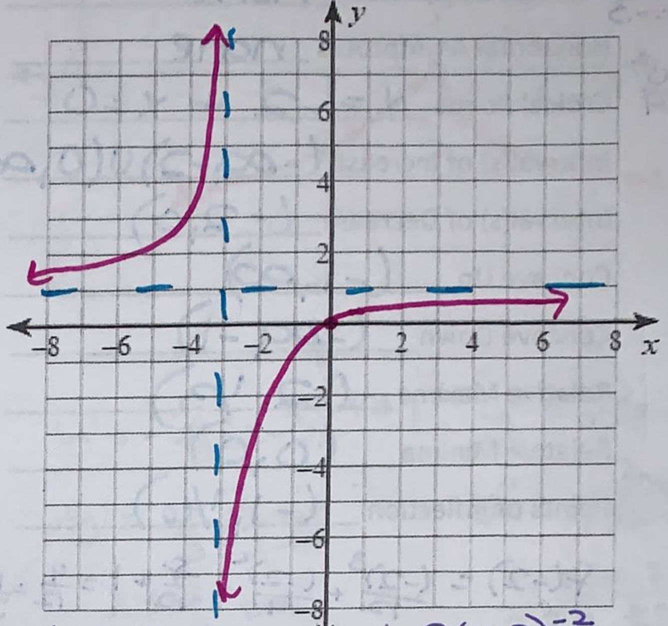
Concave Up $(-\infty,-3)$

Concave Down $(-3,\infty)$

Relative Maxima none

Relative Minima none

Points of Inflection none



$y' = \frac{(x+3)1 - x}{(x+3)^2}$

$y' = \frac{3}{(x+3)^2}$

f' + + DNE + +, -3, ↗ ↗

$y' = 3(x+3)^{-2}$

$y'' = \frac{-6}{(x+3)^3}$

f'' + + DNE − −, ∪ -3 ∩

2. $y = \frac{3}{x+3} = 3(x+3)^{-1}$

$\frac{0}{1} = \frac{3}{x+3}$

$3 \neq 0$

$y = \frac{3}{0+3}$

$y = 1$

X intercept none

Y intercept (0,1)

Vertical Asymptotes X = -3

Horizontal Asymptotes y = 0

Critical Points none

Interval(s) of Increase none

Interval(s) of Decrease $(-\infty,-3) \cup (-3,\infty)$

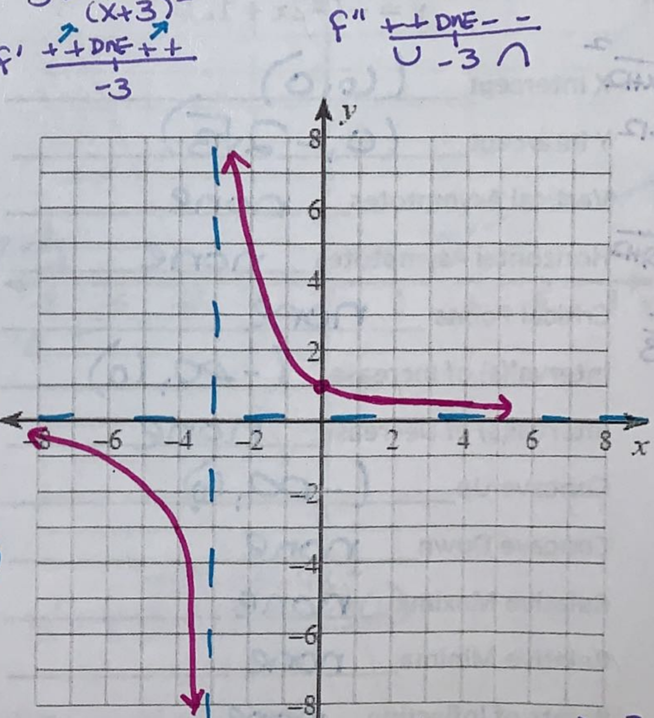
Concave Up $(-3,\infty)$

Concave Down $(-\infty,-3)$

Relative Maxima none

Relative Minima none

Points of Inflection none



$y' = -3(x+3)^{-2}$

$y' = \frac{-3}{(x+3)^2}$

y' − − DNE − −, ↘ -3 ↘

$y'' = 6(x+3)^{-3}$

$y'' = \frac{6}{(x+3)^3}$

− − und + +, -3

(9)

3.

$y = \frac{x^3}{12} + \frac{x^2}{4} = \frac{1}{12}x^3 + \frac{1}{4}x^2$

$(\frac{1}{12}x^3 + \frac{1}{4}x^2)$ X intercept $(0,0)$ $(-3,0)$

$x^3 + 3x^2$ Y intercept $(0,0)$

$x^2(x+3)$

$=0$ $x=-3$

Vertical Asymptotes none

Horizontal Asymptotes none

$\frac{0^3}{12} + \frac{0^2}{4} = 0$

Critical Points $x = -2$ + $x = 0$

Interval(s) of Increase $(-\infty, -2) \cup (0, \infty)$

Interval(s) of Decrease $(-2, 0)$

Concave Up $(-1, \infty)$

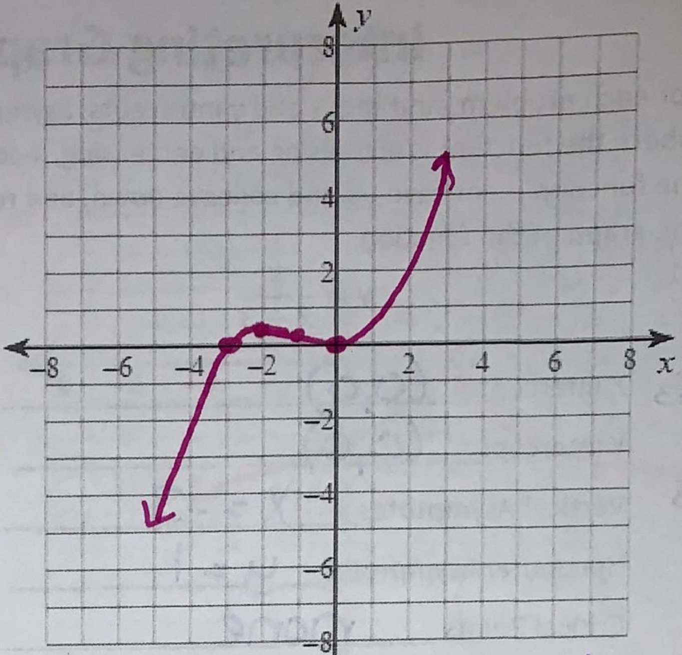
Concave Down $(-\infty, -1)$

Relative Maxima $(-2, 1/3)$

Relative Minima $(0, 0)$

Points of Inflection $(-1, 1/6)$

$f(-2) = \frac{(-2)^3}{12} + \frac{(-2)^2}{4} = \frac{-8}{12} + 1 = \frac{4}{12} = \frac{1}{3}$



$y' = \frac{3}{12}x^2 + \frac{2}{4}x$

$y' = \frac{1}{4}x^2 + \frac{1}{2}x$

$4(0 = \frac{1}{4}x^2 + \frac{1}{2}x)$

$0 = x^2 + 2x$

$0 = x(x+2)$

$x = 0$ $x = -2$

max min

$+ \; 0 \; - \; 0 \; +$

$-2 \quad 0$

$y'' = \frac{2}{4}x + \frac{1}{2}$

$y'' = \frac{1}{2}x + \frac{1}{2}$

$0 = \frac{1}{2}x + \frac{1}{2}$

$x = -1$

y'' $- \; - \; 0 \; + \; +$

-1

POI

$f(-1) = \frac{(-1)^3}{12} + \frac{(-1)^2}{4} = -\frac{1}{12} + \frac{1}{4}$

$= 1/6$

4.

$y = -(-2x + 12)^{\frac{1}{2}}$

$0^2 = -\sqrt{-2x+12}^{\,2}$ X intercept $(6,0)$

$0 = -2x + 12$ Y intercept $(0, -2\sqrt{3})$

$2x = 12$

$x = 6$ Vertical Asymptotes none

$y = -\sqrt{-2(0)+12}$ Horizontal Asymptotes none

$y = -\sqrt{12}$ Critical Points none

$y = -2\sqrt{3}$

Interval(s) of Increase $(-\infty, 6)$

Interval(s) of Decrease none

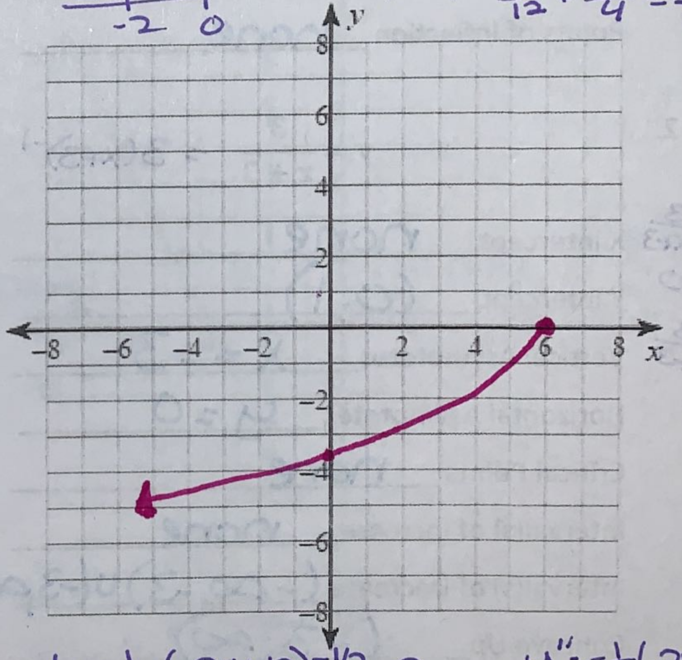
Concave Up $(-\infty, 6)$

Concave Down none

Relative Maxima none

Relative Minima none

Points of Inflection none



$y' = -\frac{1}{2}(-2x+12)^{-1/2} \cdot -2$

$y' = \frac{1}{\sqrt{-2x+12}}$ or $(-2x+12)^{-1/2}$

$0 = \frac{1}{\sqrt{-2x+12}}$

$-2x + 12 > 0$

$x < 6$

$+ \; +$ DNE DNE

$\nearrow \quad 6$

$y'' = -\frac{1}{2}(-2x+12)^{-3/2} \cdot -2$

$y'' = \frac{1}{(-2x+12)^{3/2}}$

$0 < (-2x+12)^{-3/2}$

$0 < -2x + 12$

$-2x + 12 > 0$

$x < 6$

$+ \; +$ DNE

$\cup \quad 6$

10

$(0)^{3/2} = \left(\frac{-}{-1}(x-3)^{2/3}\right)^{3/2}$

$0 = x-3$

$x = 3$

$y = -(0-3)^{2/3}$

$y = -\sqrt[3]{-3}^{2}$

$y = -2.1$

5. $y = -(x-3)^{\frac{2}{3}}$

X intercept (3,0)

Y intercept $(0, -\sqrt[3]{9})$ or (0,-2.1)

Vertical Asymptotes none

Horizontal Asymptotes none

Critical Points x = 3

Interval(s) of Increase $(-\infty, 3)$

Interval(s) of Decrease $(3, \infty)$

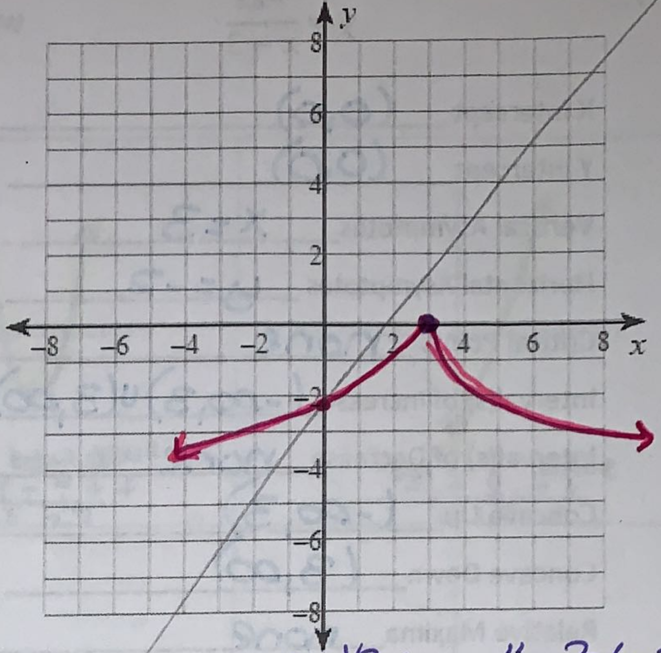
Concave Up $(-\infty, 3) \cup (3, \infty)$

Concave Down none

Relative Maxima (3,0)

Relative Minima none

Points of Inflection none



$y' = -\frac{2}{3}(x-3)^{-1/3} \cdot 1$

$y' = \frac{-2}{3\sqrt[3]{x-3}}$

$x \neq 3$

+ DNE −

↗ 3 ↘

$y'' = \frac{2}{9}(x-3)^{-4/3}$

$y'' = \frac{2}{9\sqrt[3]{x-3}^{4}}$

+ DNE +

∪ 3 ∪

6. $y = \frac{x^2}{4x+8}$

$\frac{0}{1} = \frac{x^2}{4x+8}$

$x^2 = 0$

$x = 0$

X intercept (0,0)

Y intercept (0,0)

Vertical Asymptotes x = -2

Horizontal Asymptotes none / Slant Asym $y = \frac{1}{4}x + \frac{1}{2}$

Critical Points

Interval(s) of Increase

Interval(s) of Decrease

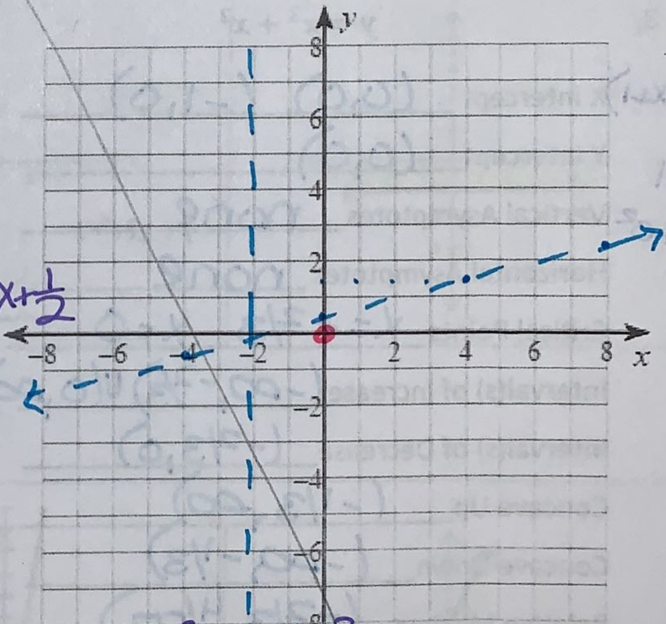
Concave Up

Concave Down

Relative Maxima

Relative Minima

Points of Inflection



$8x^2 + 16x - 4x^2$

$y' = \frac{(4x+8)(2x) - x^2(4)}{(4x+8)^2}$

$y' = \frac{4x^2 + 16x}{(4x+8)^2}$

$$\begin{array}{r} \frac{1}{4}x + \frac{1}{2} \\ 4x+8 \overline{) x^2 + 0x + 0} \\ \underline{-x^2 - 2x} \\ 2x + 0 \\ \underline{-2x - 4} \end{array}$$

$\frac{1x^2}{4x} = \frac{1}{4}x$

$\frac{2x}{4x} = \frac{1}{2}$

⑪

7. $y = \frac{-2x}{x-3}$

X intercept (0,0)

Y intercept (0,0)

Vertical Asymptotes x=3

Horizontal Asymptotes y=-2

Critical Points none

Interval(s) of Increase $(-\infty,3)\cup(3,\infty)$

Interval(s) of Decrease none

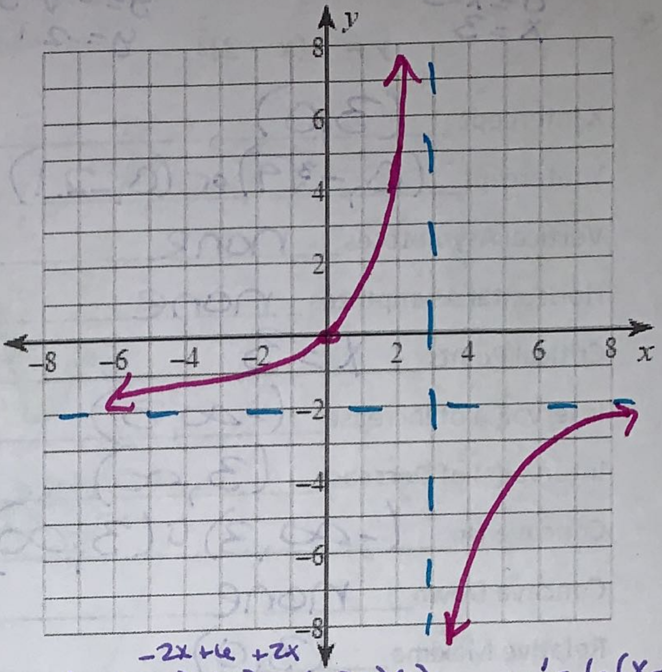
Concave Up $(-\infty,3)$

Concave Down $(3,\infty)$

Relative Maxima none

Relative Minima none

Points of Inflection none



$$y' = \frac{\overset{-2x+6}{(x-3)(-2)} - \overset{+2x}{(-2x)(1)}}{(x-3)^2}$$

$$y' = \frac{6}{(x-3)^2}$$

y' ++ DNE ++
↗ 3 ↗

$y' = 6(x-3)^{-2}$

$y'' = \frac{-12}{(x-3)^3}$

y'' ++ DNE − −
3

8. $y = x^3 + x^2$

$0 = x^2(x+1)$

$x=0$

$x=-1$

$y=0^3+0^2$

$y=0$

X intercept (0,0) (-1,0)

Y intercept (0,0)

Vertical Asymptotes none

Horizontal Asymptotes none

Critical Points x=-2/3 x=0

Interval(s) of Increase $(-\infty,-2/3)\cup(0,\infty)$

Interval(s) of Decrease (-2/3,0)

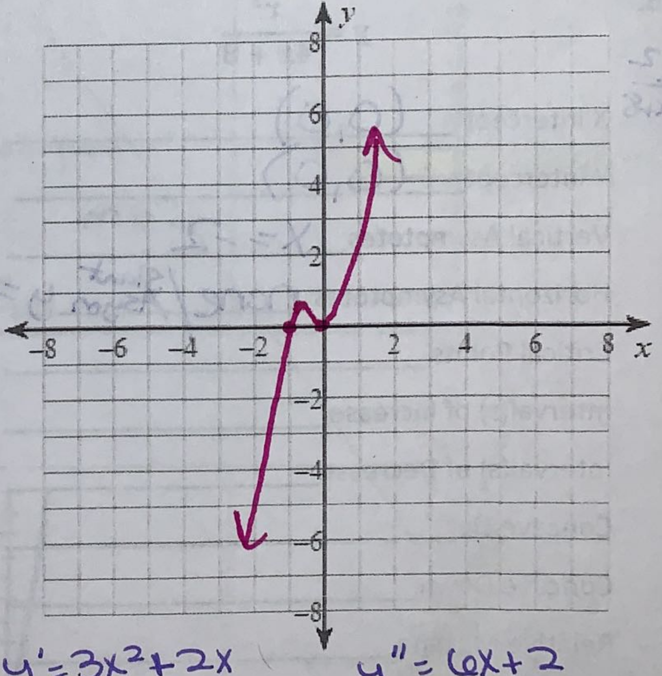
Concave Up $(-1/3,\infty)$

Concave Down $(-\infty,-1/3)$

Relative Maxima (-2/3, 4/27)

Relative Minima (0,0)

Points of Inflection (-1/3, 2/27)



$y' = 3x^2 + 2x$

$0 = x(3x+2)$

$x=0$ $x=-2/3$

max min

y' + 0 − 0 +
-2/3 0

$f(-2/3) = (-\frac{2}{3})^3 + (-\frac{2}{3})^2$

$= \frac{-8}{27} + \frac{4}{9} = \frac{+4}{27}$

$y'' = 6x+2$

$0 = 6x+2$

$x = -1/3$

POI

− − 0 + +
-1/3

$f(-1/3) = (-1/3)^3 + (-1/3)^2$

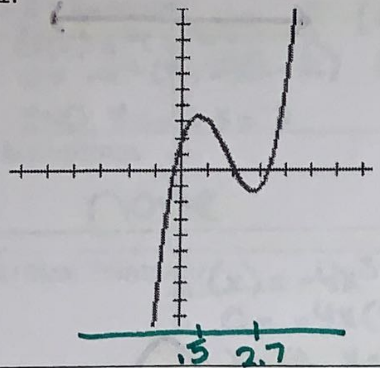
$-1/27 + 1/9 = 2/27$

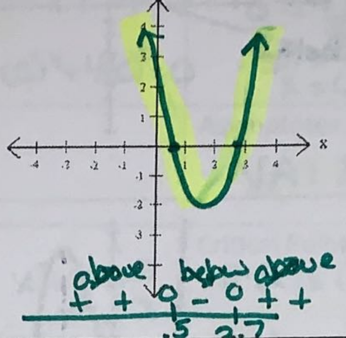
⑫

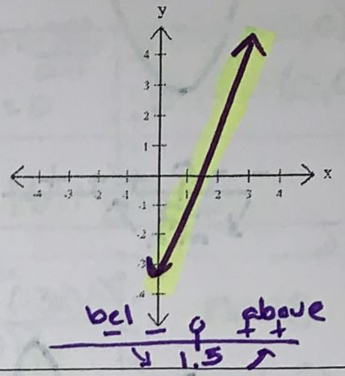
Calculus: Unit 4A Curve Sketching Quiz Review

Given f(x), sketch the graphs of f'(x) and f"(x)

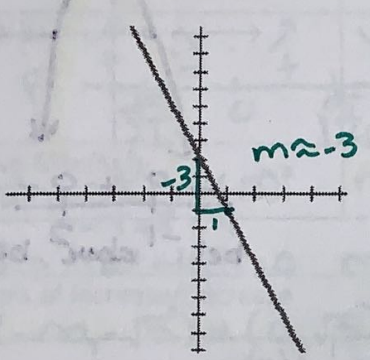
1.

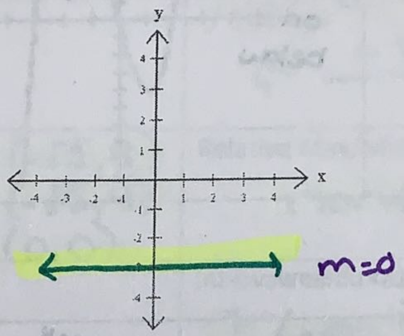


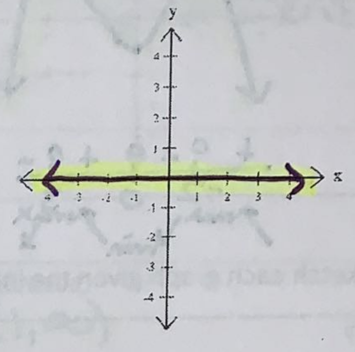




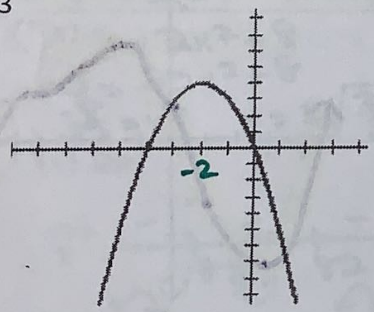
2.

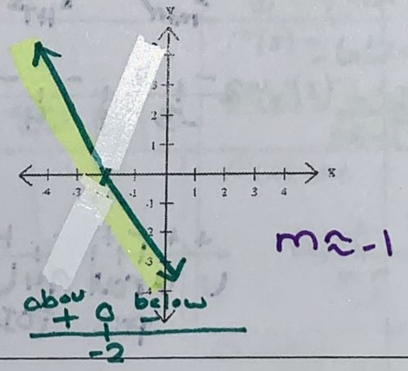


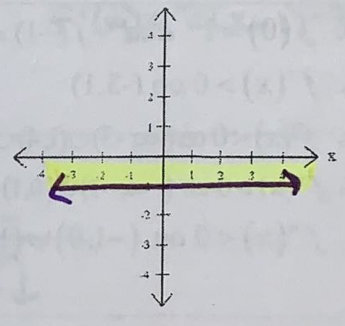




3

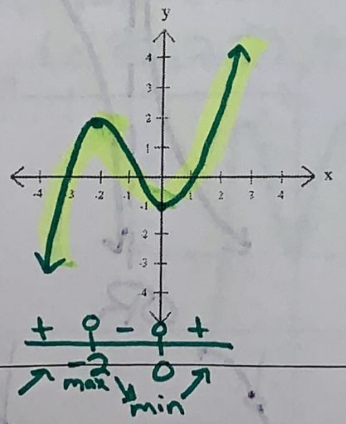


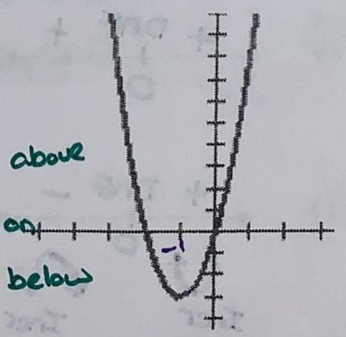


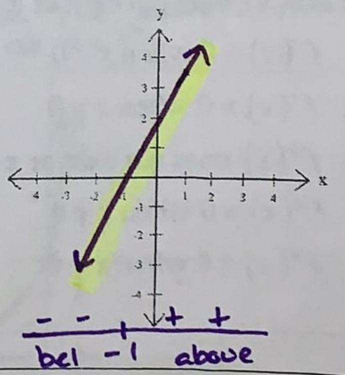


Given f'(x), sketch the graphs of f(x) and f"(x)

4.



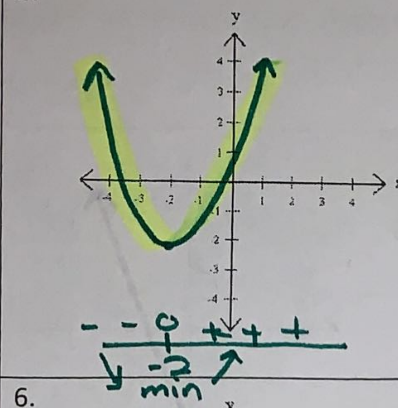


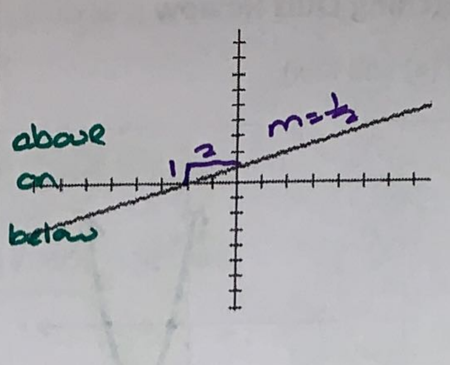


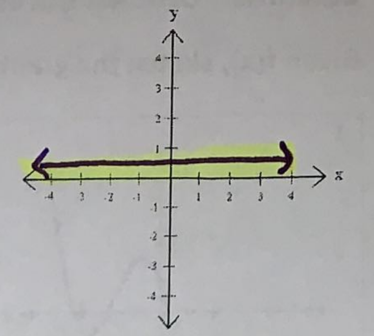
f(x)

f''(x)

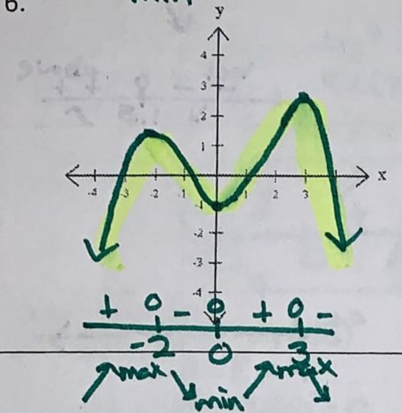
5.

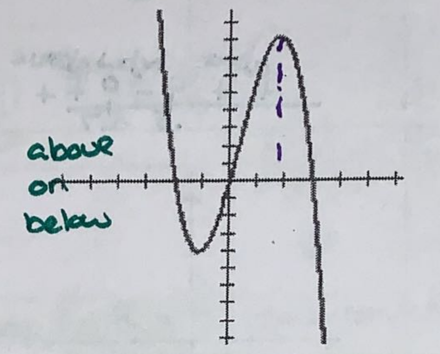


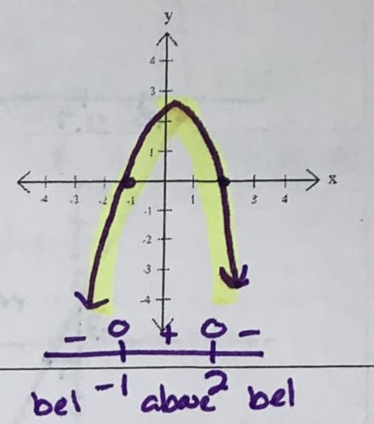




6.







Sketch each graph given the information below

7.

a. $f(x)$ is continuous

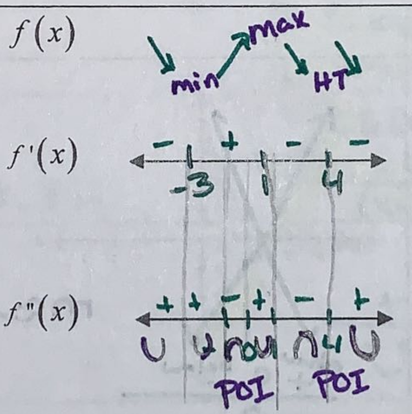
b. $f(0)=1$ *and* $f(-1)=-2$

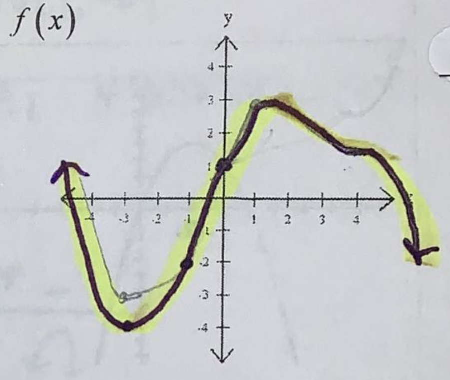
c. $f'(x)>0$ on (-3,1)

d. $f'(x)<0$ on $(-\infty,-3)\cup(1,4)\cup(4,\infty)$

e. $f''(x)>0$ on $(-\infty,-1)\cup(0,1)\cup(4,\infty)$

f. $f''(x)<0$ on $(-1,0)\cup(1,4)$





8.

a. $f(x)$ is not continuous at $x=0$

b. $f'(x)$ does not exist at $x=0$

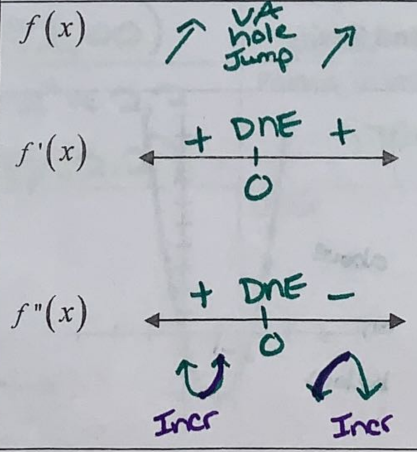
c. $f'(x)>0$ when $x<0$

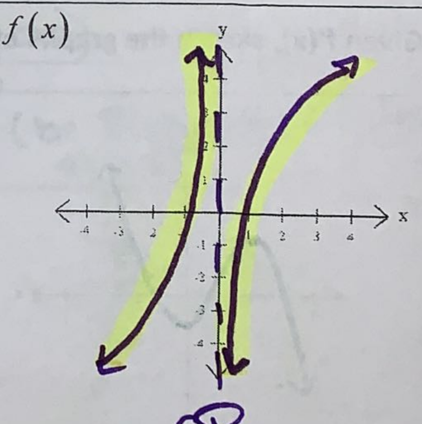
d. $f'(x)>0$ when $x>0$

e. $f''(x)$ does not exist at $x=0$

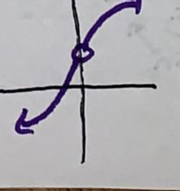
f. $f''(x)>0$ when $x<0$

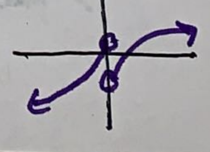
g. $f''(x)<0$ when $x>0$



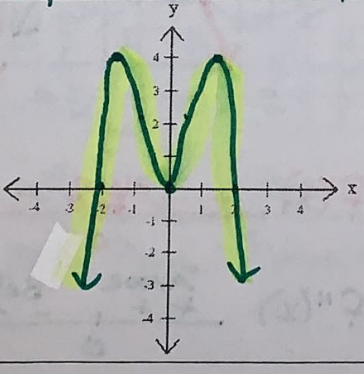
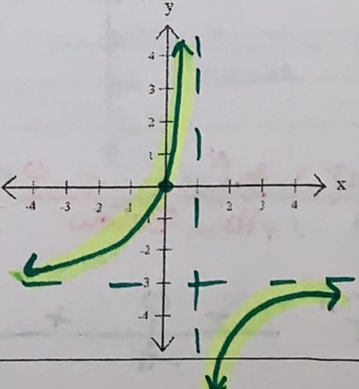


OR





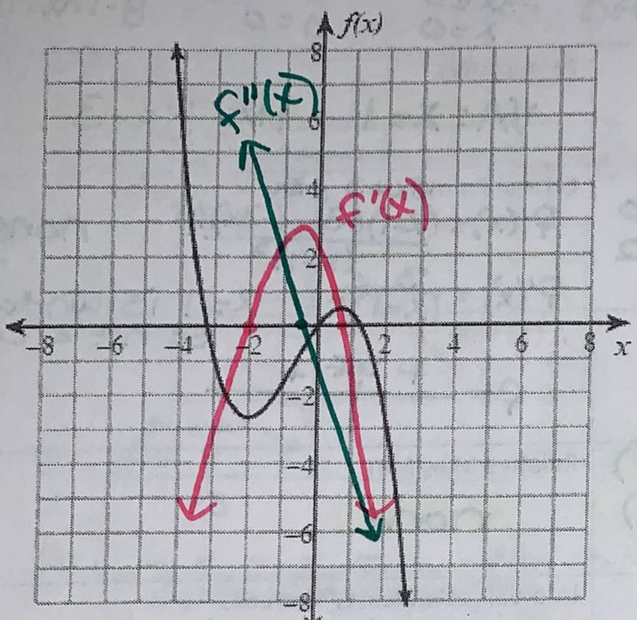
State the following information and sketch the graph.

9. $f(x) = -x^4 + 4x^2$	**10.** $f(x) = \frac{-3x}{x-1}$
X & Y Intercepts $0 = -x^2(x^2-4)$ X: (0,0) (-2,0) (2,0) $0 = -x^2(x+2)(x-2)$ y: (0,0) $f(x) = -(0)^4 + 4(0)^2 = 0$ $x=0$ $x=-2$ $x=2$	X & Y Intercepts $\frac{0}{1} = \frac{-3x}{x-1}$ $f(x) = \frac{-3(0)}{0-1}$ x-int: (0,0) $-3x = 0$ $y = 0$ y-int: (0,0) $x = 0$
Asymptotes none	Asymptotes VA: $x=1$ HA: $y=-3$
Critical Points $f'(x) = -4x^3 + 8x$ $0 = -4x(x^2-2)$ $x^2-2=0$ $x^2=2$ $x=0$ $x=\pm\sqrt{2}$	Critical Points $-3x+3 + 3x$ $f'(x) = \frac{(x-1)(-3)-(-3x)(1)}{(x-1)^2}$ none $f'(x) = \frac{3}{(x-1)^2}$ $x=1$ is undefined so not a crit. pt.
$f'(x)$ line ↗ ↘ ↗ ↘ f' + − + − $-\sqrt{2}$ 0 $\sqrt{2}$ −1.4 1.4	$f'(x)$ line f' + DNE + 1
Relative Max/Min $f(\pm\sqrt{2}) = -(\sqrt{2})^4 + 4(-\sqrt{2})^2$ max $(-\sqrt{2}, 4)$ $-4+8=4$ $(+\sqrt{2}, 4)$ $f(0) = -(0)^4 + 4(0)^2 = 0$ min (0,0)	Relative Max/Min none
Intervals of Increase/Decrease ↑ $(-\infty, -\sqrt{2}) \cup (0, \sqrt{2})$ ↓ $(-\sqrt{2}, 0) \cup (\sqrt{2}, \infty)$	Intervals of Increase/Decrease ↑ $(-\infty, 1) \cup (1, \infty)$ ↓ none
$f''(x) = -12x^2 + 8$ $0 = -12x^2 + 8$ $\frac{-8}{-12} = x^2$ $x = \pm\sqrt{\frac{2}{3}} \approx \pm .82$	$f''(x) = -6(x-1)^{-3} \cdot 1$ $f'(x) = 3(x-1)^{-2}$ $f''(x) = \frac{-6}{(x-1)^3}$
$f''(x)$ line f'' − + − −.82 .82	$f''(x)$ line + DNE − 1
Concavity up (−.82, .82) down $(-\infty, -.82) \cup (.82, \infty)$	Concavity up $(-\infty, 1)$ down $(1, \infty)$
Point(s) of Inflection $f(\pm.82) = -(-.82)^4 + 4(-.82)^2 \approx 2.2$ (−.82, 2.2) & (.82, 2.2)	Point(s) of Inflection none (bc f(x) doesn't exist at x=1)
Graph 	Graph 

Derivative Applications Review

Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$ and $f''(x)$. Be sure to label your graphs!

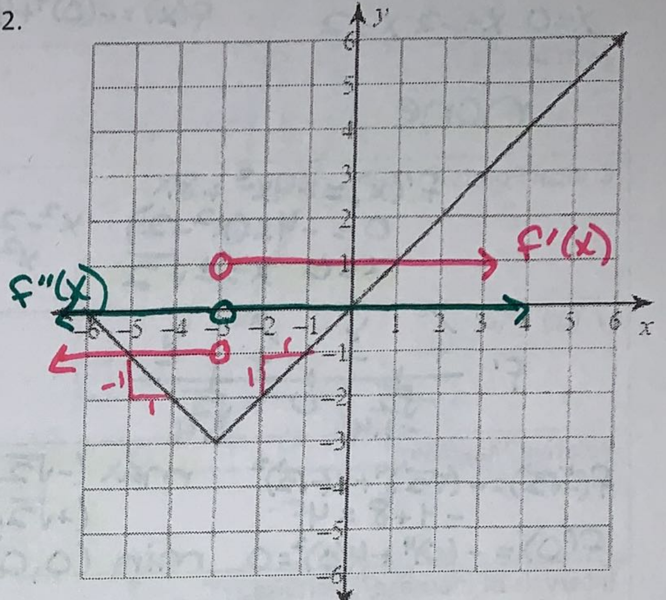
1.



f′ − 0 + 0 −
-2 .7

f″ + + 0 − −
-.5

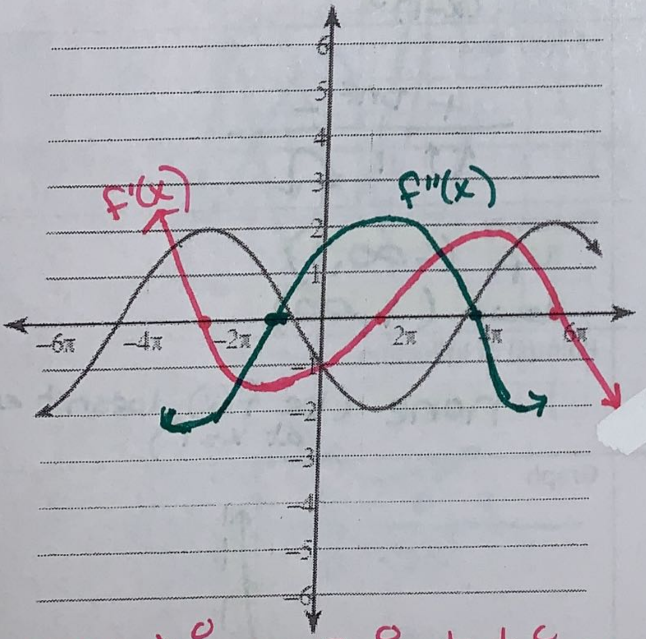
2.



f′ − DNE + +
m=-1 -3 m=1

f″ m=0 DNE m=0
-3

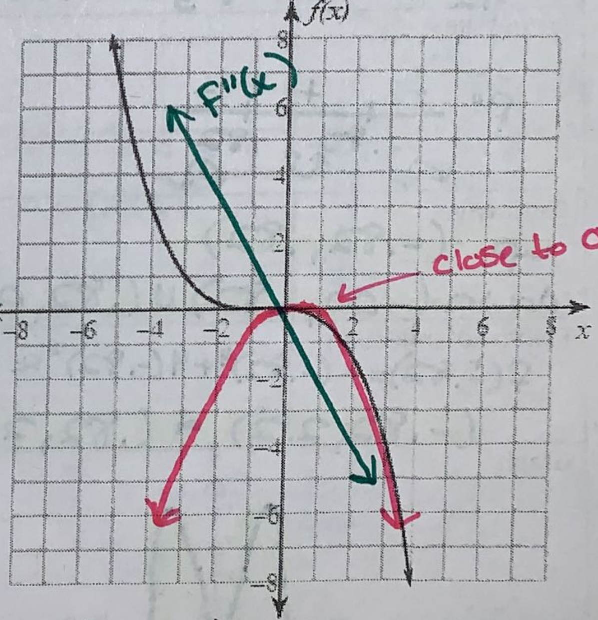
3.



f′(x) +.0 − − 0 + + 0 −
Ab Below Above Bel

f″ − 0 + 0 −
-π 4π

4.



f′(x) − − close to 0 − −
0

f″(x) Above + + Below − −
0

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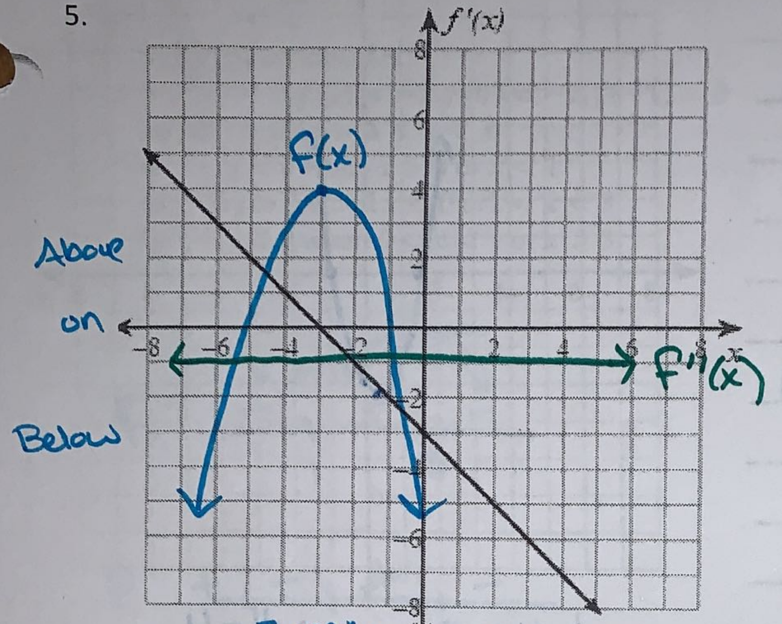
Given the graph of $f'(x)$, sketch an approximate graph of $f(x)$ and $f''(x)$. Be sure to label your graphs!

5.



↗ max ↘

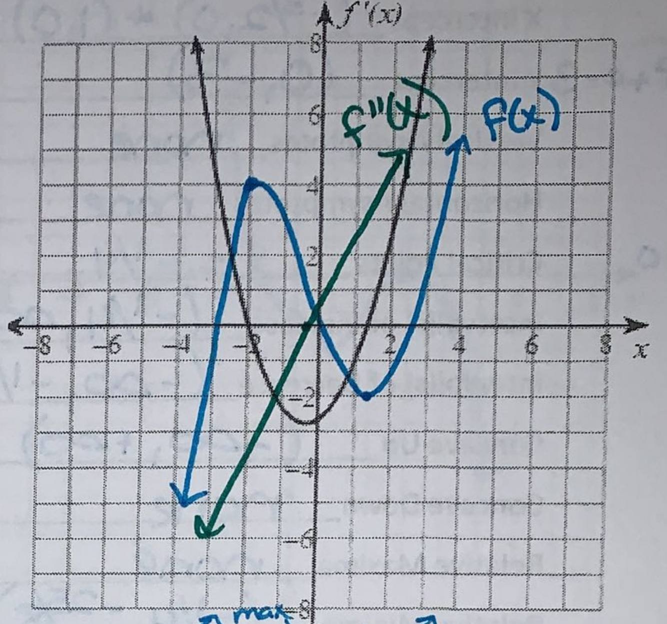
f + + o − − −

-3

f'' − − − − − m=-1

Decreasing everywhere

6.



↗ max ↘ min ↗

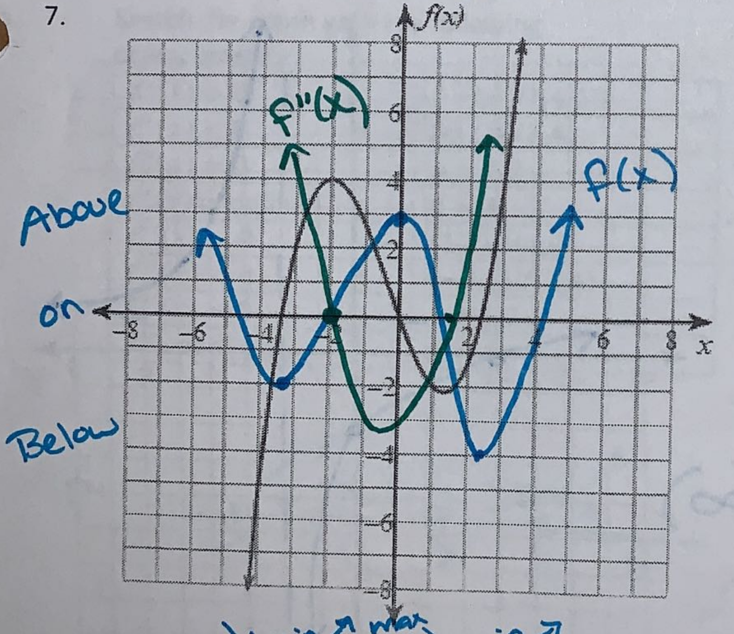
f + o − o +

-2 1.3

f'' − − o + +

Below -.5 Above

7.



↘ min ↗ max ↘ min ↗

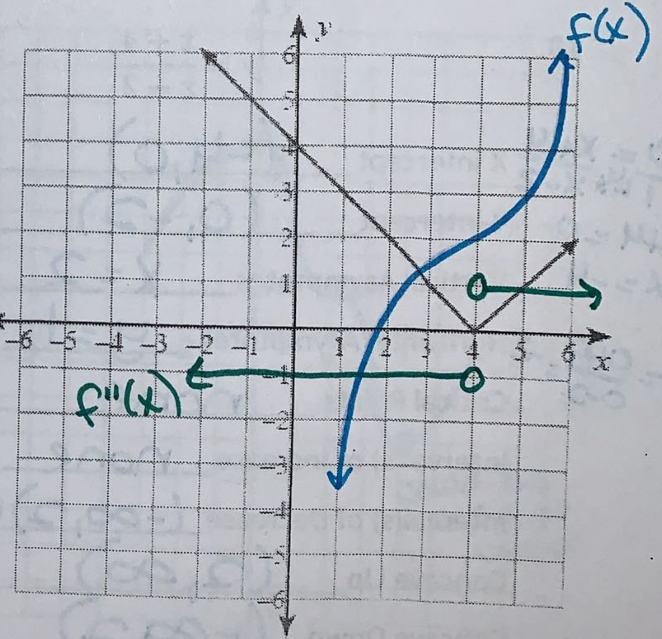
f − o + o − o +

-3.5 0 2.4

f'' + Inc o Dec − o Inc +

Above -2 Below 1.3 Above

8.



f + + + + + +

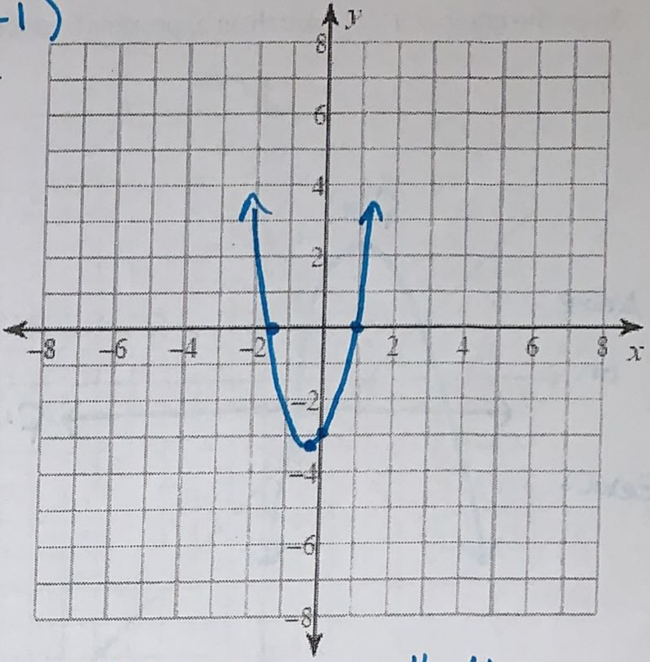
Above 4 Above

f'' − Dec DNE + Inc

m=-1 4 m=1

∩ ∪

9. $y = 2x^2 + x - 3$ $(2x+3)(x-1)$



X intercept $(-3/2,0)$ + $(1,0)$

$2(0)^2+0-3$ Y intercept $(0,-3)$

Vertical Asymptotes none

Horizontal Asymptotes none

$y'=0$ Critical Points $x=-1/4$

Interval(s) of Increase $(-1/4,\infty)$

Interval(s) of Decrease $(-\infty,-1/4)$

Concave Up $(-\infty,+\infty)$

Concave Down none

Relative Maxima none

Relative Minima $(-1/4,-25/8)$ (-3.125)

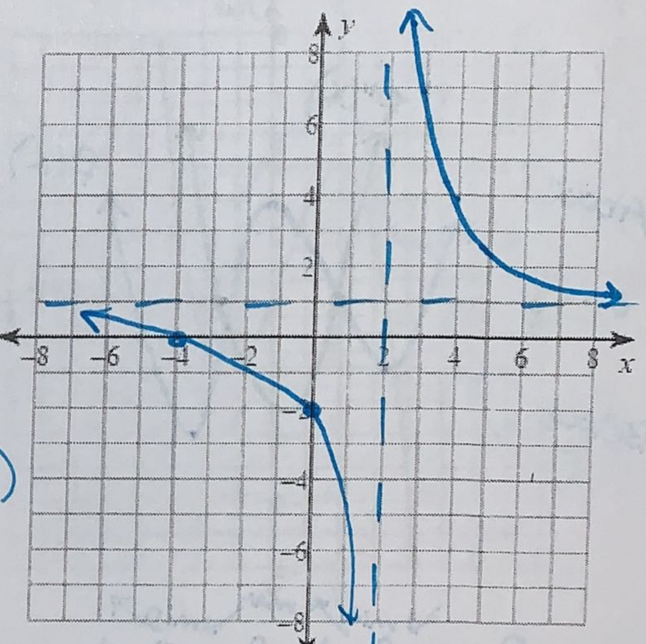
$f(-1/4) = 2(-1/4)^2 - \frac{1}{4} - 3$

$y' = 4x+1$
$0 = 4x+1$
$x = -1/4$

$- \; - \; \overset{0}{|} \; + \; +$
$\searrow \; -1/4 \; \nearrow$

$y'' = 4$
$+ \; + \; +$

10. $y = \dfrac{x+4}{x-2}$



$0 = \frac{x+4}{x-2}$ X intercept $(-4,0)$

$x+4=0$ Y intercept $(0,-2)$

$x=-4$ Vertical Asymptotes $x=2$

$y = \frac{0+4}{0-2} = -2$ Horizontal Asymptotes $y=1$

Critical Points none

Interval(s) of Increase none

Interval(s) of Decrease $(-\infty,2)\cup(2,\infty)$

Concave Up $(2,\infty)$

Concave Down $(-\infty,2)$

Relative Maxima none

Relative Minima none

$y' = \frac{\overset{x-2-x-4}{(x-2)(1)-(x+4)(1)}}{(x-2)^2}$

$y' = \frac{-6}{(x-2)^2}$ $x=2$ is undef.

$y'=-6(x-2)^{-2}$

$y''=\frac{12}{(x-2)^3}$

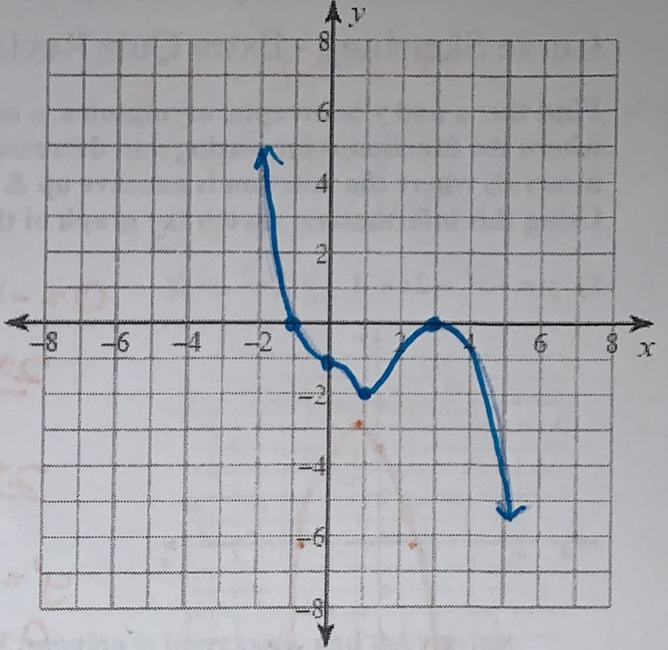
y' $- \; - \; \overset{DNE}{|} \; - \; -$
$\searrow \; 2 \; \searrow$

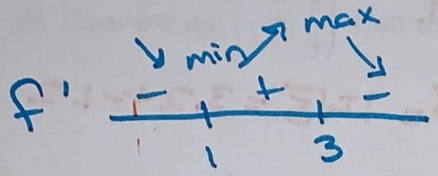
y'' $- \; - \; \overset{DNE}{|} \; + \; +$
$\frown \; 2 \; \smile$

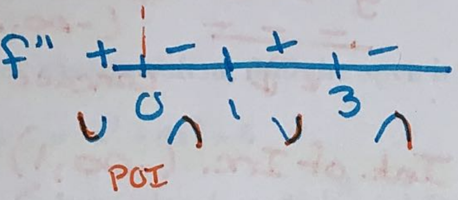
18

11. Given the following characteristics, draw an appropriate graph.

a. $f(x)$ is continuous
b. $f(3) = 0, f(1) = -2, f(0) = -1, f(-1) = 0$
c. $f'(x) > 0$ when $1 < x < 3$
d. $f'(x) < 0$ when $x > 3$ or $x < 1$
e. $f''(x) > 0$ when $x < 0$ or $1 < x < 3$
f. $f''(x) < 0$ when $0 < x < 1$ or $x > 3$



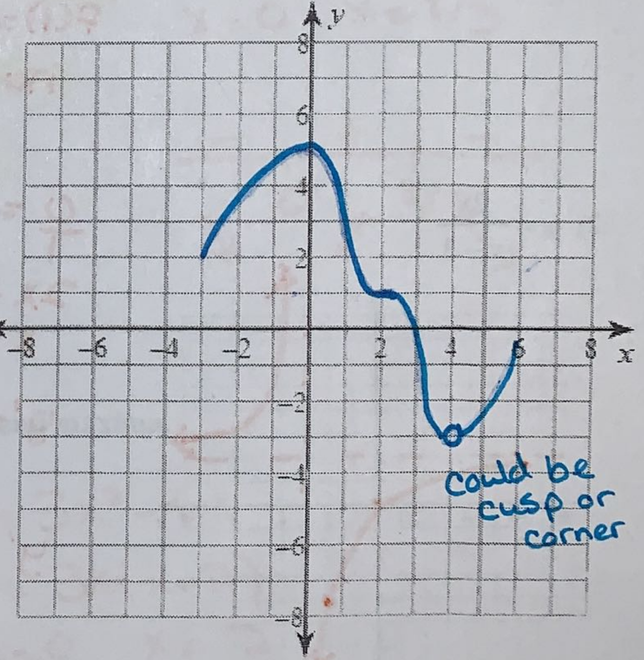




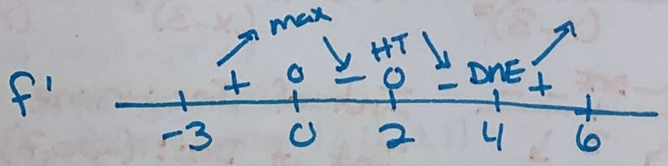
12.

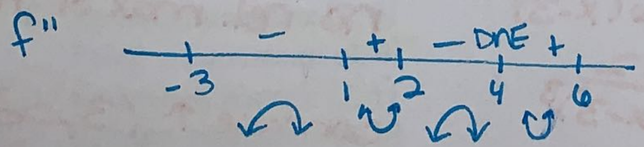
Sketch the graph with the following characteristics:

$f'(x) > 0$	$(-3,0)U(4,6)$
$f'(x) < 0$	$(0,2)U(2,4)$
$f'(x) = 0$	$x = 0,2$
$f'(x)$ undefined	$x = 4$
$f''(x) > 0$	$(1,2)U(4,6)$
$f''(x) < 0$	$(-3,1)U(2,4)$
$f''(x) = 0$	$x = 1,2$
f'' undefined	$x = 4$



could be cusp or corner





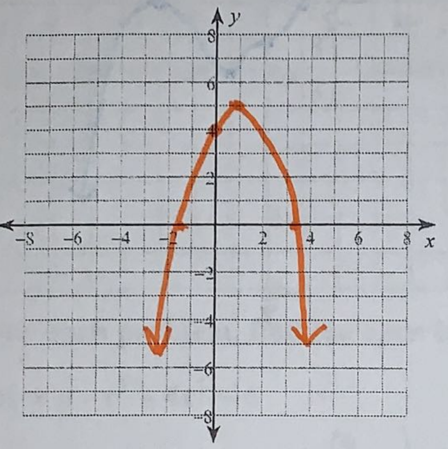
Calculus Name________________________ ID: 1

Curve Sketching - Extra Quiz Review Problems

Date________________ Period____

Find the: x and y intercepts, asymptotes, x-coordinates of the critical points, intervals where the function is increasing and decreasing, x-coordinates of the inflection points, intervals where the function is concave up & concave down, and relative minima & maxima. Using this information, sketch the graph of the function.

1) $y = -x^2 + 2x + 4$



a=1 b=-2 c=-4

$0 = -1(x^2 - 2x - 4)$

y-int: (0,4)

x-int: (3.2,0)+(-1.2,0)

$\frac{2 \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$

$\frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5} = 3.2 + -1.2$

$y' = -2x + 2$

$0 = -2x + 2$

$x = 1$

$y'' = -2$

$\underline{- \ - \ -}$

⌒

concave ↓ (-∞,∞)

concave ↑ none

$y' \ \underline{+ \ | \ -}$

↗ 1 ↘

Int. of Inc. (-∞,1)

Int of Dec (1,∞)

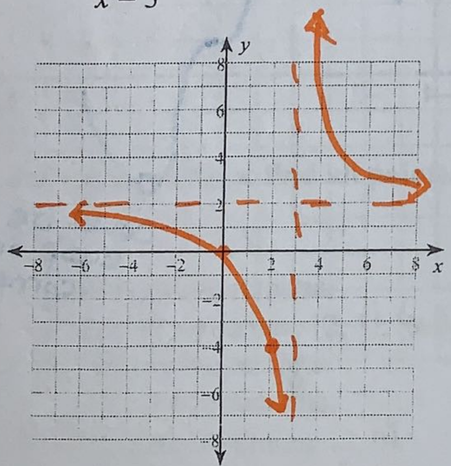
max (1,5)

$f(1) = -(1)^2 + 2(1) + 4$

no min

no point of inflection

2) $y = \frac{2x}{x - 3}$



$\frac{0}{1} = \frac{2x}{x-3}$

$2x = 0$

$x = 0$

x-int + y-int: (0,0)

VA: x=3 HA: y=2

$y' = \frac{(x-3)(2) - 2x(1)}{(x-3)^2} = \frac{2x - 6 - 2x}{(x-3)^2} = \frac{-6}{(x-3)^2}$

$y' \ \underline{- \ \text{DNE} \ -}$

↘ 3 ↘

Int of Inc: none

Int of Dec: (-∞,3)∪(3,∞)

$y' = -6(x-3)^{-2}$

$y'' = \frac{12}{(x-3)^3}$

no rel. max or min

concave ↑ (3,∞)

concave ↓ (-∞,3)

$y'' \ \underline{- \ - \ \text{DNE} \ + \ +}$

⌒ 3 ⌣

no point of inflection

⑳

For each problem, find the intervals where the function is concave up & concave down.

3) $y = -x^3 + 2x^2 - 3$

[✓ A)] Concave up: $\left(-\infty, \frac{2}{3}\right)$ Concave down: $\left(\frac{2}{3}, \infty\right)$

B) Concave up: $\left(\frac{2}{3}, \infty\right)$ Concave down: $\left(-\infty, \frac{2}{3}\right)$

C) Concave up: $\left(\frac{2}{9}, \infty\right)$ Concave down: $\left(-\infty, \frac{2}{9}\right)$

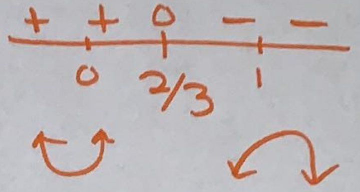
D) Concave up: $\left(-\infty, \frac{8}{3}\right)$ Concave down: $\left(\frac{8}{3}, \infty\right)$

$y' = -3x^2 + 4x$

$y'' = -6x + 4$

$0 = -6x + 4$

$x = 2/3$



For each problem, find the open intervals where the function is increasing and decreasing.

4) $y = -x^3 + 4x^2 - 6$

A) Increasing: $(-\infty, 0), \left(\frac{8}{3}, \infty\right)$ Decreasing: $\left(0, \frac{8}{3}\right)$

B) Increasing: $\left(4, \frac{32}{3}\right)$ Decreasing: $(-\infty, 4), \left(\frac{32}{3}, \infty\right)$

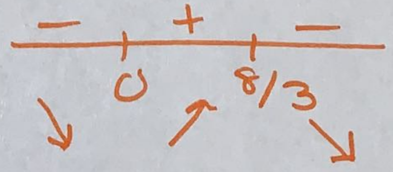
C) Increasing: $\left(-\infty, \frac{1}{3}\right), \left(\frac{8}{9}, \infty\right)$ Decreasing: $\left(\frac{1}{3}, \frac{8}{9}\right)$

[✓ D)] Increasing: $\left(0, \frac{8}{3}\right)$ Decreasing: $(-\infty, 0), \left(\frac{8}{3}, \infty\right)$

$y' = -3x^2 + 8x$

$0 = -x(3x - 8)$

$x = 0 \quad x = 8/3$



For each problem, find all points of relative minima and maxima.

5) $y = x^3 - 3x^2 + 5$

A) No relative minima.
Relative maxima: (2, 1), (0, 1)

B) No relative minima.
No relative maxima.

C) Relative minimum: (2, 0)
Relative maximum: (0, 0)

[✓ D)] Relative minimum: (2, 1)
Relative maximum: (0, 5)

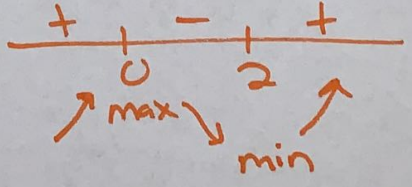
$y' = 3x^2 - 6x$

$0 = 3x(x - 2)$

$x = 0 \quad x = 2$

$f(0) = 0^3 - 3(0)^2 + 5$

$f(0) = 5$



$f(2) = 2^3 - 3(2)^2 + 5$

$f(2) = 1$