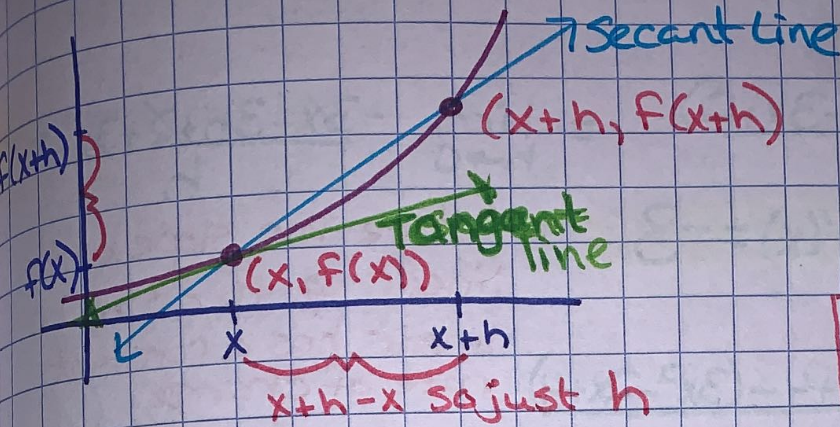
Definition of a Derivative



$$m_{sec} = \frac{f(x+h) - f(x)}{x+h-x}$$

$$m_{sec} = \frac{f(x+h) - f(x)}{h}$$

$$\boxed{m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Secant line's slope represents average rate of change

Tangent line's slope represents instantaneous rate of change

Average Rate of Change vs. Instantaneous Rate of Δ

Similarities	Differences
• Both are rates of change	• IROC is a limit; AROC is not
• Slopes of lines	• One is an average + one isn't
• $\frac{f(x+h) - f(x)}{h}$	• AROC involves 2 points + IROC involves 1 point

Derivative

- slope of a tangent line
- rate of change at one point
- an instantaneous rate of change
- denoted by y', $f'(x)$, or $\frac{dy}{dx}$

Definition of a derivative:

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Find the derivative using the definition.

1. $f(x) = -3x+2$

$f'(x) = \lim_{h\to 0} \frac{-3(x+h)+2-(-3x+2)}{h} = \lim_{h\to 0} \frac{-\cancel{3x}-3h+\cancel{2}+\cancel{3x}-\cancel{2}}{h}$

$f'(x) = \lim_{h\to 0} -3$ so $f'(x) = -3$

Notice the slope of the linear eq. is -3 ... bc linear has a constant rate of change.

2. $y = 3x^2-2x+4$

$y' = \lim_{h\to 0} \frac{3(x+h)^2-2(x+h)+4-(3x^2-2x+4)}{h}$

$\lim_{h\to 0} \frac{3(x^2+2xh+h^2)-2x-2h+4-3x^2+2x-4}{h}$

$\lim_{h\to 0} \frac{\cancel{3x^2}+6xh+3h^2-\cancel{2x}-2h+\cancel{4}-\cancel{3x^2}+\cancel{2x}-\cancel{4}}{h} = \frac{\cancel{h}(6x+3h-2)}{\cancel{h}}$

$\lim_{h\to 0} 6x+3(0)-2 \longrightarrow y' = 6x-2$

3. $y = \frac{1}{x+1}$ *Get common denominator

$\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{1}{x+h+1}-\frac{1}{x+1}}{h} = \lim_{h\to 0} \frac{\frac{x+1}{(x+h+1)(x+1)} + \frac{-1(x+h+1)}{(x+h+1)(x+1)}}{h}$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{\cancel{x}+\cancel{1}-\cancel{x}-h-\cancel{1}}{(x+h+1)(x+1)} \cdot \frac{1}{\cancel{h}} \rightarrow \frac{dy}{dx} = \lim_{h\to 0} \frac{-1}{(x+0+1)(x+1)}$

$\frac{dy}{dx} = \frac{-1}{(x+1)^2}$

4. $f(x) = 3\sqrt{x-2}$ *Rationalize the numerator

$f'(x) = \lim_{h\to 0} \frac{(3\sqrt{x+h-2}-3\sqrt{x-2})}{h} \frac{(3\sqrt{x+h-2}+3\sqrt{x-2})}{(3\sqrt{x+h-2}+3\sqrt{x-2})}$

$= \lim_{h\to 0} \frac{9(x+h-2)-9(x-2)}{h(3\sqrt{x+h-2}+3\sqrt{x-2})}$

$= \lim_{h\to 0} \frac{\cancel{9x}+9\cancel{h}-\cancel{18}-\cancel{9x}+\cancel{18}}{\cancel{h}(3\sqrt{x+h-2}+3\sqrt{x-2})} = \lim_{h\to 0} \frac{9}{3\sqrt{x+0-2}+3\sqrt{x-2}}$

$f'(x) = \frac{9}{6\sqrt{x-2}}$ $f'(x) = \frac{3}{2\sqrt{x-2}}$

Tangent + Normal Lines

To find the tangent equation:
- Find y-coordinate of point (if it's not given) by evaluating $f(x)$
- Find slope of the tangent line (find the derivative + evaluate $f'(x)$)
- Input the slope + point into point-slope form

To find the normal (perpendicular) line:
- All steps are the same except you find the slope of the line perpendicular to the tangent (opposite reciprocal)

1. Find the equation of the tangent line

$f(x) = x^2 + 2x \qquad f'(3)$

$f(3) = (3)^2 + 2(3) = 15 \qquad (3, 15)$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) - (x^2+2x)}{h} = \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h \cancel{- x^2} \cancel{- 2x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h+2)}{\cancel{h}} \qquad f'(x) = 2x+2$$

$f'(3) = 2(3) + 2 = 8 \qquad m = 8 \qquad y - 15 = 8(x-3)$

2. Find the equation of the tangent + normal line

$f(x) = \sqrt{x} \quad$ at $x = 3$

$f(3) = \sqrt{3} \qquad (3, \sqrt{3})$

$$f'(x) = \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})}{h} \cdot \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})} = \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{2\sqrt{x}}$$

$f'(3) = \frac{1}{2\sqrt{3}}$ so $m = \frac{1}{2\sqrt{3}}$ + $\perp m = -2\sqrt{3}$

Tangent $y - \sqrt{3} = \frac{1}{2\sqrt{3}}(x-3)$ normal $y - \sqrt{3} = -2\sqrt{3}(x-3)$

3. Find the equation of the normal line to $f(x) = 4x - 3x^2$ at $(2, -4)$

$$f'(x) = \lim_{h \to 0} \frac{4(x+h) - 3(x+h)^2 - (4x - 3x^2)}{h} = \frac{\cancel{4x} + 4h \cancel{- 3x^2} - 6xh - 3h^2 \cancel{- 4x} \cancel{+ 3x^2}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(4 - 6x - 3h)}{\cancel{h}} \qquad f'(x) = 4 - 6x$$

$f'(2) = 4 - 6(2) = -8 \qquad m = -8 \qquad \perp m = \frac{1}{8} \qquad y + 4 = \frac{1}{8}(x-2)$

Power + Sum-Difference Rules

- When y is a function of x, we will denote the derivative using y' or $f'(x)$ or dy/dx.
- dy/dx is read as the derivative of y with respect to x.

The Power Rule: For any real #, K

$$\frac{dy}{dx} x^{K} = K \cdot x^{K-1}$$

Find the derivative.

1. $y = x^5$ $\quad y' = 5x^4$
2. $y = x$ $\quad y' = 1x^0 \rightarrow y' = 1$
3. $y = x^{-4}$ $\quad y' = -4x^{-5} \rightarrow y' = \frac{-4}{x^5}$
4. $y = \sqrt{x}$ $\quad$ * 1st rewrite with rational exponents
   $y = x^{1/2}$ $\quad y' = \frac{1}{2}x^{-1/2}$ $\quad y' = \frac{1}{2x^{1/2}}$ or $y' = \frac{1}{2\sqrt{x}}$

The derivative of a constant is 0.

1. $y = 94$ $\quad y' = 0$ $\qquad$ 2. $y = \pi^2$ $\quad y' = 0$

The derivative of a constant times a function is the constant times the derivative of the function.

$$\frac{d}{dx}\left[c \cdot f(x)\right] = c \cdot \frac{d}{dx} f(x)$$

1. $f(x) = 7x^4$ $\quad 7\frac{d}{dx}x^4 = 7 \cdot 4x^3$ $\quad f'(x) = 28x^3$
2. $f(x) = -9x$ $\quad f'(x) = -9x^0$ or $-9(1)$ so $f'(x) = -9$
3. $f(x) = \frac{1}{5x^2}$ $\quad f(x) = \frac{1}{5} \cdot x^{-2} = \frac{1}{5} \cdot -2x^{-3}$ $\quad f'(x) = \frac{-2}{5x^3}$

Sum-Difference Rule:

$$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

$$\frac{d}{dx}\left[f(x) - g(x)\right] = \frac{d}{dx} f(x) - \frac{d}{dx} g(x)$$

Find the derivative.

1. $y = 5x^3 - 12$ $\quad y' = 5 \cdot 3x^2 - 0$ $\quad y' = 15x^2$

2. $y = 24x - \sqrt{x} + \frac{5}{x}$

$= 24 - x^{1/2} + 5x^{-1}$ $\quad y' = -\frac{1}{2}x^{-1/2} - 5x^{-2}$

$y' = \frac{-1}{2\sqrt{x}} - \frac{5}{x^2}$

3. $y = 3x^5 + 2\sqrt[3]{x} + \frac{1}{3x^2} + \sqrt{5}$

$y = 3x^5 + 2x^{1/3} + \frac{1}{3}x^{-2} + \sqrt{5}$

$y' = 15x^4 + \frac{2}{3}x^{-2/3} - \frac{2}{3}x^{-3} + 0$ $\quad y' = 15x^4 + \frac{2}{3\sqrt[3]{x^2}} - \frac{2}{3x^3}$

4. $y = 3x\left(x^2 - \frac{2}{x}\right)$

$y = 3x^3 - 6$

$y' = 9x^2$

5. $y = \frac{x^3 - 3x^2 + 4}{x^2}$ $= \frac{x^3}{x^2} - \frac{3x^2}{x^2} + \frac{4}{x^2}$

$y = x - 3 + 4x^{-2}$

$y' = 1 - 0 - 8x^{-3}$ $\quad y' = 1 - \frac{8}{x^3}$

6. $y = \frac{(x+1)^2}{4x}$ Find y', y'', & y'''.

$y = \frac{x^2 + 2x + 1}{4x} = \frac{x^2}{4x} + \frac{2x}{4x} + \frac{1}{4x}$ $\quad y = \frac{1}{4}x + \frac{1}{2} + \frac{1}{4}x^{-1}$

$y' = \frac{1}{4} - \frac{1}{4}x^{-2}$ $\quad y' = \frac{1}{4} - \frac{1}{4x^2}$

$y'' = \frac{1}{2}x^{-3}$ $\quad y'' = \frac{1}{2x^3}$

$y''' = -\frac{3}{2}x^{-4}$ $\quad y''' = \frac{-3}{2x^4}$

7. $g(t) = \frac{t^3 - 4t^2 + 6}{\sqrt{t}}$ Find dg/dt at $(4,3)$.

$g(t) = t^{5/2} - 4t^{3/2} + 6t^{-1/2}$ $\quad \frac{5\sqrt{t^3}}{2} - 6\sqrt{t} - \frac{3}{\sqrt{t^3}}$ $\quad = \frac{61}{8}$

$\frac{dg}{dt} = \frac{5}{2}t^{3/2} - 6t^{1/2} - \frac{3}{t^{3/2}}$ $\quad \frac{5\sqrt{4^3}}{2} - 6\sqrt{4} - \frac{3}{\sqrt{4^3}} = 20 - 12 - \frac{3}{8}$

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Product + Quotient Rule

Product Rule: $\frac{d}{dx}(f(x)\cdot g(x)) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$

$\frac{d}{dx}(1st \cdot 2nd) = 1st \cdot 2nd' + 2nd \cdot 1st'$

1. $f(x) = (2x^2-1)(x^3+3)$
   $f'(x) = (2x^2-1)(3x^2) + (x^3+3)(4x)$
   $= 6x^4 - 3x^2 + 4x^4 + 12x$
   $f'(x) = 10x^4 - 3x^2 + 12x$

2. $h(x) = x^3(\sqrt{x}+1)$
   $h'(x) = x^3\left(\frac{1}{2x^{1/2}}\right) + (x^{1/2}+1)(3x^2)$
   $h'(x) = \frac{1}{2}x^{5/2} + 3x^{5/2} + 3x^2$
   $h'(x) = \frac{7}{2}x^{5/2} + 3x^2$

Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}$

$\frac{d}{dx}\left(\frac{hi}{lo}\right) = \frac{lo\,de\,hi - hi\,de\,lo}{lo^2}$

1. $f(x) = \frac{x^2-3x}{x-1}$
   $f'(x) = \frac{(x-1)(2x-3)-(x^2-3x)(1)}{(x-1)^2} = \frac{2x^2-5x+3-x^2+3x}{(x-1)^2}$
   $f'(x) = \frac{x^2-2x+3}{(x-1)^2}$

2. $f(x) = (1-3x)(x^2+2)^{-1}$
   $f(x) = \frac{1-3x}{x^2+2}$
   $f'(x) = \frac{(x^2+2)(-3)-(1-3x)(2x)}{(x^2+2)^2} = \frac{-3x^2-6-2x+6x^2}{(x^2+2)^2}$
   $f'(x) = \frac{3x^2-2x-6}{(x^2+2)^2}$

The Chain Rule

Used to take the derivative of compositions.

$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$ $f(x) = \text{outside}$ $g(x) = \text{inside}$

deriv. outside · deriv. inside
(Keep inside same)

1. $y = (2x^3+4)^5$ $f(x) = x^5$ $g(x) = 2x^3+4$
$y' = 5(2x^3+4)^4 \cdot 6x^2$
$y' = 30x^2(2x^3+4)^4$

2. $y = (x^2+5x+3)^4$ $f(x) = x^4$ $g(x) = x^2+5x+3$
$y' = 4(x^2+5x+3)^3(2x+5)$
$y' = (8x+20)(x^2+5x+3)^3$

3. $f(x) = 4\sqrt[3]{x^2+3x}$ $f(x) = 4\sqrt[3]{x}$ or $4x^{1/3}$ $g(x) = x^2+3x$
$f'(x) = 4/3(x^2+3x)^{-2/3}(2x+3)$
$f'(x) = \frac{4(2x+3)}{3(x^2+3x)^{2/3}} = \frac{8x+12}{3(x^2+3x)^{2/3}}$

4. $f(x) = \sqrt{(5x+2)^3}$ $f(x) = x^{3/2}$ $g(x) = 5x+2$
$f(x) = (5x+2)^{3/2}$
$f'(x) = 3/2(5x+2)^{1/2}(5)$
$f'(x) = \frac{15}{2\sqrt{5x+2}}$ or $\frac{15}{2(5x+2)^{1/2}}$

5. $y = (\sqrt{x}-1)^2$ $f(x) = x^2$ $g(x) = \sqrt{x}-1$ or $x^{1/2}-1$
$y' = 2(\sqrt{x}-1)\frac{1}{2\sqrt{x}} = \frac{2(\sqrt{x}-1)}{2\sqrt{x}} = \frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}}$
$y' = 1 - \frac{1}{\sqrt{x}}$

6. $f(x) = \frac{1}{\sqrt{2x^3-7x^2}} = (2x^3-7x^2)^{-1/2}$ $f(x) = x^{-1/2}$ $g(x) = 2x^3-7x^2$
$f'(x) = -\frac{1}{2}(2x^3-7x^2)^{-3/2}(6x^2-14x)$
$f'(x) = \frac{-3x^2+7x}{\sqrt{2x^3-7x^2}^{\,3}}$ or $\frac{-3x^2-7x}{(2x^3-7x^2)^{3/2}}$

3.6 # Derivatives of Trig Functions

Basic Trig Derivatives

$\frac{d}{dx} \sin x = \cos x$
$\frac{d}{dx} \cos x = -\sin x$
$\frac{d}{dx} \tan x = (\sec x)^2$ or $\sec^2 x$
$\frac{d}{dx} \sec x = \sec x \tan x$
$\frac{d}{dx} \csc x = -\csc x \cot x$
$\frac{d}{dx} \cot x = -(\csc x)^2$ or $-\csc^2 x$

You will use these formulas w/ product, quotient + chain rule

★ If a trig function contains anything except 1 variable (like x), you have to use the chain rule

Find the derivative.

1. $f(x) = \sin x \cos x$ Product rule
$f'(x) = \sin x \cdot -\sin x + \cos x \cdot \cos x$
$f'(x) = -\sin^2 x + \cos^2 x$

2. $f(x) = x^2 \cos x$ Product rule
$f'(x) = x^2 \cdot -\sin x + \cos x \cdot 2x$
$f'(x) = -x^2 \sin x + 2x \cos x$

3. $f(x) = \frac{\cos x}{x \sin x}$ Simplify w/ trig identities $f(x) = \frac{1}{x} \cdot \frac{\cos x}{\sin x} = \frac{\cot x}{x}$
$f(x) = \frac{\cot x}{x}$ Quotient Rule
$f'(x) = \frac{x \cdot -\csc^2 x - \cot x (1)}{x^2}$ $f'(x) = \frac{-x \csc^2 x - \cot x}{x^2}$

4. $f(x) = (x^2+1) \sec x$ Product Rule
$f'(x) = (x^2+1)(\sec x \tan x) + \sec x (2x)$

5. $f(x) = x - 4\csc x + 2\cot x$ Power / Sum + Dif. Rules
$f'(x) = 1 - 4(-\csc x \cot x) + 2(-\csc^2 x)$
$f'(x) = 1 + 4\csc x \cot x - 2\csc^2 x$

6. $f(x) = \csc x \tan x$ Simplify 1st w/ trig identities
$f(x) = \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x} = \sec x$
$f'(x) = \sec x \tan x$

7. $\frac{d}{dx}(\sin 3x)$ Chain Rule $\sin(3x)$
$f(x) = \sin x \quad g(x) = 3x$
$\cos(3x) \cdot 3$
$3\cos(3x)$

8. $\frac{d}{dx}(\tan x^2)$ Chain Rule out: $\tan x$ in: x^2
$\sec^2(x^2) \cdot (2x)$
$= 2x \sec^2(x^2)$

9. $\frac{d}{dx}(\tan^2 x)$ means $(\tan x)^2$ so Chain Rule
$f(x) = x^2 \quad g(x) = \tan x$
out in
$2(\tan^1 x) \cdot \sec^2 x$
$2\tan x \sec^2 x$

10. $f(x) = \frac{\csc x}{1+\cot^2 x}$ Simplify w/ trig identities 1st
← pythagorean identity
$f(x) = \frac{\csc x}{\csc^2 x} = \frac{1}{\csc x} = \sin x$
$f'(x) = \cos x$

11. $f(x) = \csc\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)$ Simplify 1st
$f(x) = \frac{1}{\sin\left(\frac{x}{3}\right)} \cdot \frac{\cos\left(\frac{x}{3}\right)}{1} \rightarrow f(x) = \frac{\cos\left(\frac{x}{3}\right)}{\sin\left(\frac{x}{3}\right)}$
$f(x) = \cot(x/3)$ Chain Rule out: $\cot x$ in: $\frac{1}{3}x$
$f'(x) = -\csc^2\left(\frac{x}{3}\right) \cdot \left(\frac{1}{3}\right)$
$f'(x) = \frac{-\csc^2\left(\frac{x}{3}\right)}{3}$

12. $f(x) = \sec \pi x$ Chain Rule
$f'(x) = \sec \pi x \tan \pi x \cdot \pi$
$f'(x) = \pi \sec \pi x \tan \pi x$

13. $f(x) = \cos^2(5x)$ Double Chain Rule
$[\cos(5x)]^2$
$f'(x) = \underbrace{2(\cos(5x))}_{\text{deriv. } (\ )^2} \cdot \underbrace{-\sin(5x)}_{\text{deriv. } \cos(5x)} \cdot \underbrace{5}_{\text{deriv. } 5x}$

$f'(x) = -10\cos 5x \sin 5x$

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Properties of Logs Review:

Product Rule → $\log_a mn = \log_a m + \log_a n$

Quotient Rule → $\log_a \frac{m}{n} = \log_a m - \log_a n$

Power Rule → $\log_a m^p = p \log_a m$

Derivatives of Natural Logs

Basic : $d/dx \ \ln x = \frac{1}{x}$

Other Natural Logs: $\frac{d}{dx} \ln(f(x)) = \frac{1}{f(x)} \cdot f'(x)$

★ In other words, "1 over the function times the derivative of the function"

Find the derivative.

1. $y = \ln(2x^2+1)$

$y' = \frac{1}{2x^2+1} \cdot 4x$

$y' = \frac{4x}{2x^2+1}$

2. $y = \ln(\tan x)$

$y' = \frac{1}{\tan x} \cdot (\sec^2 x)$

$y' = \frac{\sec^2 x}{\tan x}$

3. $y = (\cos x)(2\ln x)$

$y' = \cos x \cdot \frac{2}{x} + 2\ln x \cdot -\sin x$

$y' = \frac{2\cos x}{x} - 2\ln x \sin x$

4. $\ln\left(\frac{3x}{3-x}\right) = \ln(3x) - \ln(3-x)$ use prop. of logs to expand

$y' = \frac{1}{3x} \cdot 3 - \frac{1}{3-x} \cdot -1$

$y' = \frac{1}{x} + \frac{1}{3-x}$

5. $y = \ln(7-x)^4$ Prop. of Logs

$y = 4\ln(7-x)$

$y = 4 \cdot \frac{1}{7-x} \cdot -1 = y' = \frac{-4}{7-x}$

6. $y = \ln e^{x^7}$ Prop. of Logs

$y = x^7 \ln e = x^7 \cdot 1$

$y = x^7$

$y' = 7x^6$

If you have a log of any other base, use the Change of Base formula: $\log_b a = \frac{\ln a}{\ln b}$ so $\log_7 x = \frac{\ln x}{\ln 7}$ or $\frac{1}{\ln 7} \cdot \ln x$

1. $y = \log_8 x = \frac{\ln x}{\ln 8}$

$y = \frac{1}{\ln 8} \cdot \ln x$ ★bring out constant

$y' = \frac{1}{\ln 8} \cdot \frac{1}{x} + \ln x(0)$ Product Rule

$y' = \frac{1}{x\ln 8}$

2. $\log_5(\cos x) = \frac{\ln \cos x}{\ln 5}$

$y = \frac{1}{\ln 5} \cdot \ln(\cos x)$

$y' = \frac{1}{\ln 5} \cdot \frac{1}{\cos x} \cdot -\sin x + \ln\cos x(0)$

$y' = \frac{-\sin x}{\ln 5 \cos x}$ or $-\frac{\tan x}{\ln 5}$

Derivatives of Exponentials

$\frac{d}{dx} a^u = a^u \cdot \ln a \cdot d(u)$

★★★ "copy the problem, times ln of base, times the derivative of the exponent"

Easiest one → $\frac{d}{dx} e^x = e^x \cdot 1 = e^x$

Find the derivative

1. $y = 3^x$ $\quad y' = 3^x \ln 3 \,(1)$ so $y' = 3^x \ln 3$

2. $y = 2^x(x^2+1)$ Product Rule
$y' = 2^x(2x) + (x^2+1)2^x \ln 2$

3. $y = 3^{\ln x}$
$y' = 3^{\ln x} \ln 3 \left(\frac{1}{x}\right)$ $\quad y' = \frac{3^{\ln x} \ln 3}{x}$

4. $y = e^{2x}$
$y' = e^{2x} \cdot \ln e \cdot 2$ $\quad y' = 2e^{2x}$

5. $y = e^{\cos x}$
$y' = e^{\cos x} \cdot \ln e \cdot -\sin x$ $\quad y' = -\sin x \cdot e^{\cos x}$

6. $y = e^{x^3+5x}$
$y' = e^{x^3+5x} \cdot \ln e \cdot (3x^2+5)$ $\quad y' = (3x^2+5)e^{x^3+5x}$

7. $y = e^{\ln x^5}$
$y = x^5 \ln e$
$y = x^5$
$y' = 5x^4$

Shortcut: $e^{\ln x}$ → bring exponent down + $\ln e$ cancels

bc $\ln y = \ln e^{\ln x^5}$
$\ln y = \ln x^5 \ln e$
$y = x^5$

$y = e^{\ln x}$ is $y = x$
$y = \ln e^x$ is $y = x$

L'Hopital's Rule

- used to evaluate limits of indeterminate forms such as $\frac{0}{0}$ or $\frac{\infty}{\infty}$ or $\frac{-\infty}{-\infty}$

$$\boxed{\text{Formula: } \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}}$$

- Take the derivative of numerator AND denominator (not quotient rule). Then find the limit.
- You can use it more than once as long as it's indeterminate

Find the limits.

1. $\lim_{x \to 0} \frac{\sin x}{x}$ $\frac{\sin 0}{0} = \frac{0}{0}$ so use L'Hopital's Rule

$\lim_{x \to 0} \frac{\sin x}{x} \xrightarrow[\text{deriv}]{\text{deriv}} \lim_{x \to 0} \frac{\cos x}{1} = \cos 0 = 1$

2. $\lim_{x \to 1} \frac{5x^4 - 4x^2 - 1}{10 - x - 9x^3}$ $\frac{5(1)^4 - 4(1)^2 - 1}{10 - 1 - 9(1)^3} = \frac{0}{0}$

$\lim_{x \to 1} \frac{20x^3 - 8x}{-1 - 27x^2} = \frac{20(1)^3 - 8(1)}{-1 - 27(1)^2} = \frac{12}{-28} = -\frac{3}{7}$

3. $\lim_{x \to \infty} \frac{e^x}{x^2} = \frac{e^\infty}{\infty^2} = \frac{\infty}{\infty}$

$\lim_{x \to \infty} \frac{e^x}{2x} = \frac{e^\infty}{2\infty} = \frac{\infty}{\infty}$ use L'Hopital's again

$\lim_{x \to \infty} \frac{e^x}{2} = \frac{e^\infty}{2} = \infty$

4. $\lim_{x \to 64} \frac{\sqrt[3]{x} - 4}{x - 64}$ $\frac{\sqrt[3]{64} - 4}{64 - 64} = \frac{0}{0}$

$\lim_{x \to 64} \frac{\frac{1}{3}x^{-2/3}}{1} = \frac{1}{3\sqrt[3]{64^2}} = \frac{1}{48}$