Unit 2 Test Review– Limits

True or False:

1. Suppose that $\lim_{x \to a} f(x) = L$, then $f(x)$ is a continuous function.

2. If the limit of a function at $x = a$ does not exist, then the function has a jump discontinuity at $x = a$.

3. Suppose that $\lim_{x \to a} f(x) = L$, then a is in the domain of $f(x)$.

4. If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ then $f(x)$ is a continuous function.

5. If $f(x)$ is a continuous function, then $\lim_{x \to a} f(x)$ exists.

6. If x= a is a vertical asymptote of the graph of the function $f(x)$, then $\lim_{x \to a} f(x) = \infty$

7. If $x = b$ is a horizontal asymptote of the function $f(x)$, then $\lim_{x \to \infty} f(x) = b$

8. A function has a removable discontinuity at $x = a$ if $\lim_{x \to a} f(x)$ does not exist and $f(a)$ is defined.

Evaluate the Limit:

9. $\lim_{x \to 2^-} \frac{2x-4}{|x-2|}$

10. $\lim_{x \to 3} \frac{x^2-5x+6}{x-3}$

11. $\lim_{x \to \infty} \frac{1-x^2}{2-3x^2}$

12. $\lim_{x \to \infty} \frac{3x^2-2x^5-x}{x^2-3x^5}$

13. $\lim_{x \to \infty} \frac{5x^2-2x^4-4}{2x-3x^2+x^3}$

14. $\lim_{x \to -5^-} \frac{4x+20}{|x+5|}$

15. $\lim_{x \to \infty} \frac{e^x+5}{3-2e^x}$

16. $\lim_{\theta \to \frac{\pi}{6}} \frac{1-\cos\theta}{2\sin^2\theta}$

17. $\lim_{x\to 0^-} \frac{3}{-4x^2}$

18. $\lim_{x\to \infty} \frac{3x^2+2x}{\sqrt{x^2-2x}}$

19. $\lim_{x\to -\infty} \frac{\sqrt{16x^2-3x+5}}{5x+4}$

20. $\lim_{x\to 0} \frac{2x^5-4x^3}{5x^4-2x^2}$

21. $\lim_{x\to \infty} \frac{(2x-3)(x-5)(2-x)}{(3-4x^2)(x+2)}$

22. $\lim_{x\to 4} \frac{x-\sqrt{x+12}}{x-4}$

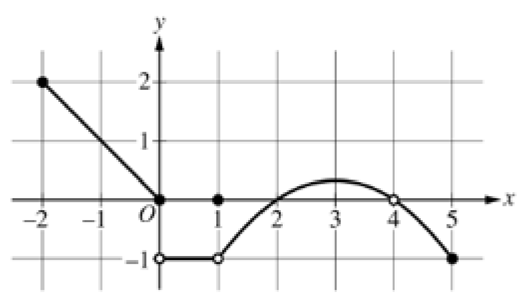
23. $\lim_{x\to 2} \frac{\sqrt{x+2}-2}{2x^2-8}$

24. $\lim_{x\to -\infty} (3-2x)(x+3)^2(x-2)$

Random Problems

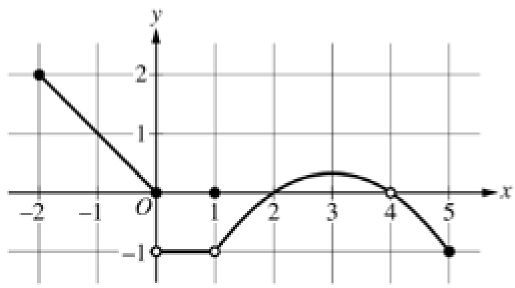
25. If the function f is continuous for all real numbers and if $f(x) = \frac{x^3-27}{x-3}$ when $x \neq 3$, then $f(3)$ must equal what?

26. For what value(s) of *c* does $\lim_{x\to c^+} f(x) = 0$?



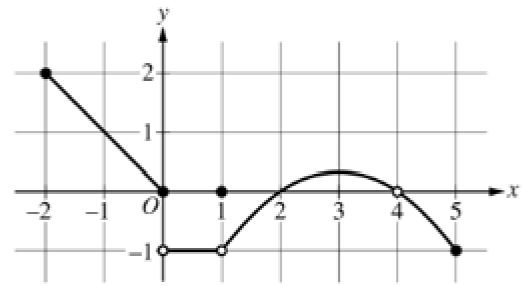
Graph of f

27. State and justify all discontinuities.



Graph of f

28. The graph of a function f is shown below. If $\lim_{x \to b} f(x)$ does not exists but $f(b)$ exists then b must equal what?



Graph of f

29. Find all discontinuities of the function:

$$\frac{x^3 + 4x^2 - 3x - 18}{x^3 - 4x^2 + x + 6}$$

30. Let

$$g(x) = \begin{cases} \dfrac{x^2 + 5x + 6}{x + 2}, & -4 \le x < -1 \\ 3 - x^2, & -1 \le x < 2 \\ -3, & 2 < x < 4 \end{cases}$$

Identify all discontinuities.

31. Let f be the piecewise function defined below. Find the value of $\lim_{x \to 7^-} f(x)$

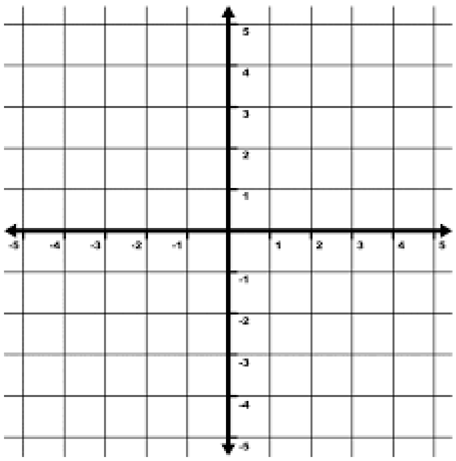
$$f(x) = \begin{cases} 3 + \sqrt{2 + x}, & x \le 7 \\ 8 - \sqrt{x - 3}, & x > 7 \end{cases}$$

32. The table below gives selected values and limits of the functions f, g, and h. What is $\lim_{x \to -1} \frac{2 + 3f(x)}{f(x)(5h(x) - g(x))}$

$f(-1) = 0$	$\lim_{x \to -1} f(x) = 1$
$g(-1) = 11$	$\lim_{x \to -1} g(x) = 4$
$h(-1) = -5$	$\lim_{x \to -1} h(x) = 0$

33. Sketch the graph of a function with the given properties:

$$\lim_{x\to 4^+} f(x) = \infty$$
$$\lim_{x\to 4^-} f(x) = 2$$
$$\lim_{x\to \infty} f(x) = -1$$
$$\lim_{x\to -\infty} f(x) = 3$$



34. Use the table to find the following limits:

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
f(x)	7.018	7.008	7.002	undefined	7.002	7.008	7.018
g(x)	4.126	4.789	4.989	8	8.0015	8.1016	8.546
h(x)	4971	8987	9972	9	8.997	8.987	8.971

$\lim_{x\to 0^+} f(x)$ $\qquad$ $\lim_{x\to 0^-} f(x)$ $\qquad$ $\lim_{x\to 0} f(x)$

$\lim_{x\to 0^+} g(x)$ $\qquad$ $\lim_{x\to 0^-} g(x)$ $\qquad$ $\lim_{x\to 0} g(x)$

$\lim_{x\to 0^+} (x)$ $\qquad$ $\lim_{x\to 0^-} h(x)$ $\qquad$ $\lim_{x\to 0} h(x)$

35. Is the function

$$f(x) = \begin{cases} 3+\sqrt{2+x}, & x \le 7 \\ 8-\sqrt{x-3}, & x > 7 \end{cases}$$

continuous at $x = 7$? Justify your reasoning.

36. Let g be the function defined below, where c is a constant

$$g(x) = \begin{cases} \dfrac{x^2+cx+6}{x^3+2}, & -4 \le x < -1 \\ 3-x^2, & -1 \le x < 2 \\ \dfrac{-3\cdot 2^x+1}{4\cdot 2^x}, & 2 < x < 4 \end{cases}$$

a. Find the value of c, if any, that makes the function continuous at $x = -1$.

b. What type of discontinuity does g have at $x = 2$? Give a reason for your answer.

c. Find all vertical and horizontal asymptotes of the graph of g.