

Unit 4 – Derivative Rules

- Homework will be assigned daily
- Daily skills checks will accumulate to form a quiz grade

Topics Covered:

- ❖ Power Rule
- ❖ Product & Quotient Rule
- ❖ Chain Rule
- ❖ Derivatives from Charts & Graphs
- ❖ Derivatives of Trig Functions
- ❖ Derivatives of Exponential & Log Functions
- ❖ L'Hopital's Rule

Quiz is _____

Test is _____

Name: _____

Constant and Power Rule Practice

Find the derivative of each function. Make sure your answers are simplified completely. If a point is given, find the value of the derivative at that point.

1. $y = 3$

2. $f(x) = x + 1$

3. $f(t) = -3t^2 + 2t - 4$

4. $s(t) = t^3 - 2t + 4$

5. $y = 4t^{\frac{4}{3}}$

6. $f(x) = 4\sqrt{x}$

7. $y = 4x^{-2} + 2x^2$

8. $y = \frac{1}{4x^3}$

9. $y = \frac{1}{(4x)^3}$

10. $y = \frac{\sqrt{x}}{x}$

11. $f(x) = x^2 - \frac{4}{x}$

12. $f(x) = x^2 - 2x - \frac{2}{x^4}$

13. $f(x) = \frac{2x^3 - 4x^2 + 3}{x^2}$

14. $y = x(x^2 + 1)$

15. $f(x) = x^{4/5}$

16. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$

17. $f(x) = \frac{4}{x^{-3}}$

18. $f(x) = \frac{\pi}{(3x)^2}$

19. $f(x) = \frac{1}{\sqrt[9]{x^7}}$

20. $f(x) = (x^2 + 2x)(x + 1)$

21. $f(x) = \frac{5x^7 + 9x^4 + 2x - 9}{10}$

22. $f(x) = (2x + 1)^2 \quad (0, 1)$

23. $f(x) = \frac{1}{x}$ at $\left(2, \frac{1}{2}\right)$

24. $f(x) = x(x^2 + 1) \quad (7, 350)$

PRODUCT & QUOTIENT RULE

PRODUCT RULE: Find the derivative 2 different ways: 1) using the Product Rule and 2) distribute first & then derive.

1. $f(x) = x^2(x^3 - 1)$

2. $f(x) = (x^3 - 1)(x^2 - 2x + 1)$

Differentiate with respect to x using the Product Rule.

3. $f(x) = (4x + 1)(3x - 2)$

4. $f(x) = (x + 3)^2$

5. $f(x) = (2x^{\frac{5}{3}} - 3)(-x^5 + 3)$

6. $f(x) = (1 + \sqrt{x})(x^3)$

When dividing by a monomial, you don't need to use the Quotient Rule. You can simplify first and then derive. Find the derivative without using the Quotient Rule.

7. $f(x) = \frac{x^2 + 3x}{6}$

8. $f(x) = \frac{5x^4 - 16x^2 + 4}{x^2}$

9. $f(x) = \frac{-3(3x - 2x^2)}{7x}$

Differentiate with respect to x using the Quotient Rule.

$$10. f(x) = \frac{x^2}{1-x^3}$$

$$11. f(x) = \frac{x^2-2}{x^2+2}$$

$$12. f(x) = \frac{x^3+3x+2}{x^2-1}$$

$$13. f(x) = (2x - 5)(4 - x)^{-1}$$

Differentiate with respect to x using the Product or Quotient Rule.

$$14. g(x) = \frac{3x^4+5x^2}{x^5+2}$$

$$15. y = \frac{\sqrt{x}}{x^3+1}$$

$$16. m(x) = \left(-4 + \frac{4}{x}\right)(3x^3 + 4)$$

$$17. h(x) = \frac{2x^4-5x^2+6x}{2x^2}$$

$$18. f(x) = (2x^3 - 5)(3x^2 + 4x - 7)$$

$$19. y = 5\sqrt{x}(3x^2 + 4x - \sqrt{x})$$

Mixed Derivatives #1

Find the derivative using the power, product, or quotient rule. If necessary, rewrite first.

1. $y = 6x^3 + 4x^2 - 2x + 5$

2. $y = \sqrt[4]{x^3}$

3. $y = 3x^2 + \frac{12}{\sqrt{x}} - \frac{1}{x^2}$

4. $y = 3 - 7x^3 + 3x^7$

5. $y = 3x^{\frac{2}{3}} + x^{\frac{3}{4}}$

6. $y = \frac{3x^3 - 5}{7}$

7. $y = \frac{4x^{\frac{3}{2}}}{x}$

8. $y = \frac{x^2 + 1}{x}$

9. $y = \frac{x^7 + 5x^6 - x^3}{x^2}$

10. $y = \frac{x+1}{\sqrt{x}}$

11. $y = (x^3 - 2)^2$

12. $y = \frac{x^2 - 4}{x + 3}$

13. $y = \frac{2x+1}{2x-1}$

14. $y = \frac{x^2 + 1}{x^2 - 1}$

15. $y = \frac{1}{1 + \sqrt{x}}$

16. $y = \frac{(x+1)(2x-5)}{(x+2)}$

17. $y = (3x^3 + 4x)(x-5)(x+1)$

Mixed Derivatives #2

Find the derivative using the power, product, or quotient rule. Remember to rewrite if necessary.

1. $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

2. $y = \sqrt{x}(\sqrt{x} + 3)$

3. $y = x(x^2 + \sqrt{x})$

4. $y = \frac{x^4 - x^3}{x^{\frac{1}{2}}}$

5. $y = \frac{4x}{3x+1}$

6. $y = \sqrt[3]{x^2} - \sqrt[3]{x}$

7. $y = (3x-2)(x^3+1)$

8. $y = x^3(2x^4 - x)$

9. $y = \frac{3x+4}{2x-3}$

10. $y = \frac{x^3 - x^2 + 2}{x^2}$

11. Find all derivatives:

$$y = x^5 + \frac{1}{6}x^2 - \frac{1}{3}x$$

12. Given: $y = x^3 - 5x^2 + 3x - 1$.

Write the equation of the tangent line to the curve at $x = 2$. (Hint: find y' first.)

Chain Rule

1. $y = (x^3 - 4)^4$

2. $y = (2x^2 + 5)^7$

3. $f(x) = (x^2 + 2x + 5)^6$

4. $f(x) = \sqrt[3]{x^2 + x}$

5. $y = \sqrt{(3x + 1)^3}$

6. $y = (\sqrt{x} + 1)^2$

7. $g(x) = \frac{1}{\sqrt{2x^3 - 7x^2}}$

8. $y = (5x^2 - 3x)^{-\frac{2}{3}}$

9. $y = \sqrt[3]{(x^2 + 4)^2}$

10. $f(x) = \frac{5}{(4x - 3)^2}$

11. $f(x) = (x^2 - 3)(5x - 1)^6$

12. $y = \sqrt{3x^2 + 5}$

Derivatives from Charts and Graphs

1. If $f(3) = 4$, $g(3) = 2$, $f'(3) = 6$ and $g'(3) = 5$ find the following.

a) $(f + g)'(3)$

b) $-5g'(3)$

c) $(f \cdot g)'(3)$

d) $\left(\frac{f}{g}\right)'(3)$

#2 – 7 Given the following chart, find the indicated derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	3	1	-5	8
-1	-9	7	4	1
0	5	9	9	-3
1	3	-3	2	6
2	-5	3	8	-4

2. If $h(x) = f(x) + g(x)$, find $h'(-1)$

3. If $h(x) = 7g(x)$, find $h'(0)$

4. If $h(x) = g(x) \cdot f(x)$, find $h'(0)$

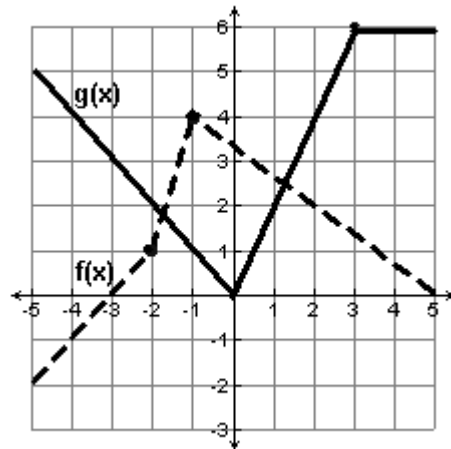
5. If $h(x) = \frac{f(x)}{g(x)}$, find $h'(0)$

6. If $h(x) = f(g(x))$, find $h'(1)$

#7 - 10 Use the graph to find the derivative.

7. If $K(x) = f(x) + g(x)$, find $K'(-3)$

8. If $K(x) = f(x) \cdot g(x)$, find $K'(2)$



9. If $K(x) = f(g(x))$, find $K'(1)$

10. If $K(x) = (g \circ f)(x)$, find $K'(-4)$

Derivatives of Trigonometric Functions

1. $y = 2 \sin x$

2. $y = \frac{\sin x}{2}$

3. $y = x + \cos x$

4. $y = x^2 - \frac{1}{2} \cos x$

5. $y = 5 + \sin x$

6. $y = \frac{1}{x} - 3 \sin x$

7. $y = \pi \cos x$

8. $y = 1 + x - \cos x$

9. $y = x^{-1} + 5 \sin x$

10. $y = \csc x - 5x + 7$

11. $y = 4\sqrt{x} + 3 \cos x$

12. $y = 2 \sin x + 3 \cos x$

13. $y = x \cdot \cos x$

14. $y = 2 \sin x - \tan x$

15. $y = 2x + \cot x$

16. $y = \cot x \cdot \sec x$

17. $y = x^2 \sin x$

18. $y = 5 + \frac{1}{\tan x}$

19. $y = \frac{\sin x}{x}$

20. $y = \sin x \cdot \cos x$

21. $y = \sin x \cdot \sec x$

22. $y = \tan x \cdot \cot x$

23. $y = \frac{4}{\cos x}$

24. $y = \sin(3x + 1)$

25. $y = \tan(2x - x^3)$

26. $y = \cos\left(-\frac{x}{3}\right)$

27. $y = 4 \sin^2 x + 5 \cos^2 x$

28. $y = \frac{1 + \tan^2 x}{\sec x}$

29. $y = (1 + \cos 3x)^2$

30. $f(x) = \cot\left(\frac{x}{2}\right) \sin\left(\frac{x}{2}\right)$

Derivatives of e^x and a^x

Find the derivative of each.

1. $y = e^{2x}$

2. $y = e^{5x^2}$

3. $y = e^{\sin x}$

4. $y = e^{\tan x}$

5. $y = e^{x^2+2x}$

6. $y = e^{\sqrt{x}}$

7. $y = 5^x$

8. $y = e^{e^x}$

9. $y = 7^{x^2+2x^3}$

10. $y = \sin e^{3x}$

11. $y = xe^x$

12. $y = (\sin x)e^x$

13. $y = x^2e^x$

14. $y = \frac{e^x}{x^2}$

15. $y = 2^x(x^2 + 1)$

16. $y = 3^{\ln x}$

17. $y = x^2 + 4^x$

18. $y = \ln e^{x^2}$

19. $y = e^{\ln x^3}$

20. $y = e^{3x} \cdot 4^{5x}$

21. $y = e^{\csc x}$

22. $y = 10^{\sin x}$

23. $y = x^2e^x - xe^x$

24. $y = xe^2 - e^x$

Derivatives of $\ln x$

Find each Derivative

1. $y = \ln(x^3 + 1)$

2. $y = \ln \sqrt{x}$

3. $y = \sqrt{\ln(x)}$

4. $y = \ln |\sin x|$

5. $y = \ln(\sec x)$

6. $y = x \cdot \ln x$

7. $y = \frac{\ln x}{x^2}$

8. $y = \ln(\ln x)$

9. $y = (\sin x)(\ln x)$

10. $y = \frac{x^2}{\ln x}$

11. $y = \ln\left(\frac{5}{5-x}\right)$

12. $y = \ln \sqrt{x^2 + 4}$

13. $y = \ln(2 - \cos x)$

14. $y = \ln(5 - x)^6$

15. $y = e^{\ln x^2}$

16. $y = \ln(3x^2 + 2)^3$

17. $y = \ln x^3 + (\ln x)^3$

18. $y = \ln \sqrt{\ln(x)}$

Derivatives Review

1. Find the Equation of the tangent line of $f(x) = 4x^2 - 5x + 2$ at $x = 3$

2. Find the Equation of the normal line of $f(x) = -x^2 + 3x - 2$ at $x = 3$

Find the Derivative.

3. $f(x) = \frac{x^2 + 3x + 2}{x^2 - 1}$

4. $f(x) = \frac{x + 1}{\sqrt{x}}$

5. $f(x) = \sin^2 3x$

6. $f(x) = \frac{x(x^2 - 1)}{x + 1}$

7. $y = (x^2 - x)(x^2 + 1)(x^2 + x + 1)$

8. $y = x \cdot \sin x + \cos x$

9. $y = 5 \sec x + \tan x$

10. $f(x) = \sqrt{x} + 4 \csc x$

11. $f(x) = \frac{1}{x} - 10 \sec x$

12. $y = \left(\frac{x + 1}{x + 2}\right)(2x - 5)$

13. $y = (2x - 7)^3$

14. $f(x) = (9 - x^2)^{\frac{2}{3}}$

15. $y = \sqrt{\frac{1}{4x^2}}$

16. $f(x) = \frac{x^2}{x^2 + 3}$

17. $y = \frac{\cos \pi x + 1}{x}$

18. $f(x) = 3 \tan 4x$

19. $y = \sin(\cos x)$

20. $y = 3x - 5 \cos^2(\pi x)$

21. $f(x) = \sqrt{x^2 + 2x + 8}$

22. $y = \frac{1}{x} + \sqrt{\cos x}$

23. $y = (x^2 + 1)e^{3x}$

24. $y = x \cdot 5^{3x}$

25. $y = \sin(e^{2x})$

26. $y = e^{e^{5x}}$

27. $y = 3^{x^2+3x}$

28. $y = (\ln x)^x$

29. $y = 3^{5x}$

30. $y = e^{\ln(5x^2)}$

31. Find the first TWO Derivatives:
 $y = 2(x^2 - 1)^3$

32. Find the f' , f'' , and f'''
 $f(x) = x^3 + 2x^2 - 4x + 5$

Evaluate the Limit using L'Hopital's Rule:

33. $\lim_{x \rightarrow 3} \frac{x^2 + 4x - 21}{x^2 - 7x + 12}$

34. $\lim_{x \rightarrow 0} \frac{\tan 3x}{\ln(1+x)}$

35. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

36. $\lim_{x \rightarrow 3} \frac{\sqrt{x+6} - 3}{x - 3}$

37. $\lim_{x \rightarrow 0} \frac{x}{\frac{1}{3+x} - \frac{1}{3}}$

38. $\lim_{x \rightarrow \infty} \frac{x - 4}{x^2 - 6x + 8}$

Unit 4 Review

Given the following functions, find:

1. $f'''(-1)$ where $f(x) = \frac{5}{x^4}$

2. $f''(-2)$ where $f(x) = \frac{1}{x+1}$

Find the following derivatives:

3. $y = \sqrt{x} + \frac{1}{x}$

4. $y = \tan(3x^2 - 5)$

5. $y = \frac{1}{\sqrt[3]{3-x^3}}$

6. $y = x^2 \sin x$

7. $y = \sqrt{\cos^3(4x)}$

8. $y = \sin^2(4x)$

9. $y = 4^x x^4$

10. $y = x(3x - 9)^4$

11. $y = x^3 \sin x - 5 \cos x$

12. $y = x^3 e^x$

13. $y = \frac{\ln x}{x^2}$

14. $y = \frac{\sin x}{x}$

15. $y = e^{-x^3}$

16. $y = 7^{\ln x}$

17. $y = e^x - xe^x$

18. $y = \frac{e^x}{x^2 - 1}$

19. $y = \ln \sqrt{x^2 - 4}$

20. $y = 7e^x$

21. $y = 4^x \sin x$

#22 – 26 Given the following chart, find the indicated derivatives.

22. If $h(x) = 3f(x) + g(x)$, find $h'(2)$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-3	4	2	7
2	1	-5	9	-8
3	7	7	-2	9

23. If $h(x) = 7g(x)$, find $h'(3)$

24. If $h(x) = g(x) \cdot f(x)$, find $h'(2)$

25. If $h(x) = \frac{f(x)}{g(x)}$, find $h'(1)$

26. If $h(x) = f(g(x))$, find $h'(1)$

#27 - 30 Use the graph to find the derivative. $F(x)$ is the solid line and $g(x)$ is the dashed line.

27. If $K(x) = 4f(x) - 3g(x)$, find $K'(-1)$

28. If $K(x) = f(x) \cdot g(x)$, find $K'(2)$

29. If $K(x) = f(g(x))$, find $K'(0)$

30. If $K(x) = (g \circ f)(x)$, find $K'(-4)$

