Unit 2 Limits Test Review
Multiple Choice Practice

1. $\lim_{x\to 0}\frac{4x-3}{7x+1}=$ $\frac{4(0)-3}{7(0)+1}=\frac{-3}{1}$

A. ∞ B. $-\infty$ C. 0 D. $\frac{4}{7}$ E. -3

2. $\lim_{x\to \frac{1}{3}}\frac{9x^2-1}{3x-1}=$ $\frac{0}{0}$ $\lim_{x\to\frac{1}{3}}\frac{(3x+1)\cancel{(3x-1)}}{\cancel{(3x-1)}}=3(\frac{1}{3})+1$

A. ∞ B. $-\infty$ C. 0 D. 2 E. 3

3. $\lim_{x\to 2}\frac{x^3-8}{x^2-4}=$ $\lim_{x\to2}\frac{\cancel{(x-2)}(x^2+2x+4)}{(x+2)\cancel{(x-2)}}=\frac{4+4+4}{4}=3$

A. 4 B. 0 C. 1 D. 3 E. 2

4. The function $G(x)=\begin{cases}x-3, & x<2\\ -5, & x=2\\ 3x-7, & x>2\end{cases}$ is not continuous at $x=2$ because…

$\lim_{x\to2^-}G(x)=-1$
$\lim_{x\to2^+}G(x)=-1$
$G(2)=-5$
$\lim_{x\to2}G(x)\neq G(2)$

A. $G(2)$ is not defined

B. $\lim_{x\to2}G(x)$ does not exist

C. $\lim_{x\to2}G(x)\neq G(2)$

D. Only reasons B and C

E. All of the above reasons.

5. $\lim_{x\to\infty}\frac{-3x^2+7x^3+2}{2x^3+3x^2+5}=$ behaves like $\lim_{x\to\infty}\frac{7x^3}{2x^3}=\frac{7}{2}$

A. ∞ B. $-\infty$ C. 1 D. $\frac{7}{2}$ E. $-\frac{3}{2}$

6. $\lim_{x\to-2}\frac{\sqrt{2x+5}-1}{x+2}=$ $\lim_{x\to-2}\frac{(\sqrt{2x+5}-1)(\sqrt{2x+5}+1)}{(x+2)(\sqrt{2x+5}+1)}$

$\lim_{x\to-2}\frac{2x+4}{(x+2)(\sqrt{2x+5}+1)}$ $\lim_{x\to-2}\frac{2}{\sqrt{2x+5}+1}=\frac{2}{\sqrt{1}+1}=1$

A. 1 B. 0 C. ∞ D. $-\infty$ E. Does Not Exist

7. If $f(x) = 3x^2 - 5x$, then find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

$\lim_{h \to 0} \frac{3(x+h)^2 - 5(x+h) - (3x^2 - 5x)}{h}$

A. ~~$3x - 5$~~

B. $6x - 5$

C. ~~$6x$~~

D. 0

E. Does not exist

$\lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 5x - 5h - 3x^2 + 5x}{h}$

$\lim_{h \to 0} 6x + 3h - 5 = 6x + 3(0) - 5 = 6x - 5$

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8. $\lim_{x \to -\infty} \frac{2 - 5x}{\sqrt{x^2 + 2}} =$ behaves like $\lim_{x \to -\infty} \frac{-5x}{|x|} \to \lim_{x \to -\infty} \frac{-5x}{-x} = 5$

A. 5 B. –5 C. 0 D. $-\infty$ E. ∞

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9. The function $f(x) = \frac{2x^2 + x - 3}{x^2 + 4x - 5}$ has a vertical asymptote at $x = -5$ because…

$(x+5)(x-1)$

$\lim_{x \to -5^+} \frac{2(-4.9)^2 + (-4.9) - 3}{(-4.9)^2 + 4(-4.9) - 5} = \frac{+}{-} = -\infty$

$\lim_{x \to -5^-} \frac{2(-5.1)^2 - 5.1 - 3}{(-5.1)^2 + 4(-5.1) - 5} = \frac{+}{+} = \infty$

A. $\lim_{x \to -5^+} f(x) = \infty$

B. $\lim_{x \to -5^-} f(x) = -\infty$

C. $\lim_{x \to -5^-} f(x) = \infty$

D. $\lim_{x \to \infty} f(x) = -5$

E. $f(x)$ does not have a vertical asymptote at $x = -5$

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10. Consider the function $H(x) = \begin{cases} 3x - 5, & x < 3 \\ x^2 - 2x, & x \geq 3 \end{cases}$. Which of the following statements is/are true?

$3(3) - 5 = 4$

$(3)^2 - 2(3) = 3$

I. $\lim_{x \to 3^-} H(x) = 4$. ✓ $3(3) - 5 = 4$

~~II.~~ $\lim_{x \to 3} H(x)$ exists.

~~III.~~ $H(x)$ is continuous at $x = 3$.

A. I only

B. II only

C. I and II only

D. I, II and III

E. None of these statements is true

Free Response Practice #1

Consider the function $h(x) = \dfrac{-2x - \sin x}{x - 1}$ to answer the following questions.

a. Find $\lim_{x \to 1^+} h(x)$. Show your numerical analysis that leads to your answer and explain what this result implies graphically about $h(x)$ at $x = 1$.

$$\lim_{x \to 1^+} \frac{-2x - \sin x}{x - 1}$$

$$\frac{-2(1.01) - \sin(1.01)}{1.01 - 1} = \frac{-}{+} = -\infty$$

There is a vertical asymptote at $x=1$

b. Find $\lim_{x \to \frac{\pi}{2}} \left[h(x) \cdot (2x - 2)\right]$. Show your analysis.

$$\lim_{x \to \frac{\pi}{2}} \left[\frac{-2x - \sin x}{x - 1} (2x-2)\right]$$

$$\frac{-2(\frac{\pi}{2}) - \sin(\frac{\pi}{2})}{\frac{\pi}{2} - 1} \cdot (2(\tfrac{\pi}{2}) - 2)$$

$$\frac{-\pi - 1}{\frac{\pi}{2} - 1} \cdot (\pi - 2)$$

$$\frac{-\pi - 1}{\frac{\pi - 2}{2}} \cdot \pi - 2$$

$$(-\pi - 1) \cdot \frac{2}{(\pi - 2)} \cdot (\pi - 2)$$

$$2(-\pi - 1)$$

$$-2\pi - 2$$

c. Explain why the Intermediate Value Theorem guarantees a value of c on the interval [1.5, 2.5] such that $h(c) = -4$. Then, find c.

Since $h(x)$ is continuous over $(1.5, 2.5)$

$f(1.5) \approx -7.99$

$f(2.5) \approx -3.73$

& $-7.99 < -4 < -3.73$

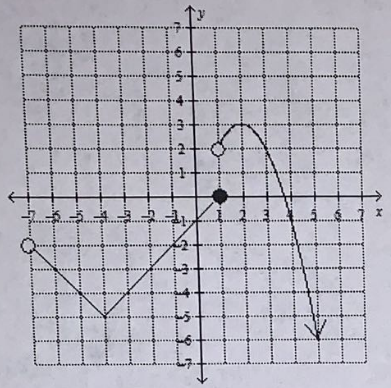
$\therefore \exists\, c \in (1.5, 2.5)$ s.t. $h(c) = -4$

$$-4 = \frac{-2c - \sin c}{c - 1}$$

$\boxed{\text{★ calc step: Solve}\left(\frac{-2x - \sin x}{x - 1} = -4, x\right)}$

$c \approx 2.354$

Free Response Practice #2



$$f(x)=\begin{cases} ax+3, & x<-3 \\ x^2-3x, & -3\le x<2 \\ bx-5, & x\ge 2 \end{cases}$$

$(1)^2-3(1)=-2$

Graph of g(x)

Equation of f(x)

Pictured above is the graph of a function g(x) and the equation of a piece-wise defined function f(x). Answer the following questions.

a. Find $\lim_{x\to 1^+}\left[2g(x)-f(x)\cdot\cos\pi x\right]$. Show your work applying the properties of limits.

$2\lim_{x\to 1^+} g(x) - \lim_{x\to 1^+} f(x)\cdot \cos \pi x$

$2(2)-(-2)\cos\pi(1)$

$4+2\cos\pi$

$4+2(-1) = 2$

b. On its domain, what is one value of x at which g(x) is discontinuous? Use the three part definition of continuity to explain why g(x) is discontinuous at this value.

$x=1$ Jump discontinuity

$\lim_{x\to 1^-} g(x)=0 \quad \lim_{x\to 1^+} g(x)=2 \quad g(1)=0$

$\lim_{x\to 1^-} g(x) \ne \lim_{x\to 1^+} g(x) \ne g(1)$ ∴ Jump discontinuity

c. For what value(s) of a and b, if they exist, would the function f(x) be continuous everywhere? Justify your answer using limits.

$ax+3 = x^2-3x$

$a(-3)+3 = (-3)^2-3(-3)$

$-3a+3=18$

$-3a=15$

$a=-5$

$x^2-3x = bx-5$

$(2)^2-3(2) = b(2)-5$

$-2 = 2b-5$

$3=2b$

$b=3/2$

$\lim_{x\to -3^-} f(x) = \lim_{x\to -3^+} f(x) = f(-3)$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$