Unit 2 Test Review– Limits (Virtual Session)

True or False:

1. Suppose that $\lim_{x\to a} f(x) = L$, then $f(x)$ is a continuous function. F; possibly $\lim_{x\to a} f(x) \neq f(a)$
2. If the limit of a function at $x = a$ does not exist then the function has a jump discontinuity at $x = a$. F; could be infinite disc.
3. Suppose that $\lim_{x\to a} f(x) = L$, then a is in the domain of $f(x)$. F; $\lim_{x\to a} f(x) \neq f(a)$
4. If $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ then $f(x)$ is a continuous function. F; $\lim_{x\to a} f(x) \neq f(a)$
5. If $f(x)$ is a continuous function, then $\lim_{x\to a} f(x)$ exists. True
6. If x= a is a vertical asymptote of the graph of the function $f(x)$, then $\lim_{x\to a} f(x) = \infty$ F; $\lim_{x\to a}$ could be $-\infty$ or DNE
7. If y $= b$ is a horizontal asymptote of the function $f(x)$, then $\lim_{x\to\infty} f(x) = b$ True; $\lim_{x\to\pm\infty} f(x) = b$
8. A function has a removable discontinuity at $x = a$ if $\lim_{x\to a} f(x)$ does not exist and $f(a)$ is defined. False

To have a removable disc., the $\lim_{x\to a} f(x)$ must be defined.

Evaluate the Limit:

9. $\lim_{x\to 2^-} \frac{2x-4}{|x-2|} =$

$\lim_{x\to 2^-} \frac{2(\cancel{x-2})}{-1(\cancel{x-2})} = -2$

10. $\lim_{x\to 3} \frac{x^2-5x+6}{x-3}$

$\lim_{x\to 3} \frac{(\cancel{x-3})(x-2)}{(\cancel{x-3})} = 3-2 = 1$

11. $\lim_{x\to\infty} \frac{1-x^2}{2-3x^2} = \frac{1}{3}$

HA

12. $\lim_{x\to\infty} \frac{3x^2-2x^5-x}{x^2-3x^5} = \frac{2}{3}$

HA: $y = 2/3$

13. $\lim_{x\to\infty} \frac{5x^2-2x^4-4}{2x-3x^2+x^3}$

behaves like $\lim_{x\to\infty} \frac{-2x^4}{x^3} = -2x$
$-2(\infty)$
$= -\infty$

14. $\lim_{x\to -5^-} \frac{4x+20}{|x+5|}$

$\lim_{x\to -5} \frac{4(\cancel{x+5})}{-1(\cancel{x+5})} = -4$

15. $\lim_{x\to\infty} \frac{e^x+5}{3-2e^x}$

$\lim_{x\to\infty} \frac{1e^x+5}{-2e^x+3} = -\frac{1}{2}$

16. $\lim_{\theta\to\frac{\pi}{6}} \frac{1-\cos\theta}{2\sin^2\theta} = \frac{1-\cos\frac{\pi}{6}}{2\sin^2\frac{\pi}{6}}$

$= \frac{1-\frac{\sqrt{3}}{2}}{2(\frac{1}{2})^2}$

$= \frac{\frac{2-\sqrt{3}}{2}}{1/2} = 2-\sqrt{3}$

17. $\lim_{x\to0^-} \frac{3}{-4x^2}$ $-\infty$

$\frac{3}{-4(-.01)^2} = \frac{+}{-}$

18. $\lim_{x\to\infty} \frac{3x^2+2x}{\sqrt{x^2-2x}}$

behaves like $\lim_{x\to\infty} \frac{3x^2}{|x|}$

$\lim_{x\to\infty} \frac{3x^2}{x} = 3(\infty) = \infty$

19. $\lim_{x\to-\infty} \frac{\sqrt{16x^2-3x+5}}{5x+4}$

behaves like $\lim_{x\to-\infty} \frac{|4x|}{5x}$

$\lim_{x\to-\infty} \frac{-4x}{5x} = \frac{-4}{5}$

20. $\lim_{x\to0} \frac{2x^5-4x^3}{5x^4-2x^2}$

$\lim_{x\to0} \frac{x^3(2x^2-4)}{x^2(5x^2-2)}$

$\lim_{x\to0} \frac{x(2x^2-4)}{5x^2-2} = \frac{0(-4)}{(-2)} = 0$

21. $\lim_{x\to\infty} \frac{(2x-3)(x-5)(2-x)}{(3-4x^2)(x+2)}$

behaves like $\lim_{x\to\infty} \frac{-2x^3}{-4x^3} = \frac{1}{2}$

22. $\lim_{x\to4} \frac{x-\sqrt{x+12}}{x-4}$

$\lim_{x\to4} \frac{(x-\sqrt{x+12})(x+\sqrt{x+12})}{(x-4)(x+\sqrt{x+12})}$

$\lim_{x\to4} \frac{x^2-x-12}{(x-4)(x+\sqrt{x+12})}$

$\lim_{x\to4} \frac{\cancel{(x-4)}(x+3)}{\cancel{(x-4)}(x+\sqrt{x+12})} = \frac{7}{8}$

23. $\lim_{x\to2} \frac{\sqrt{x+2}-2}{2x^2-8}$

$\lim_{x\to2} \frac{(\sqrt{x+2}-2)(\sqrt{x+2}+2)}{2(x+2)(x-2)(\sqrt{x+2}+2)}$

$\lim_{x\to2} \frac{x+2-4}{2(x+2)(x-2)(\sqrt{x+2}+2)}$

$\lim_{x\to2} \frac{1}{2(x+2)(\sqrt{x+2}+2)} = \frac{1}{2(4)(4)} = \frac{1}{32}$

24. $\lim_{x\to-\infty} (3-2x)(x+3)^2(x-2)$

behaves like $\lim_{x\to-\infty} -2x^4$

$-2(-\infty)^4$

$-\infty$

Random Problems

25. If the function f is continuous for all real numbers and if $f(x) = \frac{x^3-27}{x-3}$ when $x \neq 3$, then $f(3)$ must equal what?

$f(x) = \frac{(x-3)(x^2+3x+9)}{x-3}$

$f(x) = x^2+3x+9$

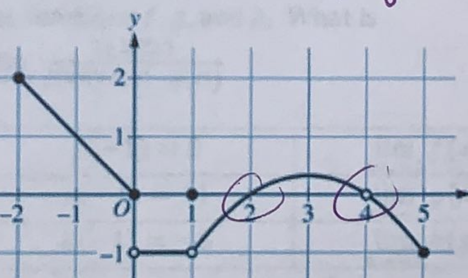
$f(3) = 9+9+9$

$f(3) = 27$

26. For what value(s) of c does $\lim_{x\to c^+} f(x) = 0$?

y

right



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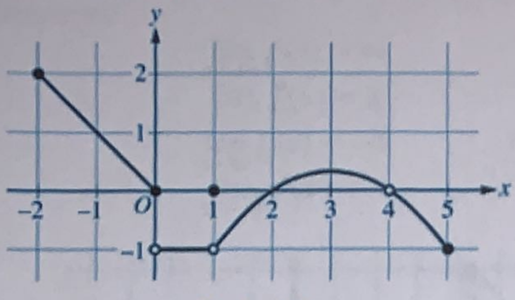
Graph of f

$c = 2 + c = 4$

27. State and justify all discontinuities.


Graph of f

$x = 0$ jump discontinuity

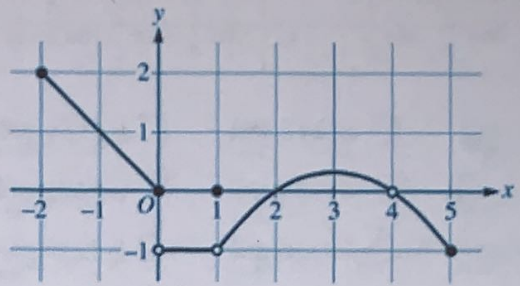
$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x) \neq \lim_{x\to 0} f(x)$

$x = 1$ removable

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \neq f(1)$

$x = 4$ removable $\lim_{x\to 4^-} f(x) \neq f(4)$

28. The graph of a function f is shown below. If $\lim_{x\to b} f(x)$ does not exists but $f(b)$ exists then b must equal what?


Graph of f

$b = 0$

29. Find all discontinuities of the function:

$$\frac{x^3 + 4x^2 - 3x - 18}{x^3 - 4x^2 + x + 6}$$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9$

$\pm 1, \pm 2, \pm 3, \pm 6$

$2 \,|\; 1 \;\; 4 \;\; -3 \;\; -18$
$\;\;\;\; \downarrow \;\; 2 \;\; 12 \;\; 18$
$\;\;\;\; 1 \;\; 6 \;\; 9 \;\; 0$

$2 \,|\; 1 \;\; -4 \;\; 1 \;\; 6$
$\;\;\;\; \downarrow \;\; 2 \;\; -4 \;\; -6$
$\;\;\;\; 1 \;\; -2 \;\; -3 \;\; 0$

$$\frac{\cancel{(x-2)}(x^2+6x+9)}{\cancel{(x-2)}(x^2-2x-3)} \quad x \neq 2$$

$$\frac{(x+3)(x+3)}{(x-3)(x+1)}$$

Removable discontinuity at $x = 2$

Infinite discontinuity at $x = 3$ + $x = -1$

30. Let

$$g(x) = \begin{cases} \dfrac{x^2+5x+6}{x+2}, & -4 \le x < -1 \\ 3 - x^2, & -1 \le x < 2 \\ -3, & 2 < x < 4 \end{cases}$$

Identify all discontinuities.

$\frac{(x+3)\cancel{(x+2)}}{\cancel{x+2}}$ Removable at $x = -2$

$\lim_{x\to -1^-} g(x) = 2$ $\lim_{x\to -1^+} g(x) = 2$ so cont.

$\lim_{x\to 2^-} g(x) = -1$ $\lim_{x\to 2^+} g(x) = -3$ so jump disc. at $x = 2$

31. Let f be the piecewise function defined below. Find the value of $\lim_{x\to 7^-} f(x)$

$$f(x) = \begin{cases} 3 + \sqrt{2+x}, & x \le 7 \\ 8 - \sqrt{x-3}, & x > 7 \end{cases}$$

$3 + \sqrt{2+7}$

$3 + 3 = 6$

$\lim_{x\to 7^-} f(x) = 6$

32. The table below gives selected values and limits of the functions $f, g,$ and h. What is $\lim_{x\to -1} \frac{2+3f(x)}{f(x)(5h(x)-g(x))}$

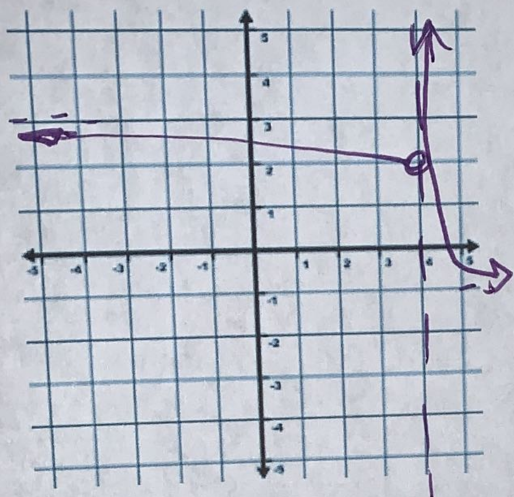
$f(-1) = 0$	$\lim_{x\to -1} f(x) = 1$
$g(-1) = 11$	$\lim_{x\to -1} g(x) = 4$
$h(-1) = -5$	$\lim_{x\to -1} h(x) = 0$

$$\frac{2 + 3\lim_{x\to -1} f(x)}{\lim_{x\to -1} f(x)\left(5\lim_{x\to -1} h(x) - \lim_{x\to -1} g(x)\right)}$$

$$\frac{2+3(1)}{1(5\cdot 0-4)} = \frac{5}{-4}$$

Sketch the graph of a function with the given properties:

$$\lim_{x\to 4^+} f(x) = \infty$$
$$\lim_{x\to 4^-} f(x) = 2$$
$$\lim_{x\to \infty} f(x) = -1$$
$$\lim_{x\to -\infty} f(x) = 3$$



34. Use the table to find the following limits:

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
f(x)	7.018	7.008	7.002	undefined	7.002	7.008	7.018
g(x)	4.126	4.789	4.989	8	8.0015	8.1016	8.546
h(x)	4971	8987	9972	9	8.997	8.987	8.971

$\lim_{x\to 0^+} f(x) = 7$ $\lim_{x\to 0^-} f(x) = 7$ $\lim_{x\to 0} f(x) = 7$

$\lim_{x\to 0^-} g(x) = 8$ $\lim_{x\to 0^-} g(x) = 5$ $\lim_{x\to 0} g(x)$ DNE

$\lim_{x\to 0^-} h(x) = 9$ $\lim_{x\to 0^-} h(x) = \infty$ $\lim_{x\to 0} h(x)$ DNE

35. Is the function

$$f(x) = \begin{cases} 3+\sqrt{2+x}, & x \le 7 \\ 8-\sqrt{x-3}, & x > 7 \end{cases}$$

continuous at x = 7? Justify your reasoning.

$\lim_{x\to 7^-} f(x) = 3+\sqrt{2+7} = 6$

$\lim_{x\to 7^+} f(x) = 8-\sqrt{7-3} = 6$

$f(7) = 6$

yes, $f(x)$ is cont. at $x=7$

$\lim_{x\to 7^-} f(x) = \lim_{x\to 7^+} f(x) = f(7) = 6$

36. Let f be the function defined below, where c is a constant

$$g(x) = \begin{cases} \dfrac{x^2+cx+6}{x^3+2}, & -4 \le x < -1 \\ 3-x^2, & -1 \le x < 2 \\ \dfrac{-3\cdot 2^x+1}{4\cdot 2^x}, & 2 < x < 4 \end{cases}$$

a. Find the value of c, if any, that makes the function continuous at $x = -1$.

$\frac{x^2+cx+6}{x^3+2} = \frac{3-x^2}{1}$ $c=5$

$\frac{1-c+6}{-1+2} = \frac{3-1}{1}$ ★ $\lim_{x\to -1^-} g(x) = \lim_{x\to -1^+} g(x) = g(-1)$

$2 = -c+7$

b. What type of discontinuity does g have at $x = 2$? Give a reason for your answer.

$\lim_{x\to 2^-} g(x) = 3-(2)^2 = -1$ Jump Discontinuity at $x=2$

$\lim_{x\to 2^+} g(x) = \frac{-3(4)+1}{4\cdot 4} = \frac{-11}{16}$

$\lim_{x\to 2^-} g(x) \neq \lim_{x\to 2^+} g(x) \neq \lim_{x\to 2} g(x)$

c. Find all vertical and horizontal asymptotes of the graph of g.

HA: $y=0 \rightarrow \frac{x^2+5x+6}{x^3+2}$ $x^3+2=0$

$y=-\frac{3}{4} \rightarrow \frac{-3\cdot 2^x+1}{4\cdot 2x^2}$ $x^3=-2$

VA: $x=\sqrt[3]{-2}$ $x=\sqrt[3]{-2}$