Creative Factoring and Other Interesting Algebra

Difference of Squares

Example: $x-16=\left(\sqrt{x}+4\right)\left(\sqrt{x}-4\right)$

1. $x-9$ 2. x^2-5 3. $x^{16}-1$ 4. $(x+5)^2-25$ 5. $9y-a^4$

1. $(\sqrt{x}+3)(\sqrt{x}-3)$

2. $(x+\sqrt{5})(x-\sqrt{5})$

3. $(x^8+1)(x^8-1)$
$(x^8+1)(x^4+1)(x^4-1)$
$(x^8+1)(x^4+1)(x^2+1)(x^2-1)$
$(x^8+1)(x^4+1)(x^2+1)(x+1)(x-1)$

4. $((x+5)+5)((x+5)-5)$
$(x+10)x$
$x(x+10)$

5. $(3\sqrt{y}-a^2)(3\sqrt{y}+a^2)$

Sums or Differences of Cubes **"SOAP"**

Example: $a^3+b^3=(a+b)(a^2-ab+b^2)$ **Example:** $a^3-b^3=(a-b)(a^2+ab+b^2)$

6. $64a^3+125b^3$ 7. $64a^3x^3-125$ 8. $(x+1)^3+64$ 9. $8c^3-(a+b)^3$

6. $a=4a$ $b=5b$
$(4a+5b)(16a^2-20ab+25b^2)$

7. $a=4ax$ $b=5$
$(4ax-5)(16a^2x^2+20ax+25)$

8. $a=x+1$ $b=4$
$(x+1+4)((x+1)^2-4(x+1)+16)=(x+5)((x+1)^2-4(x+1)+16)$

9. $a=2c$ $b=a+b$
$(2c-(a+b))(4c^2+2c(a+b)+(a+b)^2)$

Factor: x^6-y^6:

10. as a difference of squares

$(x^3+y^3)(x^3-y^3)$
$(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

11. as a difference of cubes

$(x^2-y^2)(x^4+x^2y^2+y^4)$
$(x+y)(x-y)(x^4+x^2y^2+y^4)$

Compare these two. Which way will allow you to factor completely most easily?

Rationalize the Numerator

12. $\dfrac{\sqrt{x+2}-\sqrt{2}}{x}\cdot\dfrac{(\sqrt{x+2}+\sqrt{2})}{(\sqrt{x+2}+\sqrt{2})}$

$\dfrac{x+\cancel{2}-\cancel{2}}{\cancel{x}(\sqrt{x+2}+\sqrt{2})}=\dfrac{1}{\sqrt{x+2}+\sqrt{2}}$

13. $\dfrac{\sqrt{x+3}+\sqrt{3}}{x}\cdot\dfrac{(\sqrt{x+3}-\sqrt{3})}{(\sqrt{x+3}-\sqrt{3})}$

$\dfrac{x+\cancel{3}-\cancel{3}}{x(\sqrt{x+3}-\sqrt{3})}=\dfrac{x}{x(\sqrt{x+3}-\sqrt{3})}=\dfrac{1}{\sqrt{x+3}-\sqrt{3}}$

Factor completely. **Use synthetic division to help find all factors.** ★ use graphing calculator to help get a root to use

14. $x^3+6x^2+5x-12$

```
1 | 1   6   5  -12
  |     1   7   12
  ----------------
    1   7  12 :  0
```

$x^2+7x+12$

$(x-1)(x+3)(x+4)$

15. $x^3+x^2-8x-12$

```
-2 | 1   1  -8  -12
   |    -2   2   12
   -----------------
     1  -1  -6 :  0
```

x^2-x-6

$(x+2)(x-3)(x+2)$

16. $x^3+6x^2-9x-14$

```
2 | 1   6  -9  -14
  |     2  16   14
  ----------------
    1   8   7 :  0
```

x^2+8x+7

$(x-2)(x+7)(x+1)$

Simplify:

17. $\dfrac{2x^3+7x^2+8x+3}{x+1}$

$x+1=0$
$x=-1$

```
-1 | 2   7   8   3
   |    -2  -5  -3
   ---------------
     2   5   3 : 0
```

$2x^2+5x+3$

$(x-1)(2x+3)(x+1)$

18. $\dfrac{2x^3+x^2-13x+6}{x+3}$

```
-3 | 2   1  -13   6
   |    -6   15  -6
   ----------------
     2  -5    2 : 0
```

$2x^2-5x+2$

(10) $(x+3)(2x-1)(x-2)$

Algebraic Limits Worksheet

1. $\lim_{x\to3} x^2+2x-7$ $(3)^2+2(3)-7$ 8	2. $\lim_{x\to-1}\frac{\frac{1}{x}+1}{x+1} = \frac{\frac{1}{x}+\frac{x}{x}}{x+1} = \frac{\frac{1+x}{x}}{x+1}$ $\frac{1+x}{x}\div\frac{x+1}{1} = \frac{1+x}{x}\cdot\frac{1}{x+1} = \frac{1}{x}$ $\frac{1}{-1} = -1$	3. $\lim_{x\to1}\frac{(4x^4-5x^2+1)}{x^2+2x-3}$ $\frac{(4x^2-1)(x^2-1)}{(x+3)(x-1)} = \frac{(2x+1)(2x-1)(x+1)(x-1)}{(x+3)(x-1)}$ $\frac{(2\cdot1+1)(2\cdot1-1)(1+1)}{1+3} = \frac{6}{4} = \frac{3}{2}$
4. $\lim_{x\to0}\frac{(x+1)^2-1}{x}$ ← Factor dif. of squares $\frac{((x+1)+1)((x+1)-1)}{x} = \frac{(x+2)\cancel{x}}{\cancel{x}}$ $0+2=2$	5. $\lim_{x\to1}\frac{x^2-2x-15}{x-5}$ $\frac{(1)^2-2(1)-15}{1-5} = \frac{-16}{-4}$ 4	6. $\lim_{x\to-3}\frac{2x^2+2x-12}{x^2+4x+3}$ $\frac{2(x^2+x-6)}{x^2+4x+3} = \frac{2\cancel{(x+3)}(x-2)}{\cancel{(x+3)}(x+1)}$ $\frac{2(-3-2)}{-3+1} = \frac{-10}{-2} = 5$
7. $\lim_{x\to2}\frac{(2x+1)^2-25}{x-2}$ $\frac{((2x+1)+5)((2x+1)-5)}{x-2}$ $\frac{(2x+6)(2x-4)}{x-2} = \frac{(2x+6)2\cancel{(x-2)}}{\cancel{x-2}}$ $(2\cdot2+6)2$ 20	8. $\lim_{x\to2}\frac{(3x-2)^2-(x+2)^2}{x-2}$ $\frac{((3x-2)+(x+2))((3x-2)-(x+2))}{x-2}$ $\frac{4x(2x-4)}{x-2} = \frac{8x\cancel{(x-2)}}{\cancel{x-2}} = 8\cdot2 = 16$	9. $\lim_{x\to1}\frac{\frac{2x}{x+1}-1}{x-1} = \frac{\frac{2x}{x+1}-\frac{x+1}{x+1}}{x-1}$ $\frac{\frac{2x-x-1}{x+1}}{x-1} = \frac{\cancel{x-1}}{x+1}\cdot\frac{1}{\cancel{x-1}} = \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}$
10. $\lim_{x\to2}\frac{\frac{2}{x^2}-\frac{1}{2}}{x-2} = \frac{\frac{4}{2x^2}-\frac{x^2}{2x^2}}{x-2}$ $\frac{\frac{4-x^2}{2x^2}}{x-2} = \frac{4-x^2}{2x^2}\cdot\frac{1}{x-2} = \frac{(2+x)(2-x)}{\cancel{(x-2)}2x^2}$ $-1(x-2)$ $\frac{-1(2+2)}{2(2)^2} = \frac{-4}{8} = \frac{-1}{2}$	11. $\lim_{x\to2}\frac{x^4-2x^2-8}{x^2-x-6}$ $\frac{(x^2-4)(x^2+2)}{(x-3)(x+2)} = \frac{\cancel{(x+2)}(x-2)(x^2+2)}{(x-3)\cancel{(x+2)}}$ $\frac{0}{-1} = 0$	12. Factor $\lim_{x\to1}\frac{x-1}{\sqrt{x}-1} = \frac{(\sqrt{x}+1)\cancel{(\sqrt{x}-1)}}{\cancel{\sqrt{x}-1}}$ $\sqrt{1}+1 = 2$
13. $\lim_{x\to0}\frac{x^2+7x+6}{x+3}$ $\frac{0^2+7(0)+6}{0+3} = 2$	14. $\lim_{x\to-2}\frac{\frac{x}{x+4}+1}{x+2} = \frac{\frac{x}{x+4}+\frac{x+4}{x+4}}{x+2}$ $\frac{\frac{2x+4}{x+4}}{x+2} = \frac{2\cancel{(x+2)}}{x+4}\cdot\frac{1}{\cancel{x+2}} = \frac{2}{-2+4}$ 1	15. $\lim_{x\to2}\frac{(x^3+x^2)(-4x-4)}{x^2+x-6}$ $\frac{x^2(x+1)-4(x+1)}{(x+3)(x-2)} = \frac{(x^2-4)(x+1)}{(x+3)(x-2)}$ $\frac{(x+2)\cancel{(x-2)}(x+1)}{(x+3)\cancel{(x-2)}} = \frac{4\cdot3}{5} = \frac{12}{5}$
16. $\lim_{x\to2}\frac{x^2-2x-3}{x+5}$ $\frac{(2)^2-2(2)-3}{2+5} = \frac{-3}{7}$	17. $\lim_{x\to2}(x^2-x+1)$ $(2)^2-2+1$ 3	18. $\lim_{x\to1}\frac{2x+1}{3x-2}$ $\frac{2(1)+1}{3(1)-2} = \frac{3}{1} = 3$
19. $\lim_{x\to1}\sqrt{10x-1}$ $\sqrt{10(1)-1}$ $\sqrt{9}$ $= 3$	20. $\lim_{x\to1}\frac{x^2-x-2}{x-2}$ $\frac{(1)^2-1-2}{1-2} = \frac{-2}{-1} = 2$	21. $\lim_{x\to4}\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)}$ $\frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)} = \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2}$ $1/4$

22. $\lim_{x\to 3}\frac{x^2-9}{x+3} = \frac{\cancel{(x+3)}(x-3)}{\cancel{x+3}}$

$3-3=0$

23. $\lim_{x\to 3}\frac{x^2-9}{2x^2+7x+3}$ didn't need to factor

$\frac{\cancel{(x+3)}(x-3)}{(2x+1)\cancel{(x+3)}} = 0$

24. $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9} = \frac{\cancel{\sqrt{x}-3}}{(\sqrt{x}+3)\cancel{(\sqrt{x}-3)}}$

$\frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = 1/6$

25. $\lim_{h\to 0}\frac{(1+h)^2-1^2}{h}$

$\frac{((1+h)+1)((1+h)-1)}{h} = \frac{\cancel{h}(h+2)}{\cancel{h}}$

$0+2=2$

26. $\lim_{h\to 0}\frac{(3+h)^2-3^2}{h}$

$\frac{((3+h)+3)((3+h)-3)}{h}$

$\frac{\cancel{h}(h+6)}{\cancel{h}} = 0+6 = 6$

27. $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$

$\frac{((x+h)+x)((x+h)-x)}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}}$

$2x+0=2x$

28. $\lim_{x\to 3}(5x^2-6)$

$5(3)^2-6 = 39$

29. $\lim_{x\to -1}\frac{x-2}{x^2+4x-3}$

$\frac{-1-2}{(-1)^2+4(-1)-3} = \frac{-3}{-6} = \frac{1}{2}$

30. $\lim_{x\to 4}\frac{x^2-16}{x-4} = \frac{(x+4)\cancel{(x-4)}}{\cancel{x-4}}$

$4+4=8$

31. $\lim_{x\to 0}\frac{6x-9}{x^3-12x+3}$

$\frac{6(0)-9}{0^3-12(0)+3} = \frac{-9}{3}$

-3

32. $\lim_{x\to 2}\frac{x^2-4x+4}{x^2+x-6}$

$\frac{(x-2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \frac{0}{5}$

0

33. $\lim_{x\to -2}\frac{x^3+8}{x+2}$

$\frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{x+2}}$

$(-2)^2-2(-2)+4$

12

Given $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} g(x) = 0$, *and* $\lim_{x\to a} h(x) = 8$, find each limit if it exists.

34. $\lim_{x\to a}[f(x)+h(x)]$

$-3+8=5$

35. $\lim_{x\to a}[f(x)]^2$

$(-3)^2$

9

36. $\lim_{x\to a}\sqrt[3]{h(x)}$

$\sqrt[3]{8} = 2$

2

37. $\lim_{x\to a}\frac{1}{f(x)}$

$\frac{1}{-3}$

38. $\lim_{x\to a}\frac{g(x)}{h(x)}$

$\frac{0}{8} = 0$

39. $\lim_{x\to a}\frac{h(x)}{g(x)}$

$\frac{8}{0}$ = undefined or DNE

40. $\lim_{x\to a}\frac{2f(x)}{h(x)-f(x)}$

$\frac{2(-3)}{8+3} = \frac{-6}{11}$

41. $\lim_{x\to a}[f(x)h(x)]$

$(-3)(8)$

-24

42. $\lim_{x\to a}\left[\frac{g(x)+h(x)}{f(x)}\right]$

$\frac{0+8}{-3} = \frac{-8}{3}$

Intermediate Value Theorem Worksheet

1. Verify the conditions of the IVT and find the guaranteed c value over [2,6] for
$$f(x) = x^2 + 2x - 11$$
when $f(c) = 4$.

$f(2) = -3 \quad f(6) = 37 \quad -3 < 4 < 37$ ✓

$4 = x^2 + 2x - 11$
$0 = x^2 + 2x - 15$
$0 = (x+5)(x-3)$

$x \neq 5$ not in the interval $\quad x = 3$

2. Verify the conditions of the IVT and find the guaranteed c value over $[-1,3]$ for
$$f(x) = 2x^2 + x - 4$$
when $f(c) = 2$.

$f(-1) = -3 \quad f(3) = 17 \quad -3 < 2 < 17$ ✓

$2 = 2x^2 + x - 4$
$0 = 2x^2 + x - 6$
$0 = (2x-3)(x+2)$

$x = \frac{3}{2} \quad x \neq -2$

3. Use the IVT to show that
$$f(x) = x^3 - 3x^2 - 7x + 1$$
has a root in the interval (4,5)

$f(4) = -11 \quad f(5) = 16$

$-11 < 0 < 16$ ✓

4. Use the IVT to show that
$$f(x) = x^4 + 3x^2 - 6$$
has a root in the interval (1,2) *and* $(-2,-1)$

$f(1) = -2 \quad f(2) = 22 \quad -2 < 0 < 22$ ✓

$f(-2) = 22 \quad f(-1) = -2 \quad -2 < 0 < 22$ ✓

5. Verify the conditions of the IVT and find the guaranteed c value over [0,5] for
$$f(x) = x^2 + x - 1$$
when $f(c) = 11$.

$f(0) = -1 \quad f(5) = 29 \quad -1 < 11 < 29$

$11 = x^2 + x - 1$
$0 = x^2 + x - 12$
$0 = (x+4)(x-3)$

$x \neq -4 \quad x = 3$

6. Verify the conditions of the IVT and find the guaranteed c value over [0,3] for
$$f(x) = x^2 - 6x + 8$$
when $f(c) = 0$.

$f(0) = 8 \quad f(3) = -1 \quad -1 < 0 < 8$

$0 = x^2 - 6x + 8$
$0 = (x-4)(x-2)$

$x \neq 4 \quad x = 2$

7. Verify the conditions of the IVT and find the guaranteed c value over [0,3] for
$$f(x) = x^3 - x^2 + x - 2$$
when $f(c) = 4$.

$f(0) = -2 \quad f(3) = 19 \quad -2 < 0 < 19$

$4 = x^3 - x^2 + x - 2$
$0 = (x^3 - x^2 + x - 6)$

can't factor
use synthetic ÷
find a root in calc

$x = 2$

$2 \,|\, 1 \;\; -1 \;\; 1 \;\; -6$
$\quad\;\; 2 \;\; 2 \;\; 6$
$\quad\;\; 1 \;\; 1 \;\; 3 \;:\; 0$

$x^2 + x + 3$
$(x+2$

8. Verify the conditions of the IVT and find the guaranteed c value over $[\frac{5}{2}, 4]$ for
$$f(x) = \frac{x^2 + x}{x - 1}$$
when $f(c) = 6$.

$\frac{6}{1} = \frac{x^2+x}{x-1}$
$x^2 + x = 6x - 6$
$0 = x^2 - 5x + 6$
$(x-3)(x-2)$

$x = 3$

$f(5/2) = \dfrac{\frac{25}{4} + \frac{10}{4}}{\frac{5}{2} - \frac{2}{2}} = \frac{35}{4} \div \frac{3}{2} = \frac{35}{4} \cdot \frac{2}{3} = \frac{35}{6}$

$f(4) = \frac{16+4}{3} = \frac{20}{3}$

$\frac{35}{6} < 6 < \frac{20}{3}$ ✓

9. Use the IVT to show that
$$f(x) = x^3 + x - 1$$
has a root in the interval [0,1]

$f(0) = -1 \quad f(1) = 1$

$-1 < 0 < 1$

10. Use the IVT to show that
$$f(x) = x^3 + 3x - 2$$
has a root in the interval [0,1]

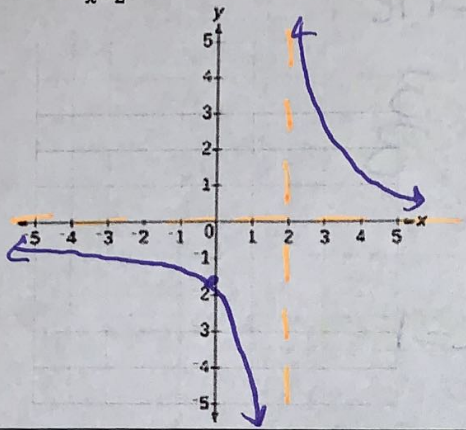
$f(0) = -2 \quad f(1) = 2$

$-2 < 0 < 2$

Vertical and Horizontal Asymptotes Worksheet

State the vertical, horizontal, or slant asymptotes for the following (justify using limits). Sketch the graph and find the end behavior.

1. $f(x) = \frac{3}{x-2}$



Vertical Asymptote: $x=2$

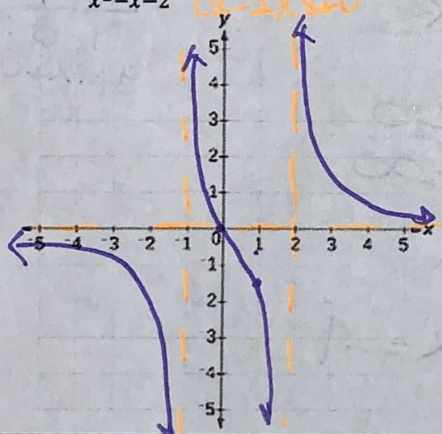
Horizontal Asymptote: $y=0$

Slant Asymptote: none

End Behavior: $\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-\infty} f(x) = 0$

2. $f(x) = \frac{3x}{x^2-x-2}$ $\frac{3x}{(x-2)(x+1)}$



Vertical Asymptote: $x=2$ $x=-1$

Horizontal Asymptote: $y=0$

Slant Asymptote: none

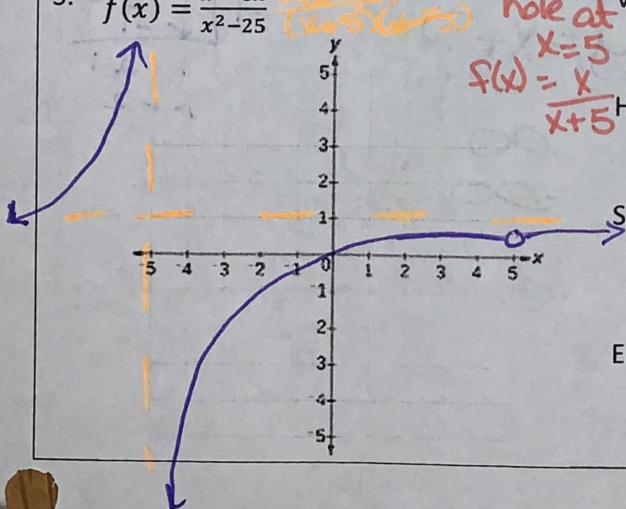
End Behavior: $\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-\infty} f(x) = 0$

3. $f(x) = \frac{x^2-5x}{x^2-25}$ $\frac{x(x-5)}{(x+5)(x-5)}$

hole at $x=5$ $(5, \frac{1}{2})$

$f(x) = \frac{x}{x+5}$



Vertical Asymptote: $x=-5$

Horizontal Asymptote: $y=1$

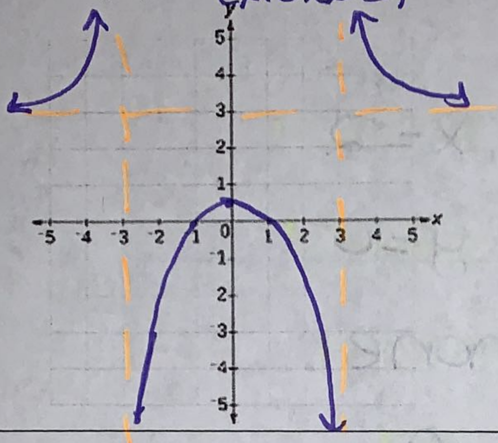
Slant Asymptote: none

End Behavior: $\lim_{x\to\infty} f(x) = 1$

$\lim_{x\to-\infty} f(x) = 1$

(14)

4. $f(x) = \frac{3x^2-4}{x^2-9}$ $\frac{3x^2-4}{(x+3)(x-3)}$



Vertical Asymptote: $x=-3$ $x=3$

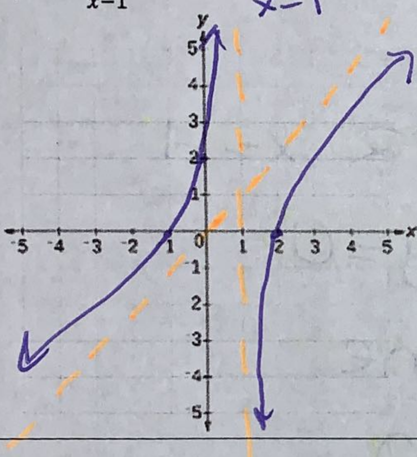
Horizontal Asymptote: $y=3$

Slant Asymptote: none

End Behavior: $\lim_{x\to\infty} f(x) = 3$

$\lim_{x\to-\infty} f(x) = 3$

5. $f(x) = \frac{x^2-x-2}{x-1}$ $\frac{(x+1)(x-2)}{x-1}$



Vertical Asymptote: $x=1$

Horizontal Asymptote: none

Slant Asymptote: $y=x$

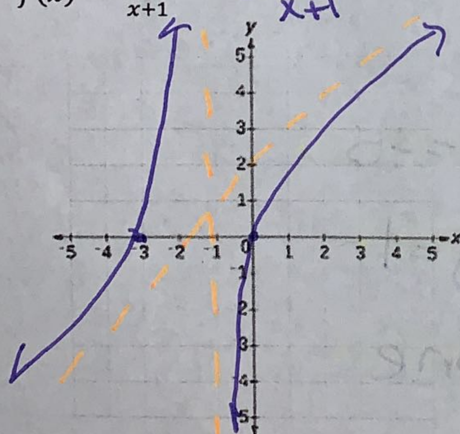
$$\begin{array}{r|rrr} 1 & 1 & -1 & -2 \\ & \downarrow & 1 & 0 \\ \hline & 1 & 0 & -2 \end{array}$$

$y=1x+0$

End Behavior: $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

6. $f(x) = \frac{x^2+3x}{x+1}$ $\frac{x(x+3)}{x+1}$



Vertical Asymptote: $x=-1$

Horizontal Asymptote: none

Slant Asymptote: $y=x+2$

$$\begin{array}{r|rrr} -1 & 1 & 3 & 0 \\ & \downarrow & -1 & \\ \hline & 1 & 2 & \end{array}$$

$x+2$

End Behavior: $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

Infinite Limits Worksheet

Find the Limit.

1. $\lim_{x\to\infty} 3$ ↔ horizontal line 3	2. $\lim_{x\to-\infty} 3$ 3	3. $\lim_{x\to-\infty} (-3)$ -3
4. $\lim_{x\to\infty} (-2x)$ odd neg. $-\infty$ or $-2(\infty) = -$	5. $\lim_{x\to\infty} (3-x)$ odd − $-\infty$ $3-\infty$	6. $\lim_{x\to\infty} \sqrt{x}$ $\sqrt{\infty} = \infty$
7. $\lim_{x\to-\infty} (4-x)$ $4 - -\infty$ $4+\infty$ ∞	8. $\lim_{x\to\infty} \frac{8}{5-3x}$ HA. y=0 0	9. $\lim_{x\to\infty} \frac{1}{x-12}$ HA y=0 0
10. $\lim_{x\to-\infty} \frac{3}{x+4}$ HA y=0 0	11. $\lim_{x\to\infty} (1+2x-3x^5)$ odd − $-\infty$	12. $\lim_{x\to\infty} (2x^3-110x+5)$ odd + ∞
13. $\lim_{x\to\infty} \frac{(3+2x^2)}{4+5x}$ no HA so $+\infty$ or $-\infty$ $\frac{3+2(\infty)^2}{4+5(\infty)} = \frac{+}{+}$ $+\infty$	14. $\lim_{x\to\infty} \frac{x^2+x}{3-x}$ no HA $\frac{\infty^2+\infty}{3-\infty} = \frac{+}{-} = -\infty$	15. $\lim_{x\to\infty} \frac{x+4}{x^2-2x+5}$ HA: y=0 0
16. $\lim_{x\to-\infty} -\frac{x-2}{x^2+2x+1}$ HA: y=0 0	17. $\lim_{x\to\infty} \frac{7-6x^5}{x+3}$ no HA $\frac{7-6(\infty)^5}{\infty+3} = \frac{-}{+} = -\infty$	18. $\lim_{x\to\infty} \frac{6-x^3}{7x^3+3}$ HA: y = $\frac{-1}{7}$ $-1/7$
19. $\lim_{x\to\infty} \frac{1}{x^2+1}$ HA: y=0 0	20. $\lim_{x\to\infty} \frac{x^4+x^2}{x^4+1}$ HA: y=1 1	21. $\lim_{x\to\infty} \frac{1+x^2}{2-x^2}$ HA: y=-1 -1
22. $\lim_{x\to\infty} \frac{2x^2}{x^2+1}$ HA: y=2 2	23. $\lim_{x\to-\infty} \frac{x+4}{3x^2-5}$ HA: y=0 0	24. $\lim_{x\to\infty} \frac{3x^3+25x^2-x+1}{4x^3-7x^2+2x+2}$ HA: y = $\frac{3}{4}$ 3/4

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Limits Review

Limit of a constant is a constant.

1. $\lim_{x\to e}\sqrt{7}$ $\sqrt{7}$

2. $\lim_{x\to\sqrt{5}}\pi$ π

Direct Substitution – ALWAYS try direct substitution first!

3. $\lim_{x\to 5}\left(2x^2-x+3\right)$

$2(5)^2-5+3$

$=48$

4. $\lim_{y\to 2^-}\dfrac{y^2-3y+2}{y+1}$

$\dfrac{(2)^2-3(2)+2}{2+1}=0$

5. $\lim_{x\to 4}\dfrac{|5-3x|}{2x+1}$

$\dfrac{|5-3\cdot 4|}{2\cdot 4+1}=\dfrac{7}{9}$

6. $\lim_{x\to 4}\cos\left(\dfrac{3\pi}{x}\right)$

$\cos\dfrac{3\pi}{4}=-\dfrac{\sqrt{2}}{2}$

If substitution results in $\frac{0}{0}$, Factor, reduce, and substitute again.

7. $\lim_{x\to 1}\dfrac{x^4-1}{x-1}=\dfrac{(x^2+1)(x+1)\cancel{(x-1)}}{\cancel{x-1}}$

$(1^2+1)(1+1)=4$

8. $\lim_{x\to 1}\dfrac{x-1}{x^3-x^2+x-1}=\dfrac{\cancel{x-1}}{(x^2+1)\cancel{(x-1)}}=\dfrac{1}{x^2+1}$

$x^2(x-1)+1(x-1)$

$\dfrac{1}{1^2+1}=\dfrac{1}{2}$

9. $\lim_{x\to 9}\dfrac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)}=\dfrac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)}$

$\dfrac{1}{\sqrt{x}+3}=\dfrac{1}{\sqrt{9}+3}=\dfrac{1}{6}$

10. $\lim_{x\to -2}\dfrac{x+2}{x^3+8}=\dfrac{\cancel{x+2}}{\cancel{(x+2)}(x^2-2x+4)}=\dfrac{1}{x^2-2x+4}$

$\dfrac{1}{(-2)^2-2(-2)+4}=\dfrac{1}{12}$

Multiply by the conjugate.

11. $\lim_{x\to 2}\dfrac{(\sqrt{5x+6}-4)(\sqrt{5x+6}+4)}{(x-2)(\sqrt{5x+6}+4)}$

$\dfrac{5x+6-16}{(x-2)(\sqrt{5x+6}+4)}=\dfrac{5x-10}{}=\dfrac{5\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{5x+6}+4)}$

$\dfrac{5}{\sqrt{16}+4}=5/8$

12. $\lim_{x\to 4}\dfrac{(3-\sqrt{x+5})(3+\sqrt{x+5})}{(x-4)(3+\sqrt{x+5})}$

$\dfrac{9-(x+5)\ -x+4}{(x-4)(3+\sqrt{x+5})}$

$\dfrac{-1}{3+\sqrt{x+5}}=\dfrac{-1}{3+\sqrt{9}}=\dfrac{-1}{6}$

13. $\lim_{x\to 0}\dfrac{(\sqrt{x+3}-\sqrt{3})(\sqrt{x+3}+\sqrt{3})}{x(\sqrt{x+3}+\sqrt{3})}=\dfrac{\cancel{x}+\cancel{3}-\cancel{3}}{\cancel{x}(\sqrt{x+3}+\sqrt{3})}$

$\dfrac{1}{\sqrt{x+3}+\sqrt{3}}=\dfrac{1}{2\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}$

0

$\dfrac{\sqrt{3}}{6}$

Complex fractions – clear the "little denominators"

14. $\lim_{h\to 0}\dfrac{\frac{2\cdot 1}{2(2+h)}-\frac{1\,(2+h)}{2(2+h)}}{h}$

$\dfrac{\frac{2}{2(2+h)}+\frac{-2-h}{2(2+h)}}{h}=\dfrac{\frac{-h}{2(2+h)}}{h}$

$\dfrac{-\cancel{h}}{2(2+h)}\cdot\dfrac{1}{\cancel{h}}=\dfrac{-1}{2(2+h)}=\dfrac{-1}{2(2+0)}$

$-1/4$

15. $\lim_{x\to 10}\dfrac{\frac{x}{5}-2\cdot\frac{5}{5}}{x-10}$

$\dfrac{\frac{x}{5}-\frac{10}{5}}{x-10}=\dfrac{\frac{x-10}{5}}{x-10}$

$\dfrac{\cancel{x-10}}{5}\cdot\dfrac{1}{\cancel{x-10}}=\dfrac{1}{5}$

16. $\lim_{h\to -2}\dfrac{(h+5)^{-1}-3^{-1}}{h+2}$

$\dfrac{\frac{3\cdot 1}{3\,h+5}+\frac{-1\,(h+5)}{3\,(h+5)}}{h+2}$

$\dfrac{\frac{3}{3(h+5)}+\frac{-h-5}{3(h+5)}}{h+2}=\dfrac{\frac{-1(h+2)}{3(h+5)}}{}\cdot\dfrac{1}{h+2}$

$\dfrac{-h-2}{3(h+5)}$

$\dfrac{-1}{3(h+5)}=\dfrac{-1}{3(-2+5)}=\dfrac{-1}{9}$

17

Rewrite the absolute value.

Reminder, if the inside is positive when you substitute in use the positive of the inside, if the inside is negative when you substitute in use the negative of the inside.

17. $\lim_{x\to 5} \frac{|2x-10|}{3x-15}$

$\frac{-2(x-5)}{3(x-5)} = -\frac{2}{3}$

18. $\lim_{x\to 7} \frac{3x-21}{|7-x|}$

$\frac{3(x-7)}{|-(-7+x)|} = \frac{3(x-7)}{-(x-7)}$

-3

19. $\lim_{x\to 2} \frac{|x-2|}{x-2}$

$\lim_{x\to 2^+} \frac{x-2}{x-2} = 1$

$\lim_{x\to 2^-} \frac{-(x-2)}{x-2} = -1$

so DNE

One sided limits when you get $\frac{\#}{0}$**, do you get** ∞ or $-\infty$**? Reason it out!**

20. $\lim_{x\to 3^-} \frac{5}{x-3}$

$\frac{5}{2.999-3} = -5000$ so

$-\infty$

21. $\lim_{x\to 3^+} \frac{-4}{x-3}$

$\frac{-4}{3.001-3} = -4000$

$-\infty$

22. $\lim_{x\to 6^+} \frac{x+6}{x^2-36} = \frac{x+6}{(x+6)(x-6)} = \frac{1}{6.001-6}$

or $\frac{6.001+6}{(6.001)^2+6}$

∞

Limits to infinity. You can do a **behaves like** only in limits to infinity. You can also divide by the highest power in the denominator, simplify, and then take the limit.

$\lim_{x\to\pm\infty}$ (polynomial) The highest power controls the behavior!

23. $\lim_{x\to\infty} (3x^2-4x+2)$ even +

∞

24. $\lim_{x\to-\infty} (5x^3-2x^2+1)$ odd +

$-\infty$

$$\lim_{x\to\pm\infty} \frac{\text{degree smaller}}{\text{DEGREE LARGER}} = 0$$

25. $\lim_{x\to\infty} \frac{3x-5}{x^2+1}$

0

26. $\lim_{x\to-\infty} \frac{4x^2-3x}{6x^5-3x+1}$

0

$$\lim_{x\to\pm\infty} \frac{\text{degree} =}{\text{degree} =} = \text{ratio of the leading coefficients}$$

27. $\lim_{x\to\infty} \frac{5x-11}{4-3x}$ $\frac{5x-11}{-3x+4}$

$-5/3$

28. $\lim_{x\to-\infty} \frac{4x^2-5x+2}{3x^2+1}$

$4/3$

29. $\lim_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)}$ $\frac{2x^2+1}{4-x^2}$

-2

$$\lim_{x\to\pm\infty} \frac{\text{DEGREE LARGER}}{\text{degree smaller}} = \infty \text{ or } -\infty$$

30. $\lim_{x\to\infty} \frac{7-6x^5}{x+3}$

$\frac{7-6(\infty)^5}{\infty+3} = \frac{-\infty}{\infty}$

$-\infty$

31. $\lim_{x\to-\infty} \frac{7-6x^5}{x+3}$

$\frac{7-6(-\infty)^5}{-\infty+3} = \frac{\infty}{-\infty}$

$-\infty$

18

32. $\lim_{x\to\infty} \frac{5+x^3-3x^4}{2x-1}$

$\frac{-3(\infty)^4}{2(\infty)} = \frac{-\infty}{\infty}$

$-\infty$

33. $\lim_{x\to-\infty} \frac{5+x^3-3x^4}{2x-1}$

$\frac{-3(\infty)^4}{2(\infty)} =$

∞

$\lim\limits_{x\to\pm\infty}$ involving square roots: Use the **behaves like** method and remember that $\sqrt{x^2}=|x|$!

34. $\lim\limits_{x\to\infty}\frac{\sqrt{4x^2-2}}{x+1}$

$\sqrt{4x^2}=|2x|$ $\frac{|2x|}{x}=2$

35. $\lim\limits_{x\to-\infty}\frac{\sqrt{4x^2-2}}{x+1}$

$\frac{-2x}{x}=-2$

36. $\lim\limits_{x\to\infty}\frac{2-x}{\sqrt{7+9x^2}}$ $\frac{-1x}{\sqrt{9x^2}}$

$\frac{-x}{3x}=-\frac{1}{3}$

37. $\lim\limits_{x\to-\infty}\frac{2-x}{\sqrt{7+9x^2}}$ $\pm\frac{1}{3}$

$\frac{-x}{-3x}=\frac{1}{3}$

Write the equations of all vertical and horizontal asymptotes.

38. $y=\frac{2x^2-5x-3}{x^2-2x-3}$ $\frac{(2x+1)\cancel{(x-3)}}{(x+1)\cancel{(x-3)}}$

HA: $y=2$
VA: $x=-1$
hole at $x=3$

39. $y=\frac{3-x}{9-x^2}=\frac{\cancel{3-x}}{(3+x)\cancel{(3-x)}}$

HA: $y=0$
VA: $x=-3$
hole at $x=3$

Continuity: Limit from right = limit from left = value of $f(x)$ at the point.

Is $f(x)$ continuous? Why?

40. $f(x)=\begin{cases}-5-x, & x>-1\\ 6x+2, & x\le -1\end{cases}$

$\lim\limits_{x\to-1^-} 6(-1)+2=-4$

$\lim\limits_{x\to-1^+} -5--1=-4$

$f(-1)=-4$

continuous

41. $f(x)=\frac{|x+2|}{x+2}$ $x+2\neq-2$

$\lim\limits_{x\to-2^-}\frac{-1(x+2)}{x+2}=-1$

$\lim\limits_{x\to-2^+}\frac{x+2}{x+2}=1$

no; at $x=-2$ Jump Discontinuity

Intermediate Value Theorem.

42. Verify the conditions of the Intermediate Value Theorem, and **find** *c* guaranteed by the theorem when $f(x)=x^2-6x+7$ over the interval $[0,3]$ and $f(c)=-1$.

$f(0)=7$ $f(3)=-2$

$-2<-1<7$

$-1=x^2-6x+7$
$0=x^2-6x+8$
$0=(x-4)(x-2)$
$x\neq4$ $x=2$

Finding values that make a function continuous.

43. Find the value of a that would make the function continuous.

$f(x)=\begin{cases}3xa+5 & \text{if } x\le -1\\ -2x+5a & \text{if } x>-1\end{cases}$

$3xa+5=-2x+5a$
$3(-1)a+5=-2(-1)+5a$
$-3a+5=2+5a$
$-8a=-3$
$a=3/8$

44. Find the value of *m* and *n* that would make the function continuous.

$g(x)=\begin{cases}3mx-4n & \text{if} & x\le -1\\ 4+nx-mx^2 & \text{if} & -1<x<2\\ x^2-mx+7n & \text{if} & x\ge 2\end{cases}$

$3mx-4n=4+nx-mx^2$
$3m(-1)-4n=4+n(-1)-m(-1)^2$
$-3m-4n=4-n-m$
$-2m=3n+4$
$m=-\frac{3}{2}n-2$

$4+nx-mx^2=x^2-mx+7n$
$4+n(2)-m(2)^2=(2)^2-m(2)+7n$
$4+2n-4m=4-2m+7n$
$-2m=5n$
$m=-\frac{5}{2}n$

$-\frac{3}{2}n-2=-\frac{5}{2}n$
$-3n-4=-5n$
$2n=4$

$n=2$ $m=-5$

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LIMITS REVIEW

Use the properties and factoring techniques to find each limit.

1. $\lim_{x\to 0}\frac{9-4x}{2x^3-4x^2+3}$

$\frac{9-4(0)}{2(0)^3-4(0)^2+3}=\frac{9}{3}=3$

2. $\lim_{x\to 2}\frac{2x^2+x-10}{x^2+x-6}$

$\frac{(2x+5)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$

$\frac{2\cdot2+5}{2+3}=9/5$

3. $\lim_{x\to -3}\frac{x^3+27}{x+3}$

$\frac{\cancel{(x+3)}(x^2-3x+9)}{\cancel{x+3}}$

$(-3)^2-3(-3)+9=27$

4. $\lim_{x\to 5}\frac{x}{x^2-25}=\frac{5}{0}$ $x=5$ is V.A.

$\lim_{x\to 5^+}=\infty$ so DNE

$x\to 5^-=-\infty$

5. $\lim_{x\to 0}\frac{x^3-8}{x^2-4}$

$\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}=\frac{4}{2}$

2

6. $\lim_{x\to 3}\frac{(3-x)^2}{x-3}=\frac{0}{0}$

$\frac{(3-x)(3-x)}{x-3}=\frac{-1\cancel{(x-3)}(3-x)}{\cancel{x-3}}$

$-1(3-3)=0$

7. $\lim_{\theta\to 0}\frac{(1-\cos\theta)(1+\cos\theta)}{2\sin^2\theta\,(1+\cos\theta)}$

$\frac{1-\cos^2\theta}{2\sin^2\theta}=\frac{\cancel{\sin^2\theta}\ 1}{\cancel{2\sin^2\theta}(1+\cos\theta)}$

$\frac{1}{2(1+\cos\theta)}=\frac{1}{2(1+1)}=\frac{1}{4}$

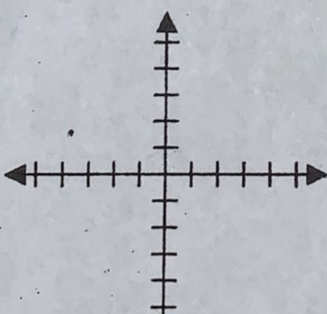
8. $\lim_{x\to -1}\frac{x^4-1}{x+1}$

$\frac{(x^2+1)(x^2-1)}{x+1}=\frac{(x^2+1)\cancel{(x+1)}(x-1)}{\cancel{x+1}}=\frac{((-1)^2+1)(-1-1)}{}$

$2\cdot-2=-4$

For problems 9-12, use the function $f(x)=\begin{cases}x-1, & x\le 1\\ 3x-7, & x>1\end{cases}$.

9. Graph the function.



10. $\lim_{x\to 1^-}f(x)$ $1-1=0$

$x<1$

11. $\lim_{x\to 1^+}f(x)$ $3(1)-7=-4$

$x>1$

12. $\lim_{x\to 1}f(x)$ DNE

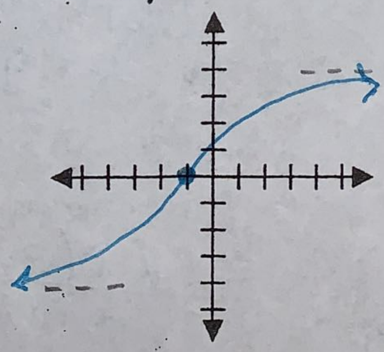
13. Draw a function that meets the following conditions. Is this function continuous? Explain.

$\lim_{x\to\infty}f(x)=4$

$\lim_{x\to -1}f(x)=0$

$\lim_{x\to -\infty}f(x)=-4$

$f(-1)=0$



yes, there are no discontinuities (holes, jumps, V.A.)

↑ removable

↑ infinite

(20)

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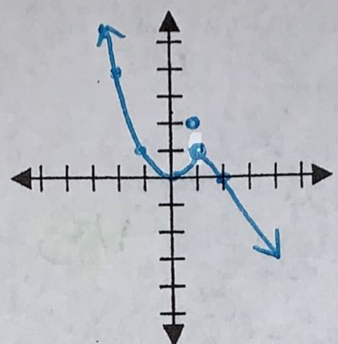
14. Draw a function that meets the following conditions. Find the indicated limit if it exists. Is this function continuous? Explain.

$$f(x)=\begin{cases} x^2, & x<1 \\ 2-x, & x>1 \\ 2, & x=1 \end{cases}$$



15. $\lim_{x\to 1^+} f(x)$ = 1

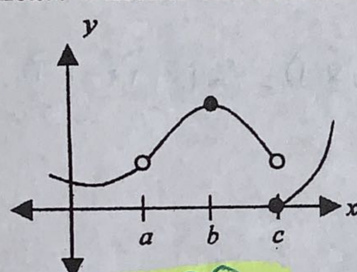
16. $\lim_{x\to 1^-} f(x)$ = 1

17. $\lim_{x\to 1} f(x)$ = 1

18. $f(1)$ = 2

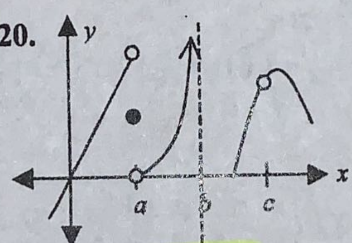
Indicate whether the function whose graph is given is continuous at each of the points *a*, *b*, and *c*.

19.



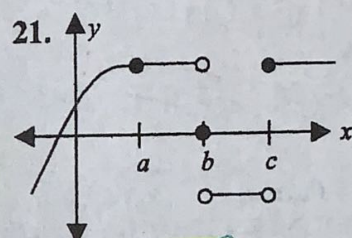
a. no
b. yes
c. no

20.



a. no
b. no
c. no

21.



a. yes
b. no
c. no

CONTINUITY REVIEW

Find a value for *k* which will cause $f(x)$ to be continuous for all real *x*.

22. $f(x)=\begin{cases} kx^2, & \text{if } x<-3 \\ 5-4x, & \text{if } x\geq -3 \end{cases}$

$Kx^2=5-4x$

$K(-3)^2=5-\cancel{4}(-3)$

$9K=17$

$K=17/9$

23. $f(x)=\begin{cases} x^3, & \text{if } x<\frac{1}{2} \\ kx^2, & \text{if } x\geq \frac{1}{2} \end{cases}$

$x^3=Kx^2$

$(1/2)^3=K(\frac{1}{2})^2$

$8(\frac{1}{8}=\frac{1}{4}K$

$1=2K$

$K=1/2$

24. Define $f(3)$ so that $f(x)=\dfrac{x^2-9}{x-3}$ is continuous at $x=3$.

hole → $\dfrac{(x-3)(x+3)}{x-3}$ $f(x)=x+3$

$x-3=0$

$x=3$

$f(3)=3+3$

$f(3)=6$

or $(3,6)$

25. Define $f(1)$ so that $f(x)=\dfrac{x^3-1}{x^2-1}$ is continuous at $x=1$.

$\dfrac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)}$

$f(1)=\frac{3}{2}$

$(1,3/2)$

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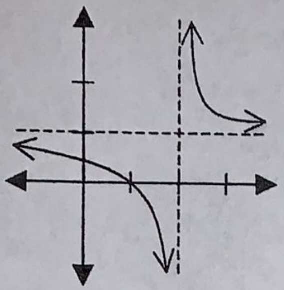
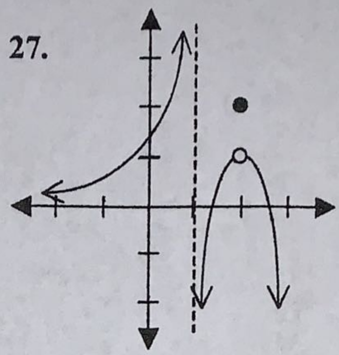
Use the graph to determine the intervals for which the function is continuous.

26.



$(-\infty,2)\cup(2,\infty)$

27.



$(-\infty,1)\cup(1,2)\cup(2,\infty)$

At what values are the following functions discontinuous? Explain the type of discontinuity.

28. $f(x)=\dfrac{x+3}{x^2-3x-10}$

$\dfrac{x+3}{(x-5)(x+2)}$ $x \neq 5$ $x \neq -2$

x=5 & x=2
Infinite discontinuities

29. $f(x)=\dfrac{x+2}{4-x^2}$

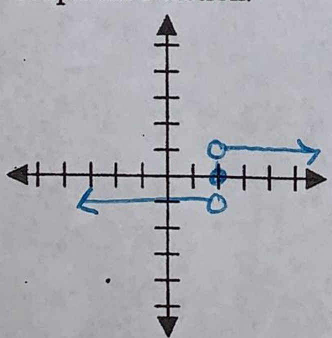
$\dfrac{\cancel{x+2}}{(\cancel{2+x})(2-x)} = \dfrac{1}{2-x}$ VA at x=2

& hole at x=-2

x=-2 Removable discontinuity
x=2 Infinite discontinuity

For problems 30-42, use the function $f(x)=\begin{cases}\dfrac{|x-2|}{x-2}, & x\neq 2\\ 0, & x=2\end{cases}$.

30. Graph the function.



31. domain: $(-\infty,\infty)$

range: $\{-1,0,1\}$

32. $f(0)=-1$

33. $f(2)=0$

34. $f(4)=1$

35. $\lim\limits_{x\to 0^+} f(x)=-1$

36. $\lim\limits_{x\to 0^-} f(x)=-1$

37. $\lim\limits_{x\to 0} f(x)=-1$

38. Is $f(x)$ continuous at $x=0$? Explain.

yes
#35-37 are same limit + f(0)=-1

39. $\lim\limits_{x\to 2^-} f(x)=-1$

40. $\lim\limits_{x\to 2^+} f(x)=1$

41. $\lim\limits_{x\to 2} f(x)=$ DNE

42. Is $f(x)$ continuous at $x=2$? Explain.

no
Jump at x=2
+ f(2)=0