# Unit 4 Graphs of Trig Functions

**KEY STANDARDS -**
- MGSE9-12.F.IF.7e Graph trigonometric functions, showing period, midline, and amplitude. Extend the domain of trigonometric functions using the unit circle
- MGSE9-12.F.TF.2 Model periodic phenomena with trigonometric functions
- MGSE9-12.F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- MGSE9-12.F.TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- MGSE9-12.F.BF.4 Find inverse functions.

| Date | Topic(s) | Assignment/HW | Credit |
|---|---|---|---|
| Mon. 9/13 | Trig Parent Graphs | | |
| Tues. 9/14 | Parent Graph Memorization Quiz & Transformations of Trig Functions | | |
| Wed. 9/15 | Writing Trig Equations | | |
| Thurs. 9/16 | Applications of Trig Functions | | |
| Fri. 9/17 | Applications of Trig Functions | | |
| Mon. 9/20 | Review | | |
| Tues. 9/21 | **Quiz** | | |
| Wed. 9/22 | Graphing Trig Functions | | |
| Thurs. 9/23 | Graphing Trig Functions | | |
| Fri. 9/24 | Graphing Trig Functions | | |
| 9/27 – 10/1 | **FALL BREAK** | | |
| Mon. 10/4 | Graphing Review | | |
| Tues. 10/5 | Inverses of Trig Functions | | |
| Wed. 10/6 | Inverses & Review | | |
| Thurs. 10/7 | Unit 4 Review | | |
| Fri. 10/8 | **Unit 4 Trig Functions Test** | | |

**Name** Key

**WS #1 Transformations of Trig Graphs**

Fill in the table.

| | | Amplitude | Period | Phase Shift | Vertical Shift |
|---|---|---|---|---|---|
| 1. | $y=-2\sin\left(\frac{1}{5}x-\pi\right)$ | 2 | $10\pi$ | right $5\pi$ | none |
| 2. | $f(\theta)=\frac{1}{3}\cos(\theta+\frac{\pi}{3})+3$ | 1/3 | $2\pi$ | left $\pi/3$ | up 3 |
| 3. | $f(x)=\cos\left(3x-\frac{5\pi}{6}\right)-\frac{1}{2}$ <br> $f(x)=\cos 3(x-\frac{5\pi}{18})-\frac{1}{2}$ | 1 | $2\pi/3$ | right $5\pi/18$ | down 1/2 |
| 4. | $y=5\cos\left(\frac{1}{2}x\right)+4$ | 5 | $4\pi$ | none | up 4 |
| 5. | $f(\theta)=-4\sin(2\theta)$ | 4 | $\pi$ | none | none |
| 6. | $f(\theta)=\frac{1}{2}\cos\left(\theta+\frac{\pi}{2}\right)+3$ | 1/2 | $2\pi$ | left $\pi/2$ | up 3 |
| 7. | $y=3\tan(x-\pi)$ | none | $\pi$ | right $\pi$ | none |
| 8. | $y=\cot\left(4x+\frac{\pi}{4}\right)-8$ <br> $y=\cot 4(x+\frac{\pi}{16})-8$ | none | $\pi/4$ | left $\pi/16$ | down 8 |
| 9. | $f(x)=-5\sin\left(\frac{1}{3}x\right)-\frac{1}{2}$ | 5 | $6\pi$ | none | down 1/2 |
| 10. | $f(x)=\csc\left(x-\frac{5\pi}{6}\right)+1$ | none | $2\pi$ | right $5\pi/6$ | up 1 |
| 11. | $y=\cos\left(\frac{3}{2}x\right)+\frac{5}{3}$ | 1 | $4\pi/3$ | none | up 5/3 |
| 12. | $y=-\frac{2}{3}\sec\left(2x-\frac{\pi}{2}\right)$ <br> $y=-\frac{2}{3}\sec 2(x-\frac{\pi}{4})$ | none | $\pi$ | right $\pi/4$ | none |

**Write the sine equation for each of the following:**

13. amplitude: 2 period: $\frac{\pi}{2}$ phase shift: $-\frac{\pi}{4}$

$\frac{2\pi}{b} = \frac{\pi}{2}$

$b = 4$

$y = \pm 2\sin 4\left(x + \frac{\pi}{4}\right)$

or

$y = \pm 2\sin(4x + \pi)$

14. amplitude: 4 period: $3\pi$ phase shift: $\frac{\pi}{2}$

$\frac{2\pi}{b} = 3\pi$

$3\pi b = 2\pi$

$b = 2/3$

$y = \pm 4\sin 2/3\left(x - \frac{\pi}{2}\right)$

or $y = \pm 4\sin\left(\frac{2}{3}x - \frac{\pi}{3}\right)$

15. amplitude: 1 period: 1 phase shift: $\frac{\pi}{4}$ displacement: 1

$\frac{2\pi}{b} = 1$

$b = 2\pi$

$y = \pm\sin 2\pi\left(x - \frac{\pi}{4}\right) + 1$

or $y = \pm\sin\left(2\pi x - \frac{\pi^2}{2}\right) + 1$

16. amplitude: 3 period: 4 phase shift: -1 displacement: -3

per.

$\frac{2\pi}{b} = \frac{4}{1}$

$4b = 2\pi$

$b = \frac{\pi}{2}$

$y = \pm 3\sin\frac{\pi}{2}(\theta + 1) - 3$

or $y = \pm 3\sin\left(\frac{\pi}{2}\theta + \frac{\pi}{2}\right) - 3$

WS #2 Write the Equations/Define Characteristics

| Amplitude: A | Period: ~~B~~ $\frac{2\pi}{B}$ | Phase Shift: C | Vertical Shift: D | y= A trig B(x-C) +D |
|---|---|---|---|---|

**Write the equation of a sine graph with the following information.**

1. amplitude = 4 period= $\pi$ phase shift= 0 vertical shift=-5

$\frac{2\pi}{b} = \pi$

$b = 2$

$y = \pm 4 \sin 2(x) - 5$

2. amplitude = 3.5 period= $2\pi$ phase shift= $\pi$ vertical shift= 3

$b = 1$

$y = \pm 3.5 \sin (x - \pi) + 3$

3. amplitude = 2 period= $\frac{\pi}{2}$ phase shift= $-\pi$ vertical shift= -2

$b = 4$

$\frac{2\pi}{b} = \frac{\pi}{2}$

$\pi b = 4\pi$

$b = 4$

$y = \pm 2 \sin 4(x + \pi) - 2$

4. amplitude = 1 period= $\frac{2\pi}{3}$ phase shift= $\frac{-5\pi}{2}$

$b = 3$

$\frac{2\pi}{b} = \frac{2\pi}{3}$

$b = 3$

$y = \pm \sin 3(\theta + \frac{5\pi}{2})$

5. amplitude = $\frac{1}{2}$ period= $4\pi$ phase shift= $\frac{\pi}{4}$

$b = \frac{1}{2}$

$\frac{2\pi}{b} = \frac{4\pi}{1}$

$4\pi b = 2\pi$

$b = \frac{1}{2}$

$y = \pm \frac{1}{2} \sin \frac{1}{2}(\theta - \frac{\pi}{4})$

6. amplitude = $\frac{2}{3}$ period= $\pi$ phase shift= $-\frac{\pi}{4}$

$b = 2$

$\frac{2\pi}{b} = \frac{\pi}{1}$

$\pi b = 2\pi$

$y = \pm \frac{2}{3} \sin 2(\theta + \frac{\pi}{4})$

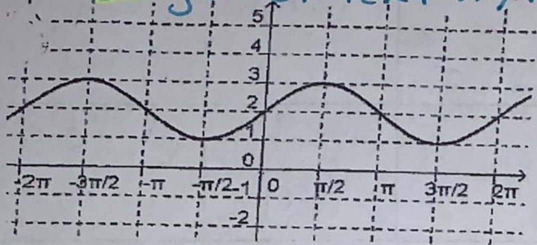
**Given the equation, define each characteristic of the graph with EXACT values only.**

| Equation | Period<br>tan pd = $\pi/b$<br>Sin/cos pd = $2\pi/b$ | Amplitude (reflection?) | Phase shift (Left/right) | Vertical shift (Up/down) |
|---|---|---|---|---|
| **7.** $y=-8\cos(3x+\pi)+2$<br>$y=-8\cos 3(x+\frac{\pi}{3})+2$ | $2\pi/3$ | 8. + reflect | right $\pi/3$ | up 2 |
| **8.** $y=5\sin(4x+\pi)-3$<br>$y=5\sin 4(x+\frac{\pi}{4})-3$ | $\pi/2$ | 5 | left $\pi/4$ | down 3 |
| **9.** $3\tan(2x)+1$ | $\pi/2$ | none | none | up 1 |
| **10.** $2\cot(3x)+2$ | $\pi/3$ | none | none | up 2 |
| **11.** $2\sec(3x+\pi)+1$<br>$y=2\sec 3(x+\frac{\pi}{3})+1$ | $2\pi/3$ | none | left $\pi/3$ | up 1 |
| **12.** $-3\sec\left(\frac{1}{2}x\right)+2$ | $4\pi$ | none reflect | none | up 2 |
| **13.** $-2\tan\left(2x-\frac{\pi}{2}\right)$<br>$y=-2\tan 2(x-\frac{\pi}{4})$ | $\pi/2$ | none reflect | right $\pi/4$ | none |
| **14.** $\frac{1}{2}\tan\left(3x+\frac{\pi}{6}\right)$<br>$y=\frac{1}{2}\tan 3(x+\frac{\pi}{18})$ | $\pi/3$ | none | left $\pi/18$ | none |
| **15.** $\frac{1}{2}\sin\left(3x+\frac{5\pi}{6}\right)$<br>$y=\frac{1}{2}\sin 3(x+\frac{5\pi}{18})$ | $2\pi/3$ | $1/2$ | left $5\pi/18$ | none |
| **16.** $-\frac{3}{4}\cos\left(4x-\frac{\pi}{2}\right)$<br>$y=-\frac{3}{4}\cos 4(x-\frac{\pi}{8})$ | $\pi/2$ | $3/4$ + reflect | right $\pi/8$ | none |
| **17.** $-\csc\left(\frac{x}{2}+2\pi\right)$<br>$y=-\csc\frac{1}{2}(x+4\pi)$ | $4\pi$ | none reflect | left $4\pi$ | none |
| **18.** $3\sec\left(\frac{x}{3}+\frac{\pi}{2}\right)$<br>$y=3\sec\frac{1}{3}(x+\frac{3\pi}{2})$ | $6\pi$ | none | left $3\pi/2$ | none |

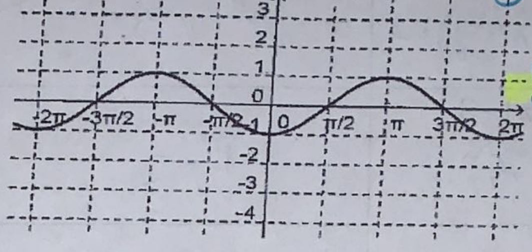
# WS #3 Write the Equations from Graphs

Write the equation of each function in **terms of sine.** The axes labels vary from graph to graph.

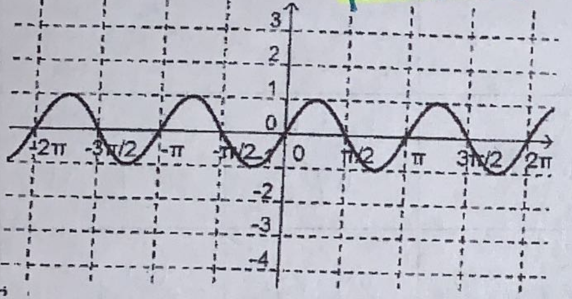
1. $y = \sin(x) + 2$ or $y = -\sin(x+\pi) + 2$



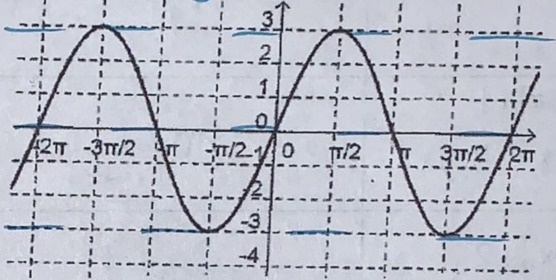
2. $y = \sin(x + \frac{3\pi}{2})$ or $y = -\sin(x + \frac{\pi}{2})$ or $y = \sin(x - \pi/2)$



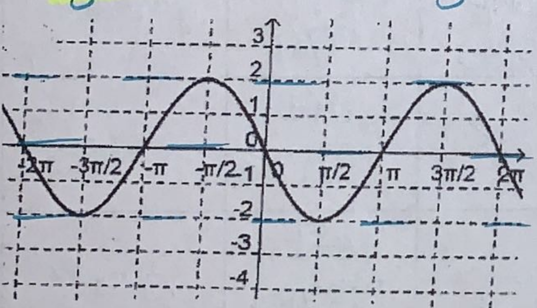
3. $y = \sin 2(x)$ or other possible answers



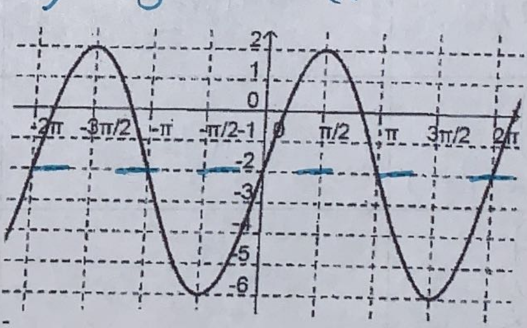
4. $y = 3\sin x$ or $y = -3\sin(x+\pi)$ or $y = 3\sin(x+2\pi)$



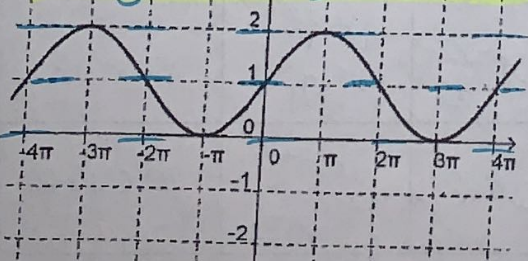
5. $y = -2\sin x$ or $y = 2\sin(x+\pi)$



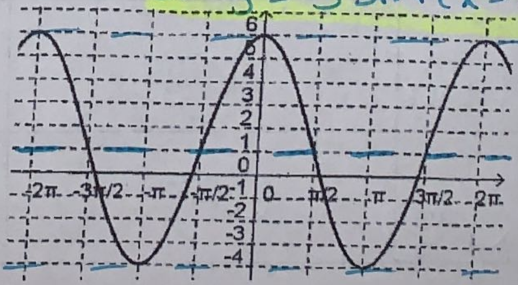
$y = 4\sin(x) - 2$ or $y = -4\sin(x+\pi) - 2$



7. $y = \sin \frac{1}{2}(x) + 1$ or $y = -\sin \frac{1}{2}(x + 2\pi) + 1$

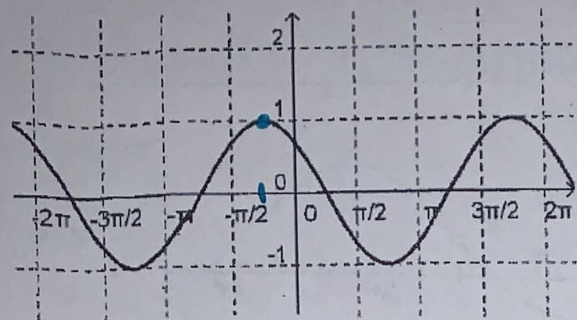


8. $y = 5\sin(x + \frac{\pi}{2}) + 1$ or $y = -5\sin(x - \frac{\pi}{2}) + 1$ or $y = 5\sin(x - \frac{3\pi}{2}) + 1$

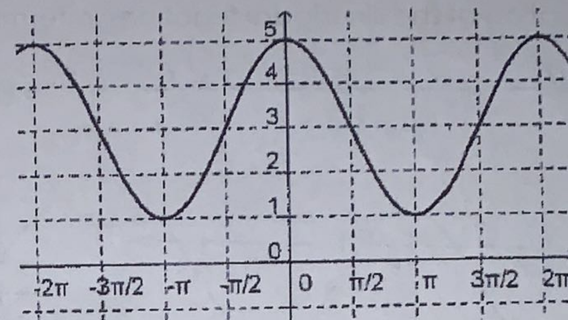


Write the equation in **terms of cosine.** The axes labels vary from graph to graph.

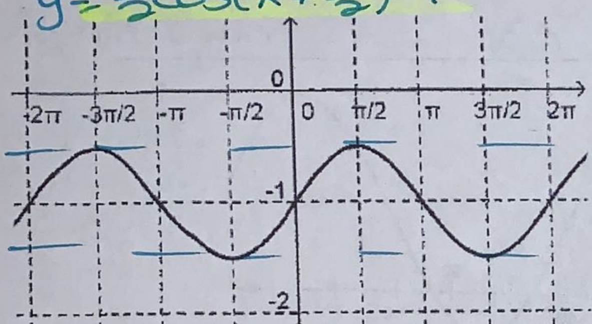
9. $y = \cos(x + \frac{\pi}{4})$



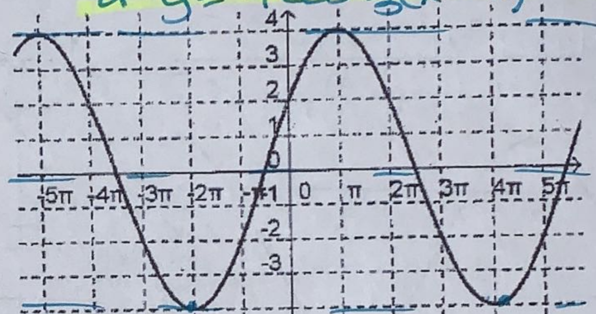

10. $y = 2\cos(x) + 3$ or $y = -2\cos(x + 2\pi) + 3$



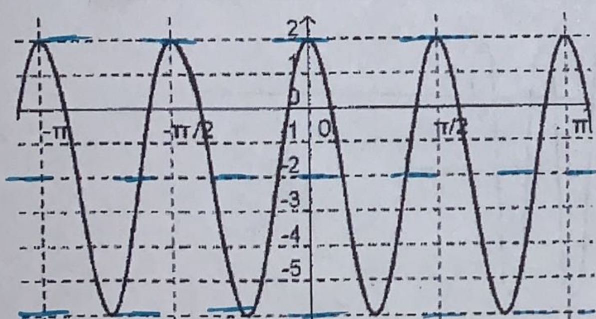

11. $y = \frac{1}{2}\cos(x - \frac{\pi}{2}) - 1$ or $y = \frac{1}{2}\cos(x + \frac{3\pi}{2}) - 1$
$y = -\frac{1}{2}\cos(x + \frac{\pi}{2}) - 1$



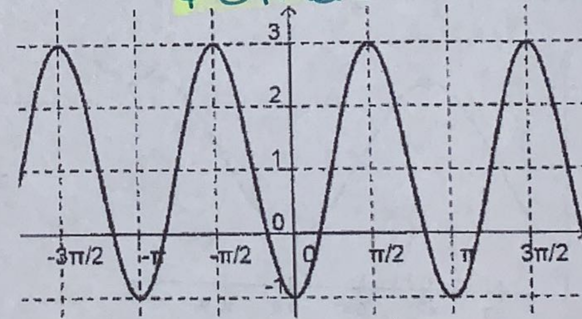

12. $y = -4\cos\frac{1}{3}(x + 2\pi)$ or $y = 4\cos\frac{1}{3}(x + 5\pi)$
or $y = 4\cos\frac{1}{3}(x - \pi)$




13. $y = 4\cos 4(x) - 2$ + other possibilities

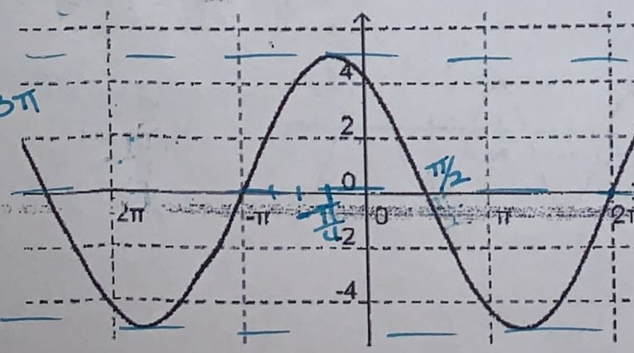



14. $y = -2\cos 2(x) + 1$ or $y = 2\cos 2(x - \frac{\pi}{2}) + 1$
+ others

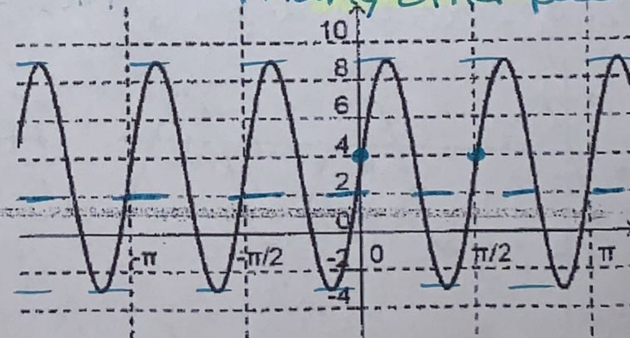



Sine 15. $y = 5\sin\frac{2}{3}(x + \pi)$

per $= 3\pi$



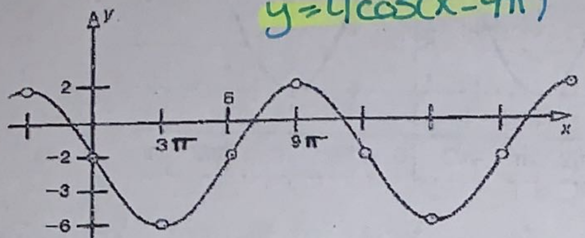

16. *In terms of sine $y = 6\sin 4(x) + 3$ +
many other poss. ans.



## WS #4 More: Write the Equations from Graphs

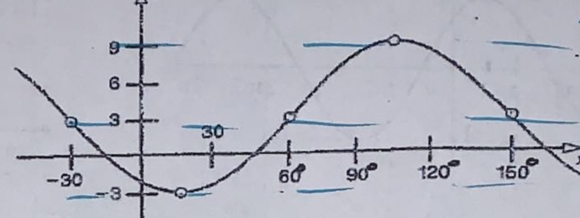
Write the equation of the sinusiodal functions in terms of Sine or cosine

1. $y=-4\sin(x)-2$ or $y=-4\cos(x-3\pi)$
$y=4\cos(x-9\pi)$



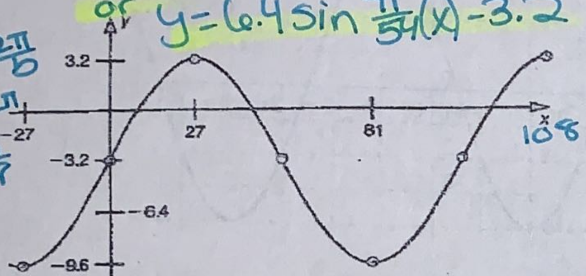

2. $y=-6\sin 2(x+30°)+3$ or $y=-6\cos 2(x-15°)+3$

$\frac{360°}{b}=180°$
$180b=360$
$b=2$



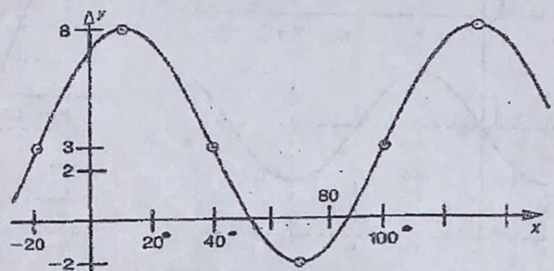

3. $y=6.4\cos\frac{\pi}{54}(x-27)-3.2$
or $y=6.4\sin\frac{\pi}{54}(x)-3.2$

$108=\frac{2\pi}{b}$
$108b=2\pi$
$b=\frac{2\pi}{108}$



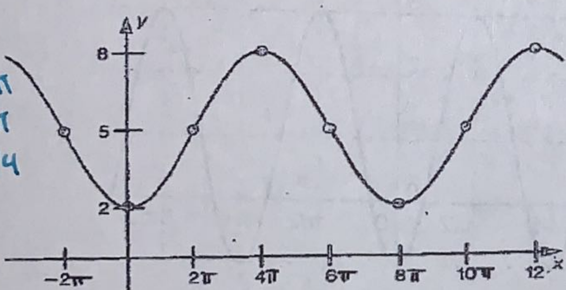

4. $y=5\sin 3(x+20°)+3$ or $y=-5\sin 3(x-40°)+3$

per $=120$
$\frac{360}{b}=120$
$b=3$



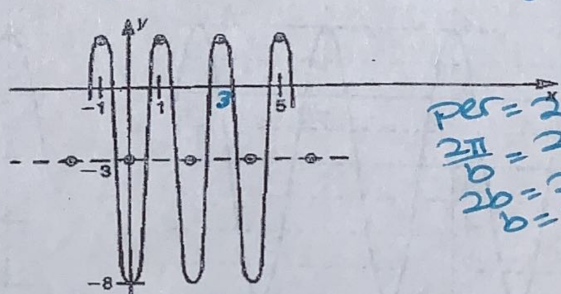

5. $y=-3\cos\frac{1}{4}(x)+5$ or $y=3\sin\frac{1}{4}(x-2\pi)+5$

per $=8\pi$
$\frac{2\pi}{b}=8\pi$
$b=1/4$

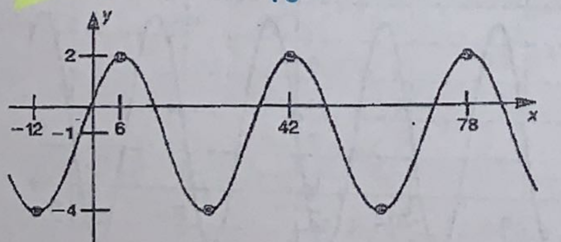



6. $y=-5\cos\pi(x)-3$ or $y=5\cos\pi(x-1)+3$

per $=2$
$\frac{2\pi}{b}=2$
$2b=2\pi$
$b=\pi$

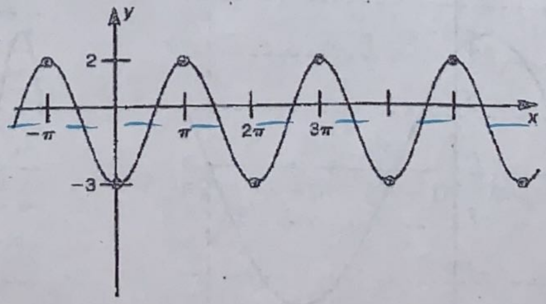



7. $y=-3\cos\frac{\pi}{18}(x+12)-1$
or $y=3\cos\frac{\pi}{18}(x-6)-1$



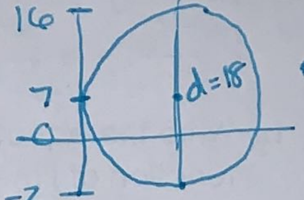

per $=36$
$\frac{2\pi}{b}=36$
$36b=2\pi$
$b=\frac{2\pi}{36}=\frac{\pi}{18}$

8. $y=-2.5\cos(x)-.5$ or $y=2.5\cos(x-\pi)-.5$



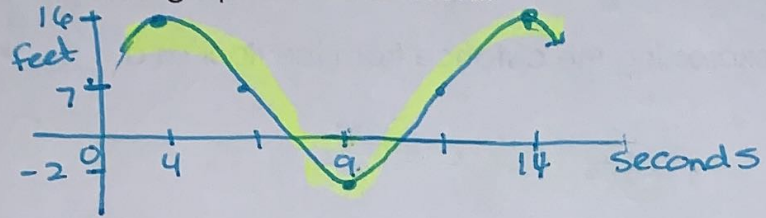

per $=2\pi$
$b=1$
$d=-1/2$
$a=2.5$

1. Mark Twain sat on the deck of a river steamboat. As the paddle wheel turned, a point on the blade moved in such a way that its distance, d, from the water's surface was a sinusoidal function of time. When Twain's stopwatch read 4 seconds, the point was at its highest, 16 ft above the water's surface. The wheel's diameter was 18 ft, and it completed a revolution every 10 seconds.




   a. Sketch a graph of the sinusoid.

$a=9$
$b=\pi/5$
$c=4$
$d=7$




   b. Write the equation of the sinusoid.

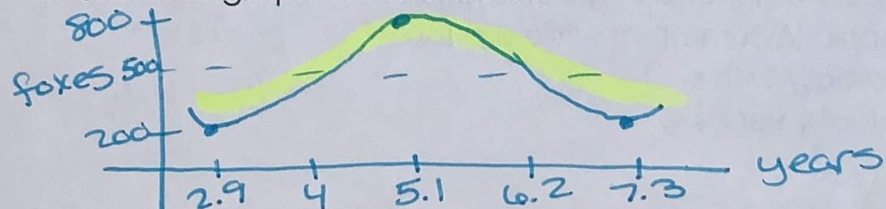
$$y=9\cos\frac{\pi}{5}(x-4)+7$$

   c. How far above the surface was the point on the blade when Mark's stopwatch read 17 seconds?

$$y=9\cos\left[\frac{\pi}{5}(17-4)\right]+7$$
$$\approx 4.2 \text{ feet}$$

2. Naturalists find that populations of some kinds of predatory animals vary periodically with time. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept at time, t = 0 years. A minimum number of 200 foxes appeared when t = 2.9 years. The next maximum, 800 foxes, occurred at t = 5.1 years.

   a. Sketch a graph of the sinusoid.




$a=300$
$b=5\pi/11$
$c=2.9$
$d=500$

   b. Write the equation expressing the number of foxes as a function of time.

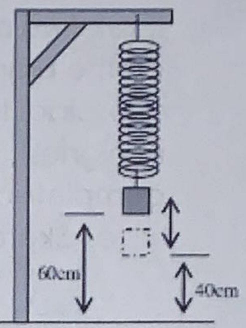
$4.4=\frac{2\pi}{b}$
$4.4b=2\pi$
$b=\frac{2\pi}{4.4}=\frac{5\pi}{11}$

$$y=-300\cos\frac{5\pi}{11}(x-2.9)+500$$

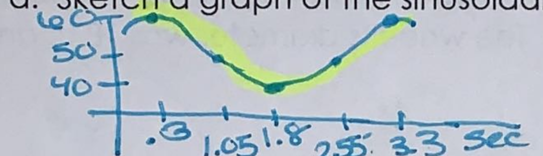
   c. Predict the fox population when t = 7, 8, 9, & 10 years.

7 yrs → 227 foxes
8 yrs → 337 foxes
9 yrs → 726 foxes
10 yrs → 726 foxes

3. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. Start a stopwatch. When the stopwatch reads 0.3 sec, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 sec.




a. Sketch a graph of the sinusoidal function.




b. Write the equation expressing the distance from the floor as a function of time.

a = 10
b = 2π/3
c = .3
d = 50

per = 3
$\frac{2\pi}{b} = 3$
$b = \frac{2\pi}{3}$

$y = 10\cos\frac{2\pi}{3}(x - .3) + 50$

c. What is the distance from the floor when the stopwatch reads 17.2 sec?

$y = 10\cos\left(\frac{2\pi}{3}(17.2 - .3)\right) + 50$

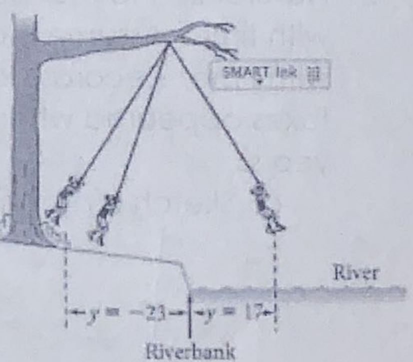
y = 43.3 cm

d. What is the distance from the floor when you started the stopwatch?

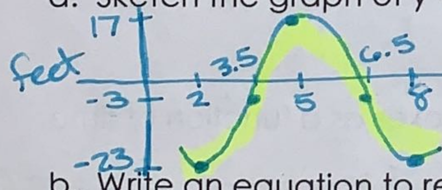
$y = 10\cos\left(\frac{2\pi}{3}(0 - .3)\right) + 50$

y = 58.1 cm

4. Zoey is at summer camp. One day she is swinging on a rope tied to a tree branch, going back and forth alternately over land and water. Nathan starts a stopwatch. When x = 2 seconds, Zoey is at one end of her swing, y = -23 feet from the riverbank. When x = 5 seconds, she is then at the other end of her swing, y = 17 feet from the riverbank. Assume that while she is swinging y varies sinusoidally with x.




a. Sketch the graph of y versus x.




b. Write an equation to represent Zoey's distance from the riverbank with respect to time.

a = 20
b = π/3
c = 2
d = -3

per = 6
$\frac{2\pi}{b} = 6$
$6b = 2\pi$
$b = \pi/3$

$y = -20\cos\frac{\pi}{3}(x - 2) - 3$

c. How far is Zoey from the riverbank after 13.2 sec? Was she over land or water at this time?

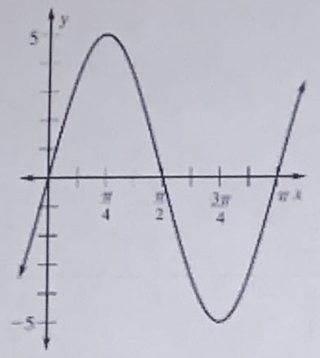
$y = -20\cos\left(\frac{\pi}{3}(13.2 - 2)\right) - 3$

y = -16.4 ft from the riverbank

She is over land.

**Examine the graph below and determine the amplitude, period, phase shift, and vertical shift of each using COSINE as the parent function. Then write an equation of the function.**

**1.**




Amplitude: 5

Period: π

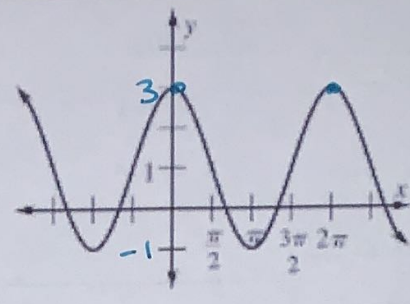
Phase Shift: π/4

Vertical Shift: 0

Function: $y=5\cos 2(x-\frac{\pi}{4})$

$\frac{2\pi}{b}=\pi$
$\pi b=2\pi$
$b=2$

**2.**




Amplitude: 2

Period: 2π   $b=1$

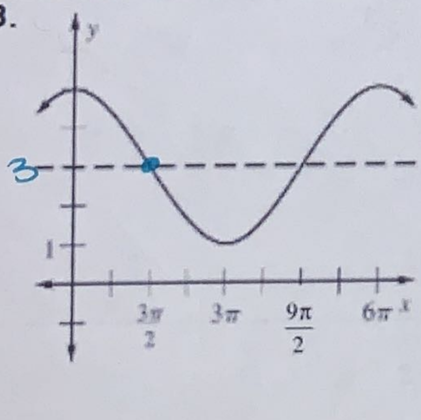
Phase Shift: none

Vertical Shift: 1

Function: $y=2\cos(x)+1$

**Examine the graph below and determine the amplitude, period, phase shift, and vertical shift of each using SINE as the parent function. Then write an equation of the function.**

**3.**




Amplitude: 2

Period: 6π

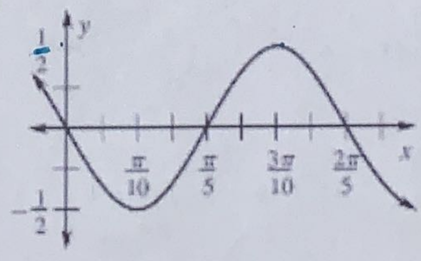
Phase Shift: 3π/2

Vertical Shift: 3

Function: $y=-2\sin\frac{1}{3}(x-\frac{3\pi}{2})+3$

$\frac{2\pi}{b}=6\pi$
$6\pi b=2\pi$
$b=1/3$

**4.**




Amplitude: 1/2

Period: 2π/5

Phase Shift: none

Vertical Shift: none

Function: $y=-\frac{1}{2}\sin 5(x)$

$\frac{2\pi}{b}=\frac{2\pi}{5}$
$b=5$

**Identify the amplitude, period, phase shift and vertical shift of the following trig functions.**

**5.** $y=-10\cos\left(\frac{x}{6}\right)$

$y=-10\cos\frac{1}{6}(x)$

$b=1/6$

$\frac{2\pi}{1/6}=\text{per}$

Amplitude: 10

Period: $12\pi$

Phase Shift: none

Vertical Shift: none

**6.** $y=5-2\sin\left(\frac{2x}{3}\right)$

$y=-2\sin\frac{2}{3}(x)+5$

$b=2/3$

$\frac{2\pi}{2/3}=\text{per}$

$2\pi\cdot\frac{3}{2}$

Amplitude: 2

Period: $3\pi$

Phase Shift: none

Vertical Shift: up 5

**7.** $y=3\cos(6x+\pi)$

$y=3\cos 6\left(x+\frac{\pi}{6}\right)$

$b=6$

$\text{per}=\frac{2\pi}{6}$

Amplitude: 6

Period: $\pi/3$

Phase Shift: left $\pi/6$

Vertical Shift: none

**8.** $y=-4\sin\left(\frac{2}{3}x-\frac{\pi}{3}\right)$

$y=-4\sin\frac{2}{3}\left(x-\frac{\pi}{2}\right)$

$b=2/3$

$\text{per}=\frac{2\pi}{2/3}$

$2\pi\cdot\frac{3}{2}$

Amplitude: 4

Period: $3\pi$

Phase Shift: right $\frac{\pi}{2}$

Vertical Shift: none

**Given the following information about each trig function, write a possible equation for each.**

**9.** Sine Function

amplitude $=\frac{1}{2}$

period $=\frac{\pi}{3}$

vertical shift $=-4$

$\frac{\pi}{3}=\frac{2\pi}{b}$

$6\pi=\pi b$

$b=6$

$y=\pm\frac{1}{2}\sin 6(x)-4$

**10.** Sine Function

amplitude $=7$

period $=4\pi$

phase shift $=-\frac{\pi}{3}$

$4\pi=\frac{2\pi}{b}$

$4\pi b=2\pi$

$b=1/2$

$y=\pm 7\sin\frac{1}{2}\left(\theta+\frac{\pi}{3}\right)$

**11.** Cosine Function

amplitude $=1$

period $=2\pi$

phase shift $=\frac{5\pi}{6}$

vertical shift $=3$

$b=1$

$y=\pm\cos\left(x-\frac{5\pi}{6}\right)+3$

**12.** Cosine Function

amplitude $=3$

period $=\pi$

phase shift $=-\pi$

vertical shift $=-1.5$

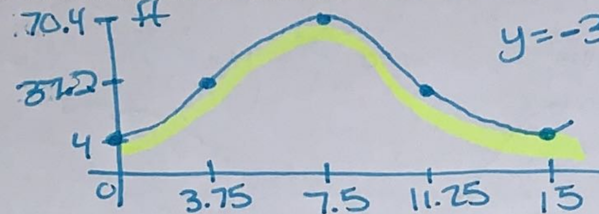
$\pi=\frac{2\pi}{b}$

$b=2$

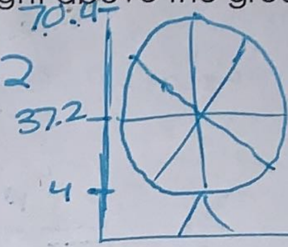
$y=\pm 3\cos 2(\theta+\pi)-1.5$

**13. There is a Ferris wheel every year at the North Georgia State Fair. This year the wheel has a radius of 33.2 feet and makes a complete revolution every 15 seconds. For clearance, the bottom of the Ferris wheel is 4 feet above the ground.**

a) Graph and write a function to show how one passenger's height above the ground varies with the time on the Ferris wheel.




$y = -33.2\cos\frac{2\pi}{15}(x) + 37.2$




$r = 33.2$
$d = 66.4$

per = 15 sec
$15 = \frac{2\pi}{b}$
$15b = 2\pi$
$b = \frac{2\pi}{15}$

b) Determine how far above the ground the passenger is 52 seconds into the ride.

$y = -33.2\cos\left(\frac{2\pi}{15}(15)\right) + 37.2$

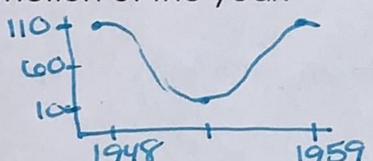
$y = 4$ ft

**14. For several hundred years, astronomers have kept track of the number of solar flares, or "sunspots", that occur on the surface of the Sun. The number of sunspots each year varies periodically, from a minimum of about 10 per year to a maximum of about 110 per year. Between 1750 and 1948, there were exactly 18 complete cycles.**

a) What is the period of the sunspot cycle?

$\frac{1948 - 1750}{18} = \frac{198}{18} = 11$ years

b) Assume that the number of sunspots per year is a sinusoidal function of time and that a maximum occurred in 1948. Find an equation expressing the number of sunspots per year as a function of the year.

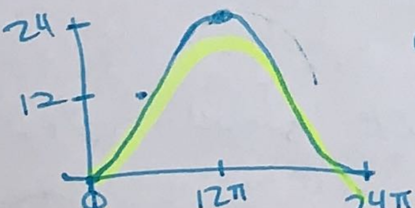



$a = 50$
$b = 2\pi/11$
$c = 1948$
$d = 60$

$y = 50\cos\frac{2\pi}{11}(x - 1948) + 60$

**15. As you stop your car at a traffic light, a pebble becomes wedged between your tire treads. When you start moving again, the distance between the pebble and the pavement varies simulateously with the distance you have traveled. The period is the circumference of the tire. The diameter of the tire is 24 inches.**

a) Graph and write an equation of the function that has NO phase shift.




$C = 2\pi r$ or $\pi d$
$C = 2\pi(12)$
$C = 24\pi$

$y = -12\cos\frac{1}{12}(x) + 12$

per $= 24\pi$
$\frac{2\pi}{b} = 24\pi$
$b = \frac{2\pi}{24\pi} = \frac{1}{12}$

$a = 12$
$b = \frac{1}{12}$
$c = 0$
$d = 12$

b) What is the pebble's distance from the pavement when you have moved 15 inches?

$y = -12\cos\left(\frac{1}{12} \cdot 15\right) + 12$

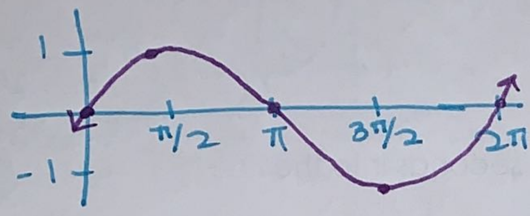
8.2 inches

# WS #7 Graphing Sine and Cosine Curves

Define the characteristics and then graph at least one full period. Label the axes appropriately.

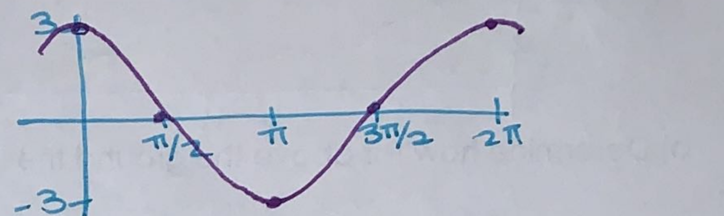
1. $y = 2\sin x$

Amp= 2 Period= $2\pi$ PS= none VS= none



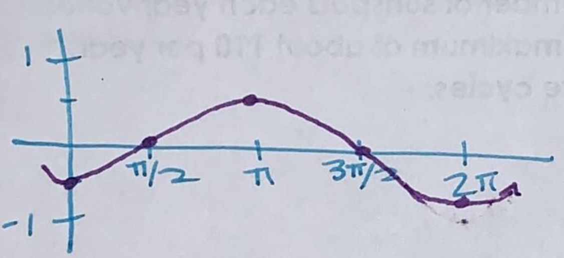

2. $y = 3\cos x$

Amp= 3 Period= $2\pi$ PS= none VS= none



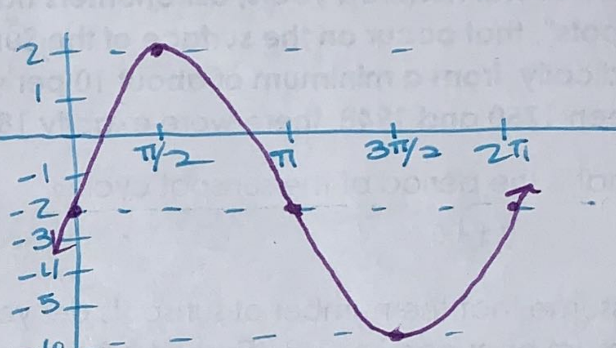

3. $y = -\frac{1}{2}\cos x$

Amp= 1/2 Period= $2\pi$ PS= n/a VS= n/a



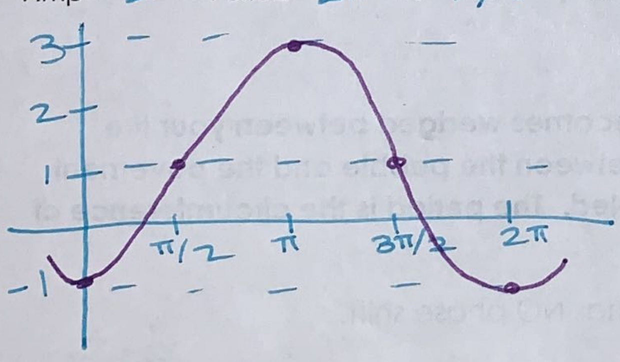

4. $y = 4\sin x - 2$

Amp= 4 Period= $2\pi$ PS= n/a VS= down 2



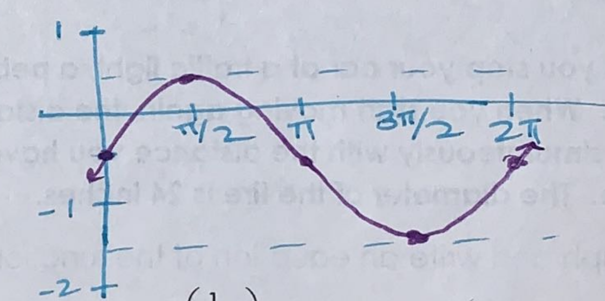

5. $y = -2\cos x + 1$

Amp= 2 Period= $2\pi$ PS= n/a VS= up 1



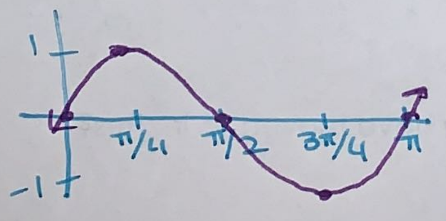

6. $y = \sin x - \frac{1}{2}$

Amp= 1 Period= $2\pi$ PS= n/a VS= down 1/2



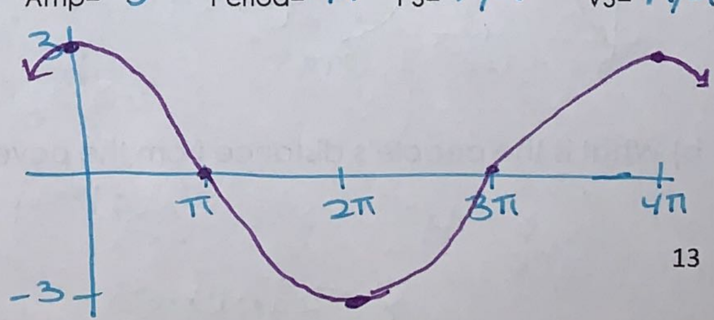

7. $y = \sin 2x$

Amp= 1 Period= $\pi$ PS= n/a VS= n/a



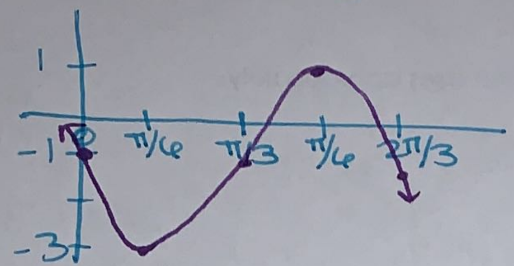

8. $y = 3\cos\left(\frac{1}{2}x\right)$

Amp= 3 Period= $4\pi$ PS= n/a VS= n/a



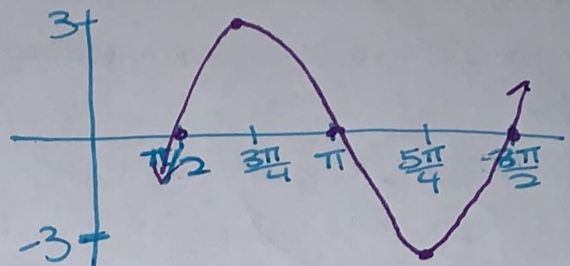
9. $y=-2\sin(3x)-1$

Amp= 2 Period= $\frac{2\pi}{3}$ PS= n/a VS= down 1



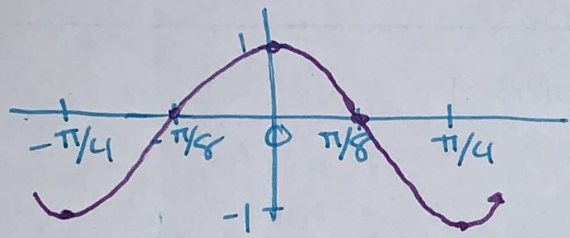

10. $y=3\sin(2x-\pi)$ $y=3\sin 2(x-\frac{\pi}{2})$

Amp= 3 Period= $\pi$ PS= right $\pi/2$ VS= n/a



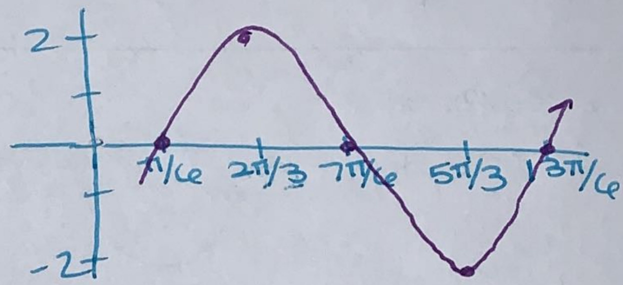

11. $y=-\cos\left(2x+\frac{\pi}{2}\right)$ $y=-\cos 2(x+\frac{\pi}{4})$

Amp= 1 Period= $\pi$ PS= left $\pi/4$ VS= n/a



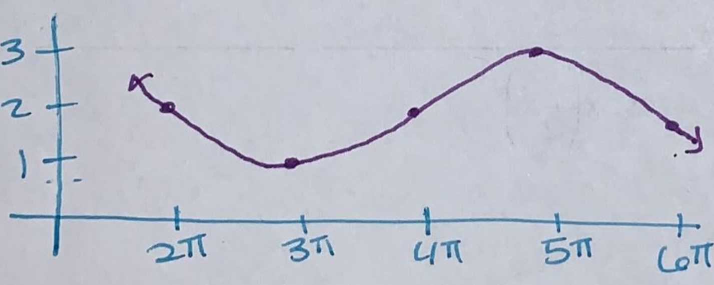

12. $y=2\sin\left(x-\frac{\pi}{6}\right)+1$

Amp= 2 Period= $2\pi$ PS= right $\pi/6$ VS= n/a



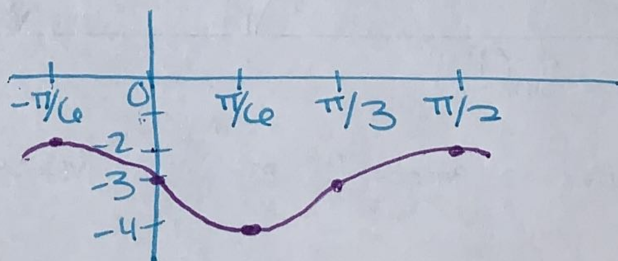

13. $y=-\sin\left(\frac{1}{2}x-\pi\right)+2$ $y=-\sin\frac{1}{2}(x-2\pi)+2$

Amp= 1 Period= $4\pi$ PS= right $2\pi$ VS= up 2



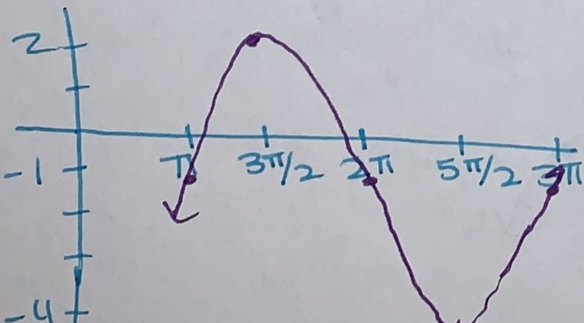

14. $y=\cos\left(3x+\frac{\pi}{2}\right)-3$ $y=\cos 3(x+\frac{\pi}{6})-3$

Amp= 1 Period= $\frac{2\pi}{3}$ PS= left $\pi/6$ VS= down 3



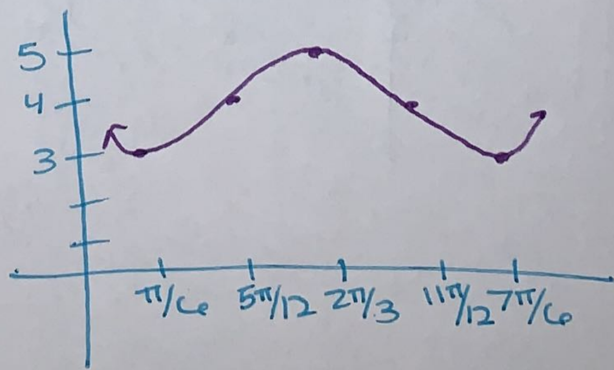

15. $y=3\sin(x-\pi)-1$

Amp= 3 Period= $2\pi$ PS= right $\pi$ VS= down 1




16. $y=-\cos\left(2x-\frac{\pi}{3}\right)+4$ $y=-\cos 2(x-\frac{\pi}{6})+4$

Amp= 1 Period= $\pi$ PS= right $\pi/6$ VS= up 4



## WS #8 Graphing Secant/Cosecant from Cosine/Sine Curves

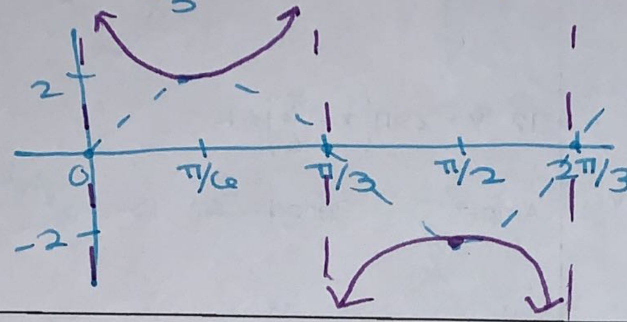
$y = a\sec(bx+c)+d$ No amplitude Period = $2\pi/b$

$y = a\csc(bx+c)+d$ Phase shift = $c$ Vertical Shift = $d$

Define the characteristics and then graph at least one full period. Label the axes appropriately.

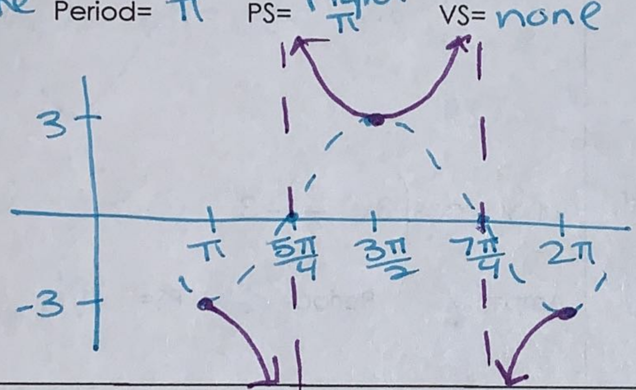
1. ~~$y=2\sin(3x)$~~ $y = 2\csc(3x)$

Amp= none Period= $\frac{2\pi}{3}$ PS= n/a VS= n/a



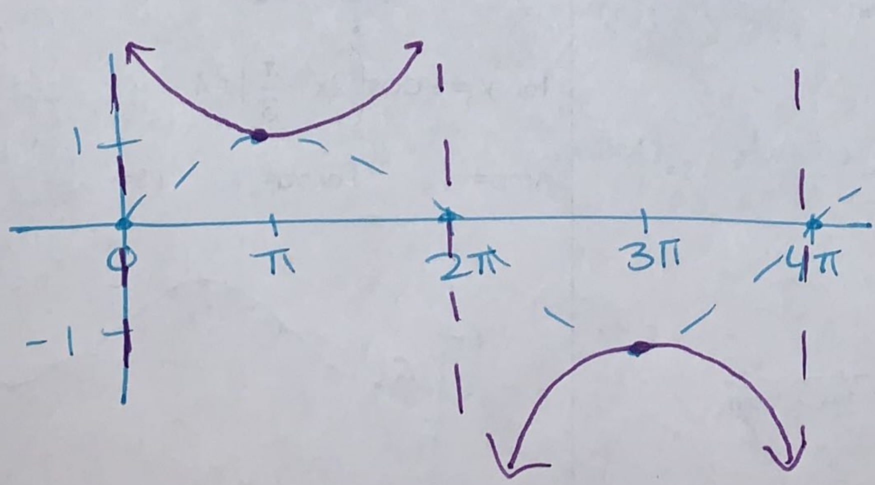

2. ~~$y=-3\cos(2x-\pi)$~~ $y = -3\sec(2x-\pi)$

Amp= none Period= $\pi$ PS= right $\pi$ VS= none




3. ~~$y=-\sin\left(\frac{1}{2}x\right)$~~ $y = -\csc\left(\frac{1}{2}x\right)$

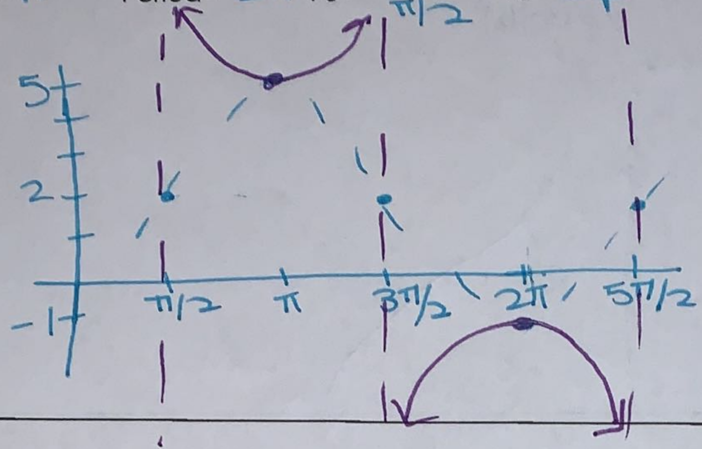
Amp= none Period= $4\pi$ PS= n/a VS= n/a



4. $\cancel{y = 2 + 3\sin\left(x - \frac{\pi}{2}\right)}$

$y = 2 + 3\csc\left(x - \frac{\pi}{2}\right)$ $y = 3\csc\left(x - \frac{\pi}{2}\right) + 2$

Amp= none Period= $2\pi$ PS= right $\pi/2$ VS= up 2




$x - \frac{\pi}{2} = 0$

$x = \frac{\pi}{2}$ beg.

$x - \frac{\pi}{2} = 2\pi$

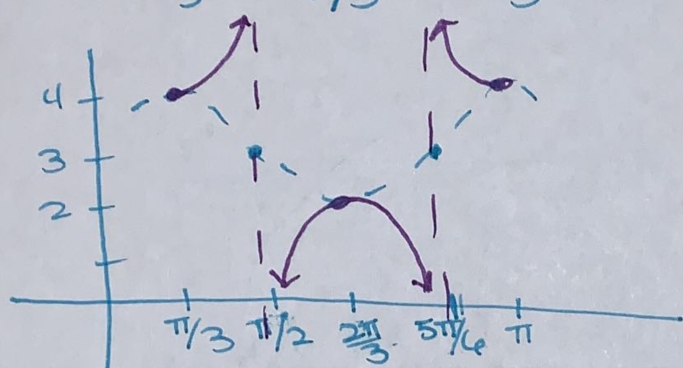
$x = \frac{5\pi}{2}$ end

5. $\cancel{y = \cos(3x - \pi) + 3}$

$y = \sec(3x - \pi) + 3$

$y = \sec 3\left(x - \frac{\pi}{3}\right) + 3$

Amp= none Period= $\frac{2\pi}{3}$ PS= right $\pi/3$ VS= up 3




$3x - \pi = 0$

$x = \frac{\pi}{3}$ beg

$3x - \pi = 2\pi$

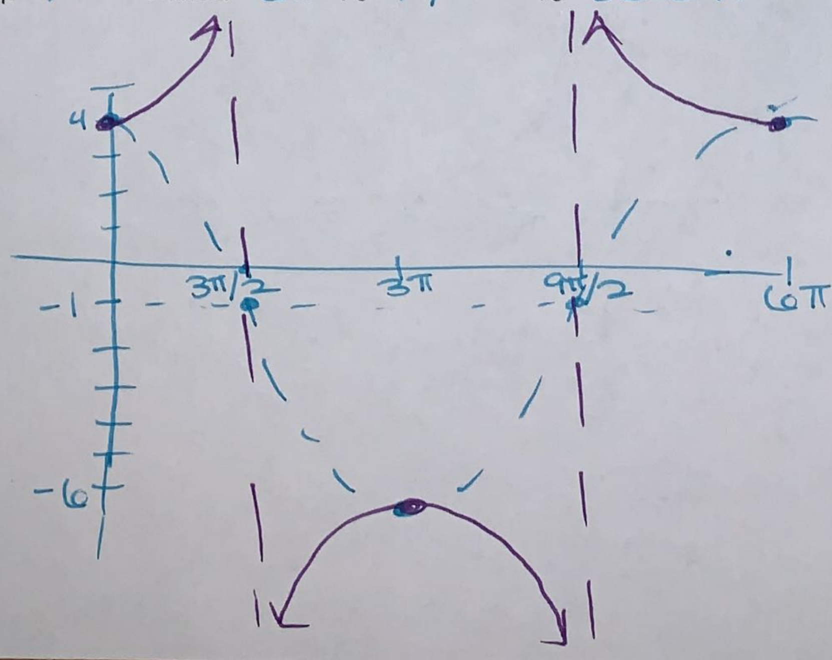
$3x = 3\pi$

$x = \pi$ end

6. $\cancel{y = 5\cos\left(\frac{1}{3}x\right) - 1}$

$y = 5\sec\left(\frac{1}{3}x\right) - 1$

Amp= none Period= $6\pi$ PS= n/a VS= down 1



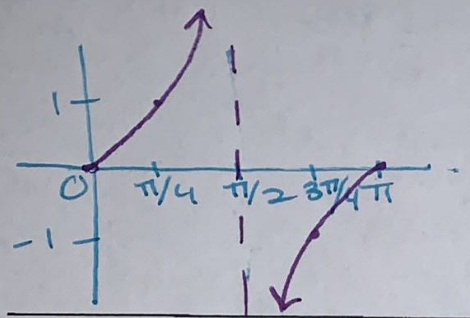
## WS #9 Graphing Tangent and Cotangent Curves

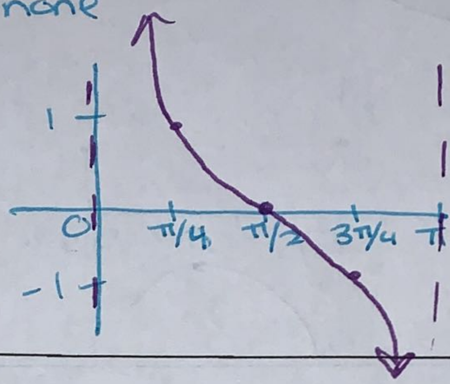
* Period = $\pi$ $\quad y = a\tan(bx+c)+d \quad y = a\cot(bx+c)+d$

**Define the characteristics and then graph at least one full period. Label the axes!**

1. $y = \tan(x)$ $\qquad$ $y = \cot(x)$

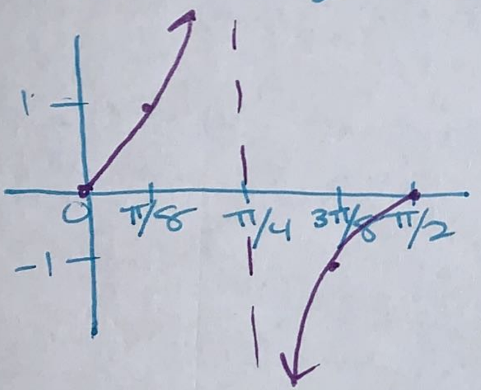
Amp= none Period= $\pi$ PS= none VS= none

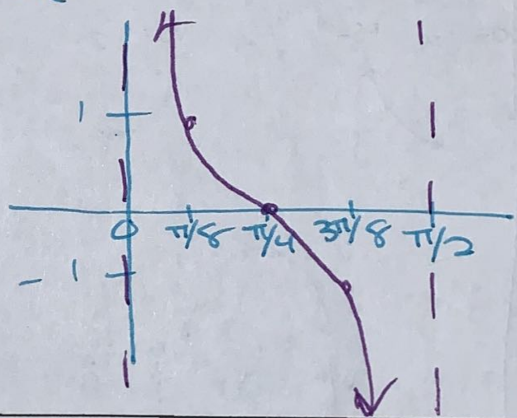



2. $y = \tan(2x)$ $\qquad$ $y = \cot(2x)$

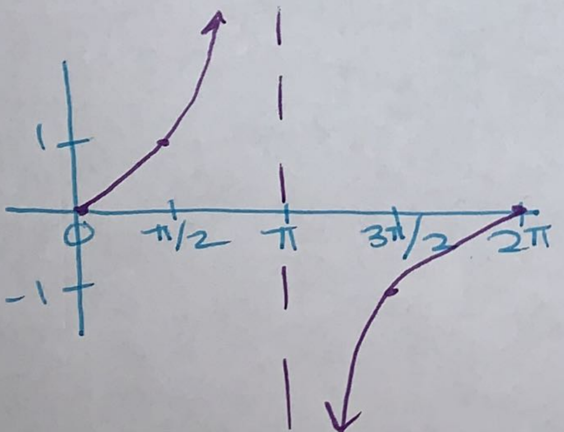
Amp= none Period= $\frac{\pi}{2}$ PS= none VS= none

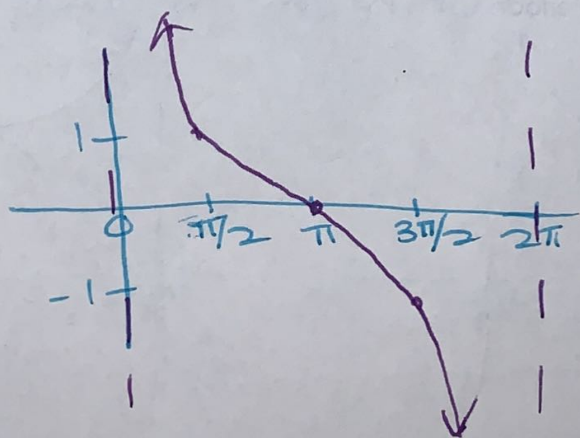



3. $y = 3\tan\left(\frac{1}{2}x\right)$ $\qquad$ $y = 3\cot\left(\frac{1}{2}x\right)$

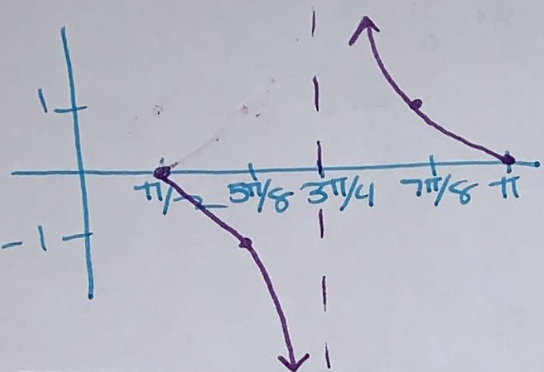
Amp= none Period= $2\pi$ PS= none VS= none

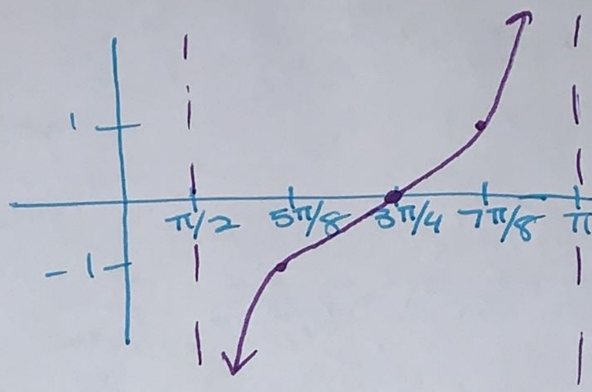


4. $y = -\tan(2x - \pi)$ $y = -\tan 2(x - \frac{\pi}{2})$ $y = -\cot(2x - \pi)$

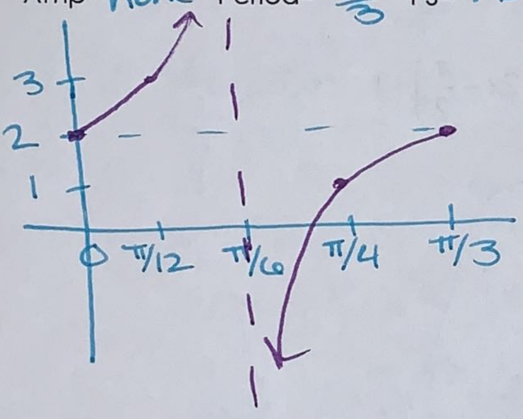
Amp= none Period= $\frac{\pi}{2}$ PS= right $\pi/2$ VS= none

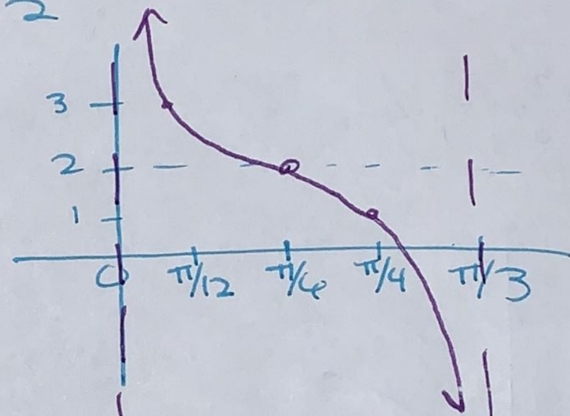



5. $y = \tan(3x) + 2$ $y = \cot(3x) + 2$

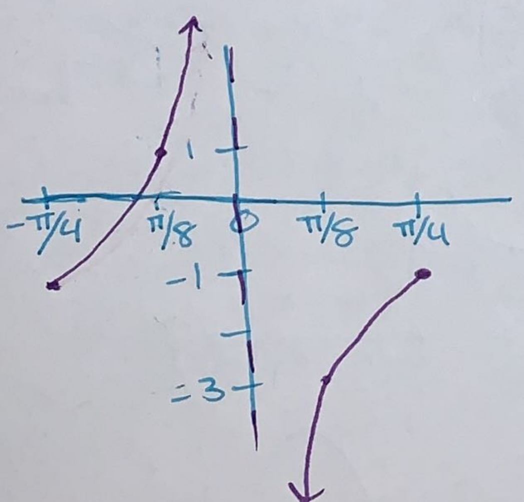
Amp= none Period= $\frac{\pi}{3}$ PS= none VS= up 2

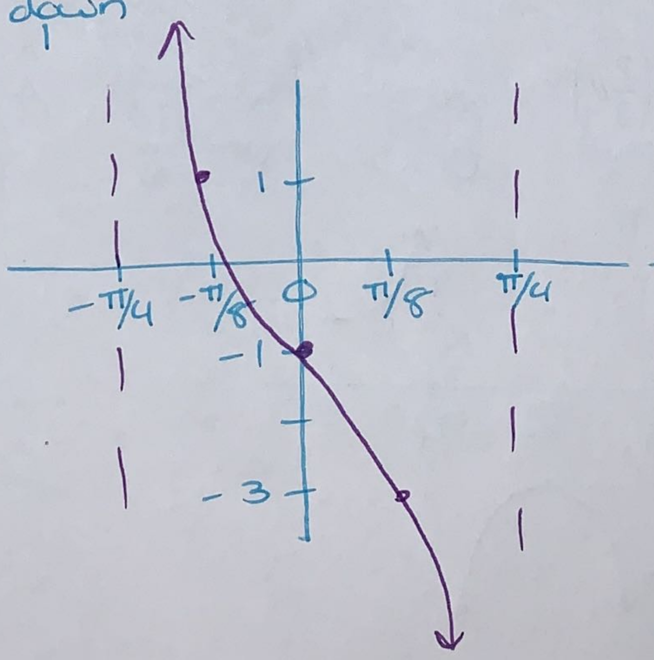



6. $y = 2\tan\left(2x + \frac{\pi}{2}\right) - 1$ $y = 2\tan 2(x + \frac{\pi}{4}) - 1$ $y = 2\cot\left(2x + \frac{\pi}{2}\right) - 1$

Amp= none Period= $\frac{\pi}{2}$ PS= left $\pi/4$ VS= down 1



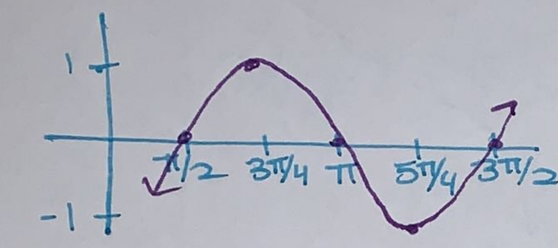
# WS #10 Graphing All Trig Functions

Graph each function. Label the x and y axis. Include a minimum of one period.

1. $y=\sin(2x-\pi)$

$2x-\pi=0$ $\quad$ $2x-\pi=2\pi$

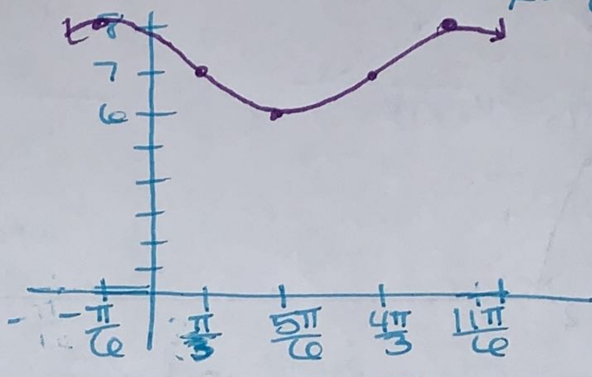
$x=\frac{\pi}{2}$ $\quad$ $x=\frac{3\pi}{2}$




2. $y=7+\cos(x+\frac{\pi}{6})$

$x+\frac{\pi}{6}=0$ $\quad$ $x+\frac{\pi}{6}=2\pi$

$x=-\frac{\pi}{6}$ $\quad$ $x=\frac{11\pi}{6}$




$-\frac{\pi}{6}$ $\quad$ $\frac{2\pi}{6}$ $\quad$ $\frac{5\pi}{6}$ $\quad$ $\frac{8\pi}{6}$ $\quad$ $\frac{11\pi}{6}$

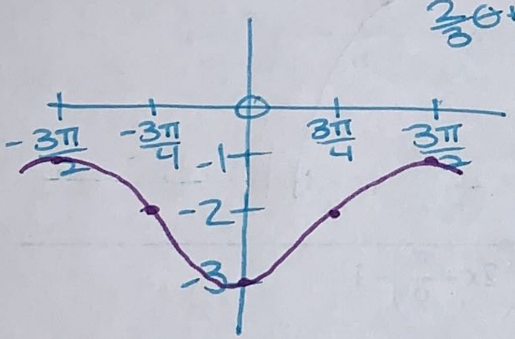
3. $y=\cos\left(\frac{2}{3}\theta+\pi\right)-2$

$\frac{2}{3}\theta+\pi=0$

$\theta=-\frac{3\pi}{2}$

$\frac{2}{3}\theta+\pi=2\pi$

$\theta=\frac{3\pi}{2}$

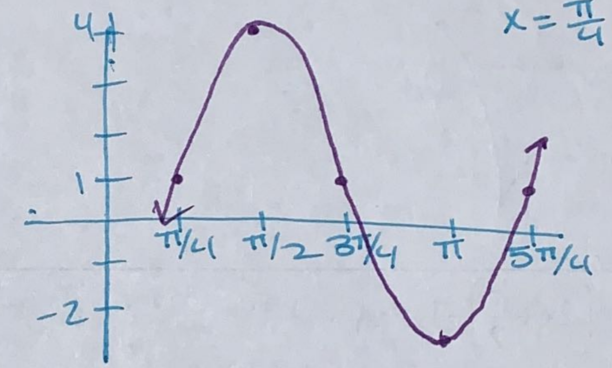



4. $y=3\sin\left(2x-\frac{\pi}{2}\right)+1$

$2x-\frac{\pi}{2}=0$ $\quad$ $2x-\frac{\pi}{2}=2\pi$

$2x=\frac{\pi}{2}$ $\quad$ $2x=\frac{5\pi}{2}$

$x=\frac{\pi}{4}$ $\quad$ $x=\frac{5\pi}{4}$

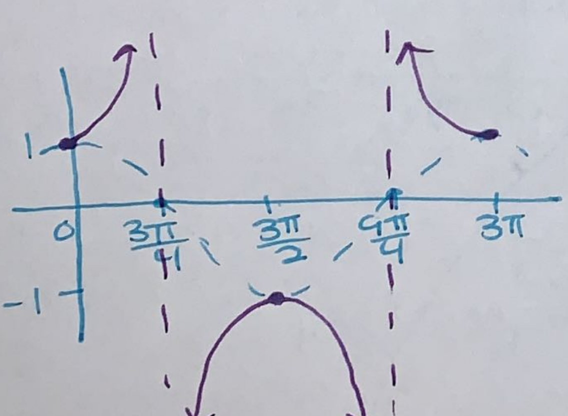



5. $y=\sec\left(\frac{2}{3}\theta\right)$

$\cos(\frac{2}{3}\theta$

$\frac{2}{3}\theta=0$ $\quad$ $\frac{2}{3}\theta=2\pi$

$\theta=0$ $\quad$ $\theta=3\pi$

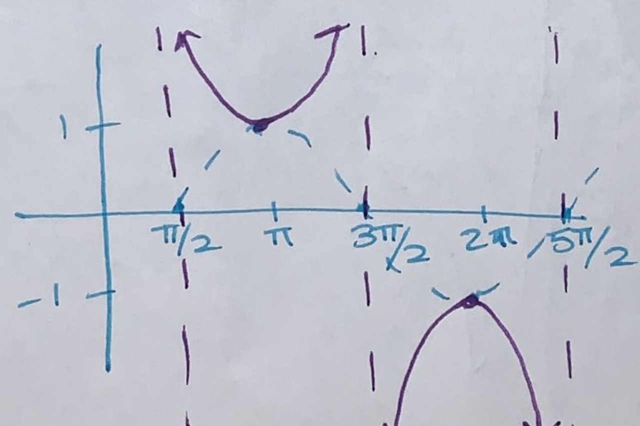



6. $y=\csc\left(x-\frac{\pi}{2}\right)$

$y=\sin(x-\frac{\pi}{2})$

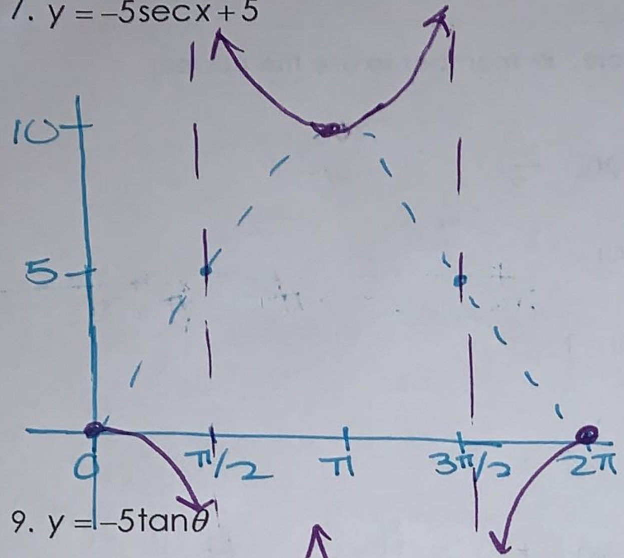
$x-\frac{\pi}{2}=0$ $\quad$ $x-\frac{\pi}{2}=2\pi$

$x=\frac{\pi}{2}$ $\quad$ $x=\frac{5\pi}{2}$



cos

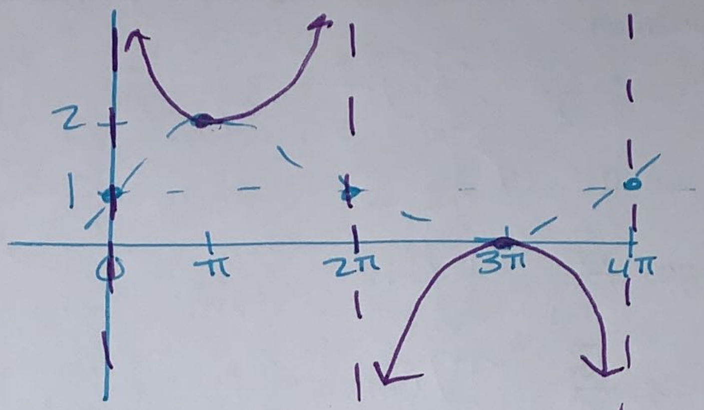
7. $y = -5\sec x + 5$



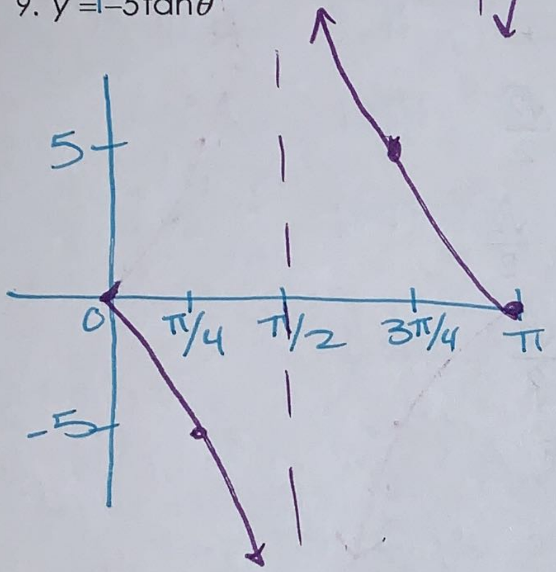

sin

8. $y = \csc\left(\frac{1}{2}x\right) + 1$ $\frac{1}{2}x = 0$ $\frac{1}{2}x = 2\pi$

$x = 0$ $x = 4\pi$

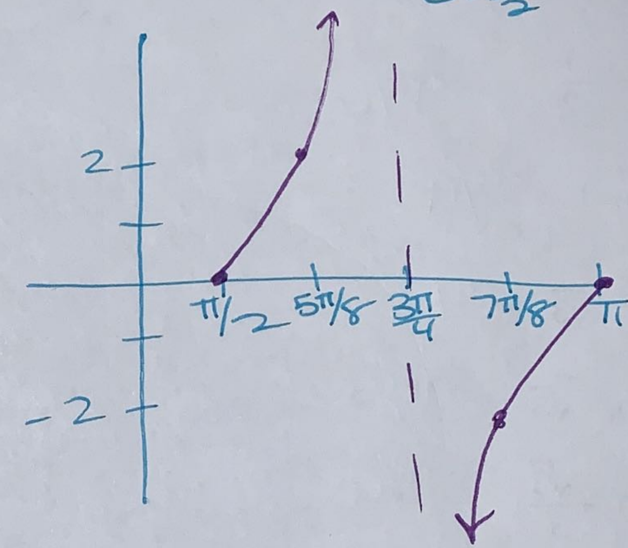



9. $y = -5\tan\theta$



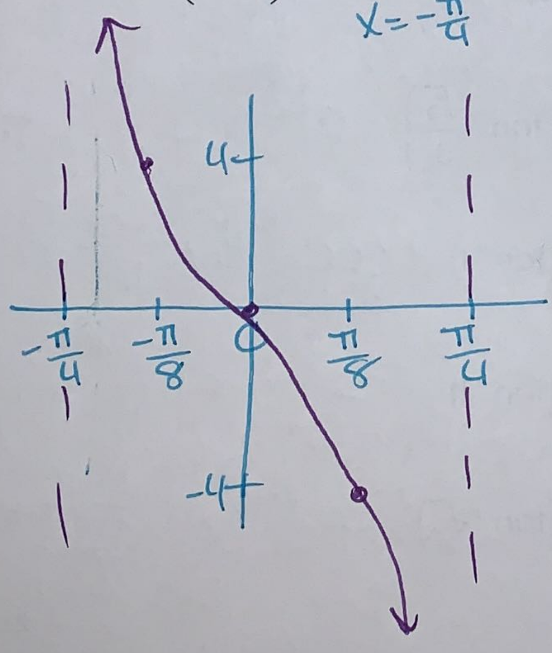

10. $y = 2\tan(2\theta - \pi)$ $2\theta - \pi = 0$ $2\theta - \pi = \pi$

$\theta = \frac{\pi}{2}$ $\theta = \pi$



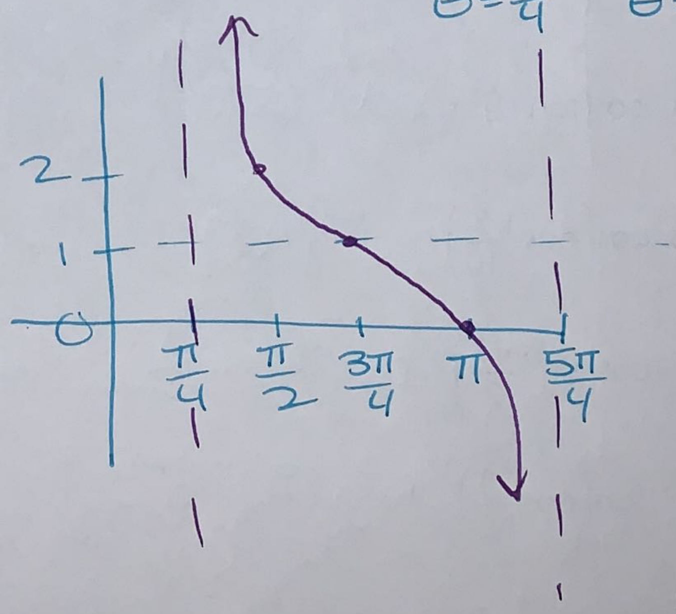

11. $y = 4\cot\left(x + \frac{\pi}{4}\right)$ $x + \frac{\pi}{4} = 0$ $x + \frac{\pi}{4} = \pi$

$x = -\frac{\pi}{4}$ $x = \frac{\pi}{4}$




12. $y = \cot\left(\theta - \frac{\pi}{4}\right) + 1$ $\theta - \frac{\pi}{4} = 0$ $\theta - \frac{\pi}{4} = \pi$

$\theta = \frac{\pi}{4}$ $\theta = \frac{5\pi}{4}$



# WS #11 Finding Exact Values – Inverses

**Find the exact value of each expression. Use your unit circle. Remember to use the correct quadrants.**

1. $\cos^{-1}-\frac{\sqrt{2}}{2} = 3\pi/4$

2. $\tan^{-1}-\frac{\sqrt{3}}{3} = -\pi/6$

3. $\tan^{-1}0 = 0$

4. $\tan^{-1}1 = \pi/4$

5. $\cos^{-1}\frac{\sqrt{2}}{2} = \pi/4$

6. $\sin^{-1}1 = \pi/2$

7. $\cos^{-1}\frac{\sqrt{3}}{2} = \pi/6$

8. $\cos^{-1}-1 = \pi$

9. $\cos^{-1}0 = \pi/2$

10. $\sin^{-1}-1 = -\pi/2$

11. $\sin^{-1}-\frac{1}{2} = -\pi/6$

12. $\tan^{-1}\sqrt{3} = \pi/3$

13. $\tan^{-1}-\sqrt{3} = -\pi/3$

14. $\sin^{-1}\frac{\sqrt{3}}{2} = \pi/3$

15. $\cos^{-1}1 = 0$

16. $\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

17. $\cos\left(\sin^{-1}\frac{\sqrt{2}}{2}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

18. $\sin\left(\sin^{-1}\frac{\sqrt{11}}{6}\right) = \frac{\sqrt{11}}{6}$

19. $\cot(\tan^{-1}1) = \cot\left(\frac{\pi}{4}\right) = 1$

20. $\csc\left(\tan^{-1}\frac{\sqrt{3}}{3}\right) = \csc\left(\frac{\pi}{6}\right) = 2$

21. $\cos\left(\cos^{-1}\frac{9}{10}\right) = \frac{9}{10}$

22. $\csc(\tan^{-1}1) = \csc\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

23. $\tan\left(\cos^{-1}\frac{1}{2}\right) = \tan\left(\pi/3\right) = \sqrt{3}$

24. $\cos(\tan^{-1}\sqrt{3}) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

## WS #12 Finding Inverses of Exact Values

1. $\arcsin(0) = x$ $0$

2. $\sin^{-1}\left(\frac{1}{2}\right) = x$ $\pi/6$

3. $\arccos(-1) = x$ $\pi$

4. $\cos^{-1}\left(-\frac{1}{2}\right) = x$ $2\pi/3$

5. $\arccos(0) = x$ $\pi/2$

6. $\cos^{-1}\left(\frac{1}{2}\right) = x$ $\pi/3$

7. $\arccos\left(\frac{1}{2}\right) = x$ $\pi/3$

8. $\arctan\sqrt{3} = x$ $\pi/3$

9. $\sin^{-1}(0) = x$ $0$

10. $\arccos\left(\frac{\sqrt{3}}{2}\right) = x$ $\pi/6$

11. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = x$ $\pi/6$

12. $\arccos\left(-\frac{1}{2}\right) = x$ $2\pi/3$

13. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$ $\pi/6$

14. $\arctan(1) = x$ $\pi/4$

15. $\arctan\left(-\frac{\sqrt{3}}{3}\right) = x$ $-\pi/6$

16. $\arcsin\left(-\frac{1}{2}\right) = x$ $-\pi/6$

17. $\arcsin\left(\frac{\sqrt{2}}{2}\right) = x$ $\pi/4$

18. $\sin^{-1}1 = x$ $\pi/4$

19. $\tan^{-1}\left(-\sqrt{3}\right) = x$ $-\pi/3$

20. $\cos^{-1}(0) = x$ $\pi/2$

**Simplify each expression.**

21. $\sin\left(\arccos\frac{1}{2}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

22. $\sin\left(\tan^{-1}(-1)\right) = \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

23. $\tan\left(\arcsin\frac{\sqrt{3}}{2}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

24. $\cos\left(\arctan\sqrt{3}\right) + \cos\left(\sin^{-1}0\right)$

$\cos(\pi/3) + \cos(1) = \frac{1}{2} + 1 = \frac{3}{2}$ or $1\frac{1}{2}$

25. $\arccos\left(\sin\frac{\pi}{6}\right) = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

26. $\tan\left(\cos^{-1}\frac{\sqrt{2}}{2}\right) - \cos(\arcsin 1)$

$\tan(\pi/4) - \cos(\pi/2)$

$1 - 0 = 1$

# WS #13 Trig Curves Review

**State the exact values for the amplitude, period, phase shift, and vertical shift for each function**

1. $y = 2\sin\left(2\theta - \frac{\pi}{2}\right) + 1$ $2\sin 2(\theta - \frac{\pi}{4}) + 1$

Amplitude: 2 Period: $\pi$

Phase Shift: right $\frac{\pi}{4}$ Vertical Shift: up 1

2. $y = -3\cos(\theta + \pi) - 4$

Amplitude: 3 Period: $2\pi$

Phase Shift: left $\pi$ Vertical Shift: down 4

3. $y = 3\tan(2\theta)$

Amplitude: none Period: $\pi/2$

Phase Shift: none Vertical Shift: none

4. $y = 2\cot\left(3\theta - \frac{\pi}{4}\right) - 2$ $y = 2\cot 3(\theta - \frac{\pi}{12}) - 2$

Amplitude: none Period: $\pi/3$

Phase Shift: right $\frac{\pi}{12}$ Vertical Shift: down 2

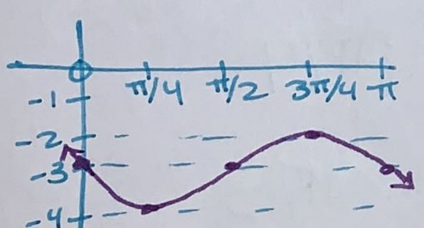
5. $y = -3\sec\left(\frac{1}{2}\theta\right) + 2$

Amplitude: none Period: $4\pi$

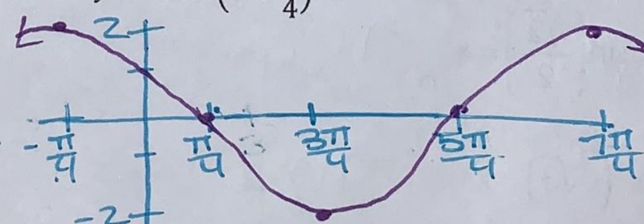
Phase Shift: none Vertical Shift: up 2

**Graph each function. Label the x and y axis.**

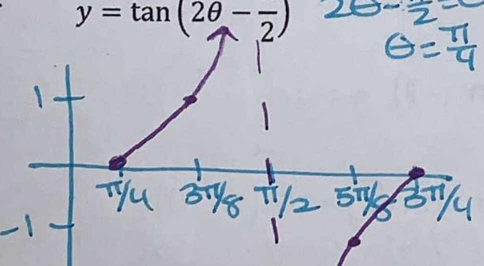
6. $y = -\sin(2\theta) - 3$



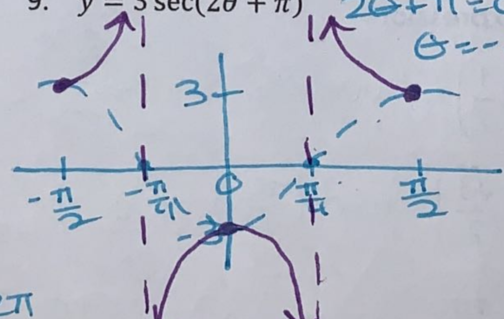

7. $y = 2\cos\left(\theta + \frac{\pi}{4}\right)$



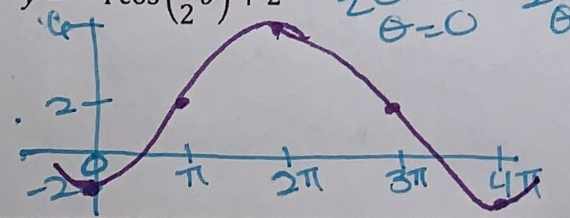

8. $y = \tan\left(2\theta - \frac{\pi}{2}\right)$ $2\theta - \frac{\pi}{2} = 0$ $\theta = \frac{\pi}{4}$ $2\theta - \frac{\pi}{2} = \pi$ $\theta = \frac{3\pi}{4}$




9. $y = 3\sec(2\theta + \pi)$ $2\theta + \pi = 0$ $\theta = -\frac{\pi}{2}$ $2\theta + \pi = 2\pi$ $\theta = \frac{\pi}{2}$




10. $y = -4\cos\left(\frac{1}{2}\theta\right) + 2$ $\frac{1}{2}\theta = 0$ $\theta = 0$ $\frac{1}{2}\theta = 2\pi$ $\theta = 4\pi$



**Write the equation of the sine function given the following information.**

11. amp $= 2$ period $= \pi$ p.s. $= 0$ v.s. $= 1$

$\pi = \frac{2\pi}{b}$

$a=2$ $b=2$ $c=0$ $d=1$

$y = \pm 2\sin 2(x)+1$

12. amp $= 1$ period $= 4\pi$ p.s. $= \frac{\pi}{3}$ v.s. $= 0$

$4\pi = \frac{2\pi}{b}$

$a=1$ $b=1/2$ $c=\frac{\pi}{3}$ $d=0$

$y = \pm \sin \frac{1}{2}(\theta - \frac{\pi}{3})$

13. amp $= \frac{1}{2}$ period $= \frac{\pi}{2}$ p.s. $= -\frac{\pi}{6}$ v.s. $= -3$

$\frac{\pi}{2} = \frac{2\pi}{b}$

$a=\frac{1}{2}$ $b=4$ $c=-\frac{\pi}{6}$ $d=-3$

$y = \pm \frac{1}{2}\sin 4(\theta + \frac{\pi}{6}) - 3$

**Find the values of each of the following. Remember principal values.**

14. $\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

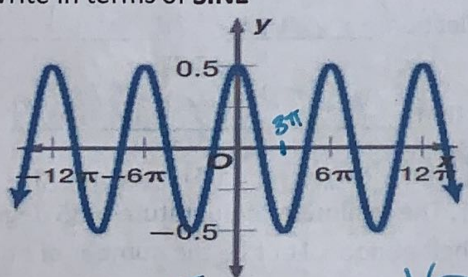
15. $\theta = \tan^{-1}(-1) = -\frac{\pi}{4}$

16. $\theta = \arctan(\sqrt{3}) = \frac{\pi}{3}$

17. $\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

**Write the equations for the following graphs.**

18. Write in terms of **SINE**




Amplitude: .5

Period: $6\pi$

Phase Shift: right $3\pi/2$

Vertical Shift: none

Reflection: yes

a $= 1/2$

b $= 1/3$

c $= 3\pi/2$

d $= 0$

$6\pi = \frac{2\pi}{b}$

$6\pi b = 2\pi$

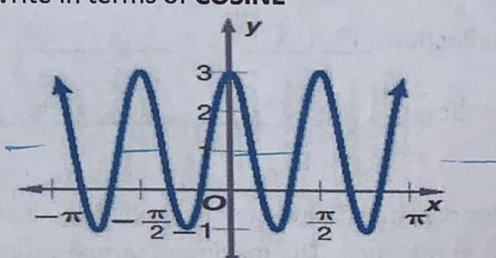
$b = \frac{2\pi}{6\pi}$

Function: $y = -\frac{1}{2}\sin\frac{1}{3}(x - \frac{3\pi}{2})$

or

$y = \frac{1}{2}\sin\frac{1}{3}(x + \frac{3\pi}{2})$

19. Write in terms of **COSINE**




Amplitude: 2

Period: $\pi/2$

Phase Shift: none

Vertical Shift: up 1

Reflection: none

a $= 2$

b $= 4$

c $= 0$

d $= 1$

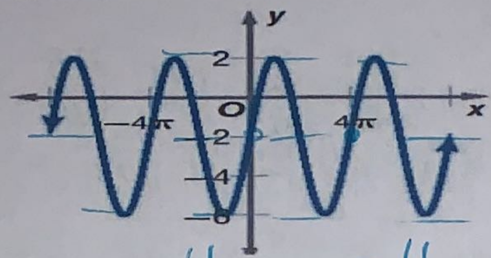
$\frac{\pi}{2} = \frac{2\pi}{b}$

$b=4$

Function: $y = 2\cos 4(x) + 1$

Sine

20.



Amplitude: 4  a = 4

Period: $4\pi$  b = 1/2  $4\pi = \frac{2\pi}{b}$

Phase Shift: none  c = 0

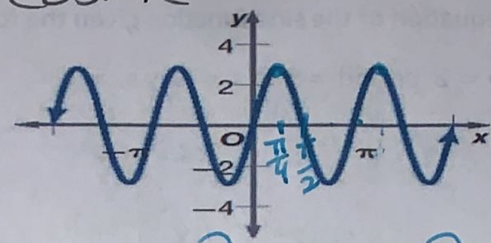
Vertical Shift: down 2

Reflection: none  d = −2

Function: $y = 4\sin\frac{1}{2}(x) - 2$

Cosine

21.



Amplitude: 2  a 2  $\pi = \frac{2\pi}{b}$

Period: $\pi$  b 2

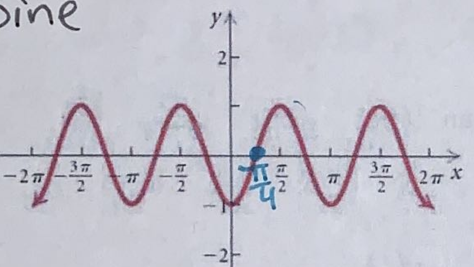
Phase Shift: right $\frac{\pi}{4}$  c $\frac{\pi}{4}$

Vertical Shift: none

Reflection: none  d 0

Function: $y = 2\cos 2(x - \frac{\pi}{4})$

22. Sine



Amplitude: 1  a 1

Period: $\pi$  b 2

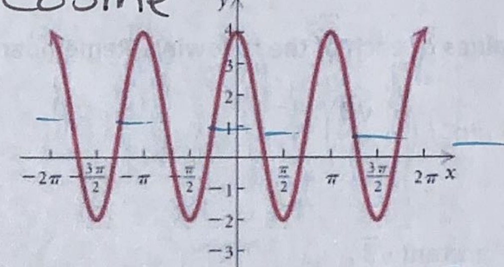
Phase Shift: right $\frac{\pi}{4}$  c $\pi/4$

Vertical Shift: none

Reflection: none  d 0

Function: $y = \sin 2(x - \frac{\pi}{4})$

23. Cosine



Amplitude: 3  a 3

Period: $\pi$  b 2

Phase Shift: none  c 0

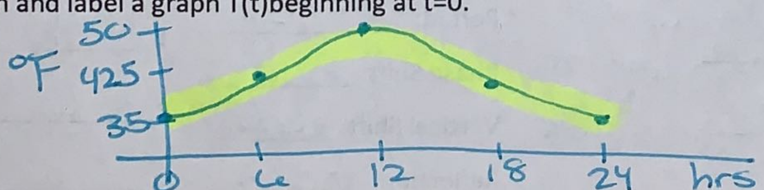
Vertical Shift: up 1

Reflection: none  d 1

Function: $y = 3\cos 2(x) + 1$

24. The temperature T(t) varies sinusoidally on a certain day in December. The minimum temperature is 35 degrees Fahrenheit at midnight. The maximum temperature is 50 degrees Fahrenheit at noon. Let t be the number of hours since midnight (t=o at midnight).

a.) Sketch and label a graph T(t)beginning at t=0.



per = 24

$\frac{2\pi}{b} = \frac{24}{1}$

$24b = 2\pi$

$b = \frac{\pi}{12}$

b.)Determine a function that represents the graph.

$y = -7.5\cos\frac{\pi}{12}(\theta) + 42.5$

c.) Find the temperature at 1 am.

$y = -7.5\cos(\frac{\pi}{12} \cdot 1) + 42.5$

35.2°F