Unit 2 Limits

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- ❖ Limits from Graphs
- ❖ Graphs from Limits
- ❖ One-Sided Limits & Continuity
- ❖ Creative Factoring
- ❖ Algebraic Limits
- ❖ Intermediate Value Theorem
- ❖ Asymptotes, End Behavior & Infinite Limits

Quiz is ______________________

Test is ______________________

Name: Bonanni

F19

Limits from Table of Values

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
f(x)	1.971	1.987	1.997	undefined	1.997	1.987	1.971
g(x)	2.018	2.008	2.002	2	2.002	2.008	2.018
h(x)	1	1	1	2	2	2	2

Find the following:

(a) $\lim_{x \to 0} f(x) = 2$

(b) $\lim_{x \to 0} g(x) = 2$

(c) $\lim_{x \to 0} h(x) =$ DNE

x	2.75	2.9	2.99	2.999	3	3.001	3.01	3.1	3.25
f(x)	5.313	5.710	5.970	5.997	6	6.003	6.030	6.310	6.813
g(x)	1.99499	1.99950	1.99995	1.99999	und	2.00005	2.00050	2.00499	2.01
h(x)	1.99499	1.99950	1.99995	1.99999	2	6.003	6.030	6.310	6.813

Find the following:

(a) $\lim_{x \to 3} f(x) = 6$

(b) $\lim_{x \to 3} g(x) = 2$

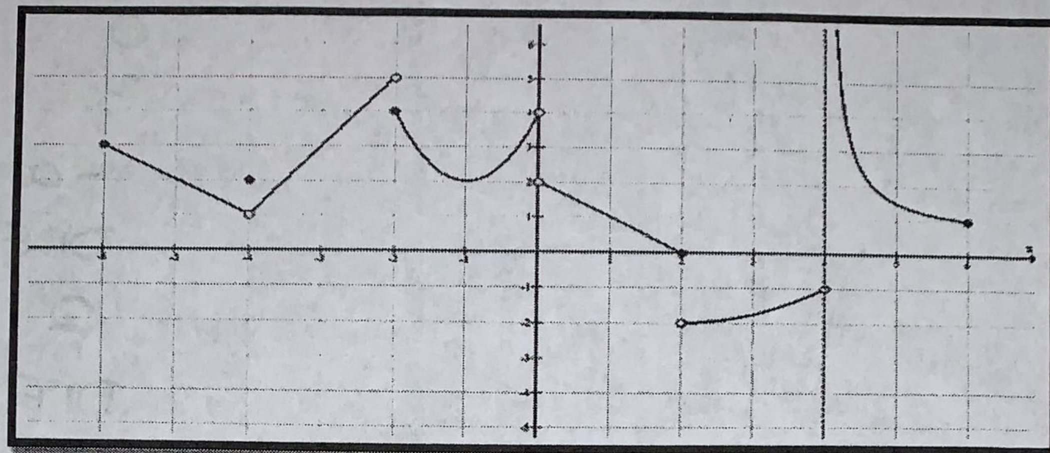
(c) $\lim_{x \to 3} h(x) =$ DNE

Honors Calculus

Name ____________________

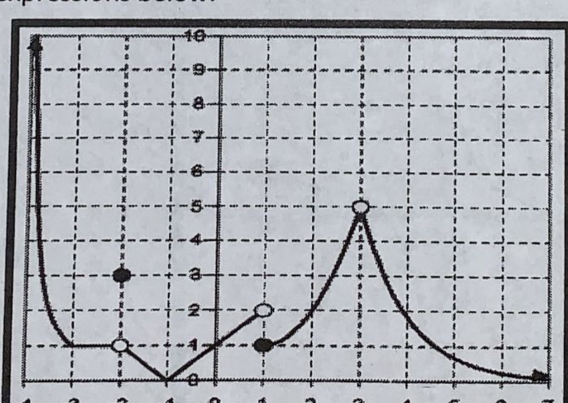
Finding Limits from a Graph

1. Use the graph to evaluate the limits below



a.	$f(-4)$	2	b.	$\lim_{x\to -4^-} f(x)$	1	c.	$\lim_{x\to -4^+} f(x)$	1	d.	$\lim_{x\to -4} f(x)$	1
e.	$f(-2)$	4	f.	$\lim_{x\to -2^-} f(x)$	5	g.	$\lim_{x\to -2^+} f(x)$	4	h.	$\lim_{x\to -2} f(x)$	DNE
i.	$f(0)$	DNE	j.	$\lim_{x\to 0^-} f(x)$	4	k.	$\lim_{x\to 0^+} f(x)$	2	l.	$\lim_{x\to 0} f(x)$	DNE
m.	$f(2)$	0	n.	$\lim_{x\to 2^-} f(x)$	0	o.	$\lim_{x\to 2^+} f(x)$	-2	p.	$\lim_{x\to 2} f(x)$	DNE
q.	$f(4)$	DNE	r.	$\lim_{x\to 4^-} f(x)$	-1	s.	$\lim_{x\to 4^+} f(x)$	∞	t.	$\lim_{x\to 4} f(x)$	DNE

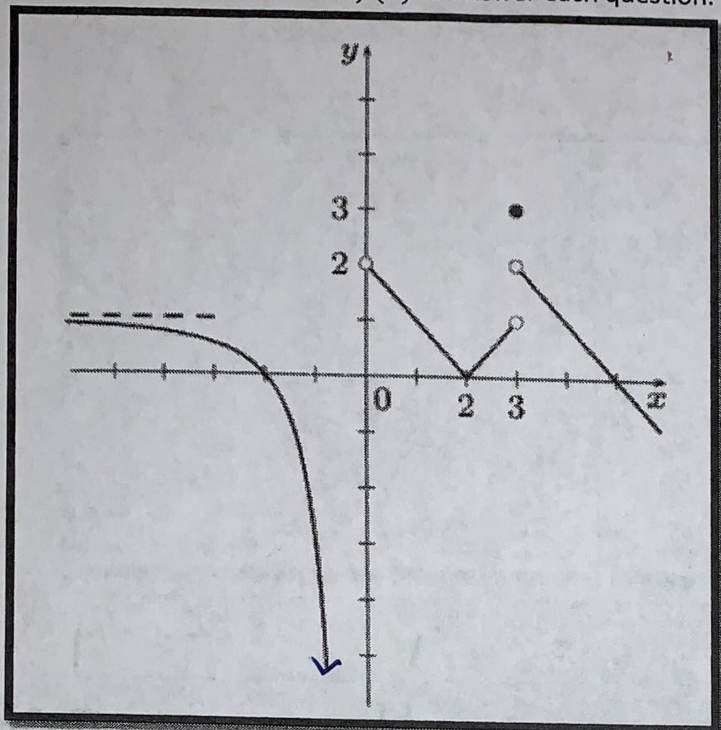
2. Use the graph to evaluate the expressions below.



a.	$f(-2)$	3	b.	$\lim_{x\to -2^+} f(x)$	1	c.	$\lim_{x\to -2} f(x)$	1
d.	$\lim_{x\to -1^+} f(x)$	0	e.	$\lim_{x\to -1^-} f(x)$	0	f.	$\lim_{x\to -1} f(x)$	0
g.	$\lim_{x\to 1^+} f(x)$	1	h.	$\lim_{x\to 1^-} f(x)$	2	i.	$\lim_{x\to 1} f(x)$	DNE
j.	$f(3)$	DNE	k.	$\lim_{x\to 3^+} f(x)$	5	l.	$\lim_{x\to 3^-} f(x)$	5
m.	$\lim_{x\to 3} f(x)$	5	n.	$\lim_{x\to -4^+} f(x)$	∞	o.	$\lim_{x\to \infty} f(x)$	0
p.	$f(1)$	1	q.	$\lim_{x\to -3} f(x)$	1	r.	$f(-4)$	DNE

2

3. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty, or\ DNE$ where appropriate.



a. $f(0) =$ DNE

b. $f(2) =$ 0

c. $f(3) =$ 3

d. $\lim_{x \to 0^-} f(x) =$ $-\infty$

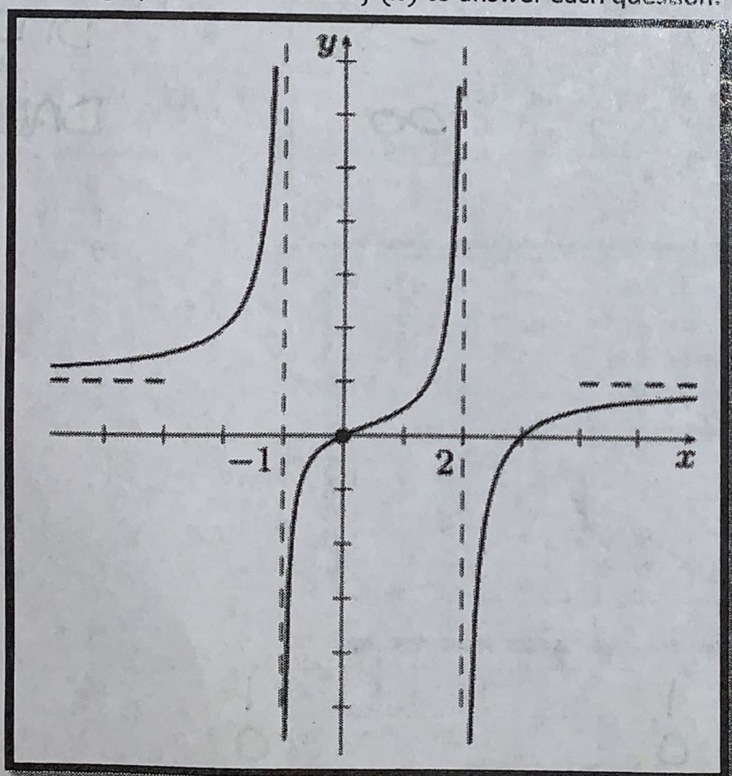
e. $\lim_{x \to 0} f(x) =$ DNE

f. $\lim_{x \to 3^+} f(x) =$ 2

g. $\lim_{x \to 3} f(x) =$ DNE

h. $\lim_{x \to -\infty} f(x) =$ 1

4. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty, or\ DNE$ where appropriate.



a. $f(0) =$ 0

b. $f(2) =$ DNE

c. $f(3) =$ 0

d. $\lim_{x \to -1} f(x) =$ DNE

e. $\lim_{x \to 0} f(x) =$ 0

f. $\lim_{x \to 2^+} f(x) =$ $-\infty$

g. $\lim_{x \to \infty} f(x) =$ 1

Graphs from Limit Worksheet

Draw a graph of a function with the give limits.

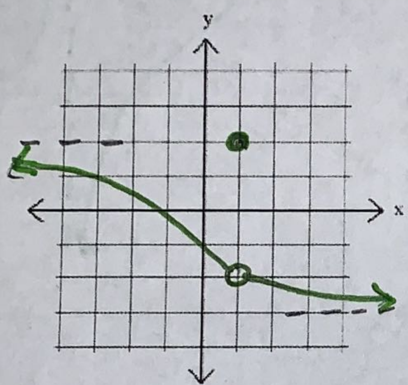
1.

$\lim_{x\to\infty} f(x) = -3$ HA

$\lim_{x\to 1} f(x) = -2$ (1,-2) open touches

$\lim_{x\to -\infty} f(x) = 2$ HA

$f(1) = 2$ (1,2) closed



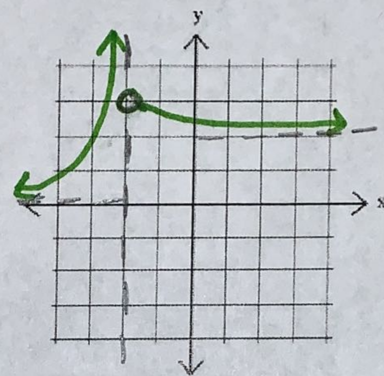
2.

$\lim_{x\to\infty} f(x) = 2$ HA

$\lim_{x\to -2^+} f(x) = 3$ (-2,3) from right

$\lim_{x\to -2^-} f(x) = \infty$ VA

$\lim_{x\to -\infty} f(x) = 0$ HA



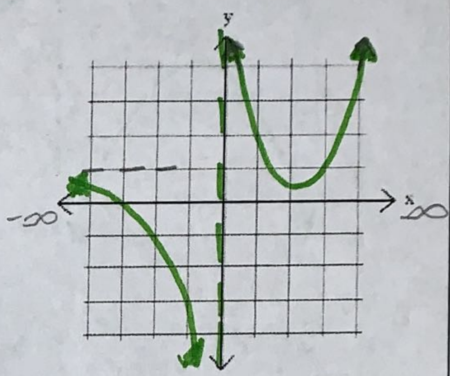
3.

$\lim_{x\to\infty} f(x) = \infty$ end behavior

$\lim_{x\to 0^+} f(x) = \infty$ VA

$\lim_{x\to 0^-} f(x) = -\infty$ VA

$\lim_{x\to -\infty} f(x) = 1$ HA



4.

$\lim_{x\to -\infty} f(x) = -\infty$ end behavior

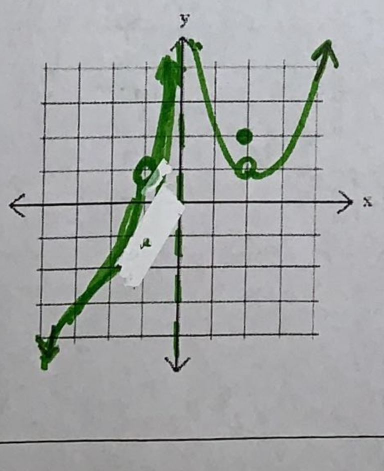
$\lim_{x\to -1} f(x) = 1$ both sides (-1,1)

$\lim_{x\to 0} f(x) = \infty$ both sides V.A

$\lim_{x\to 2} f(x) = 1$ both sides (2,1)

$f(2) = 2$ pt at (2,2)

$\lim_{x\to\infty} f(x) = \infty$ end behavior



5.

$\lim_{x\to -\infty} f(x) = -\infty$ E.B.

$\lim_{x\to -2^-} f(x) = \infty$ VA

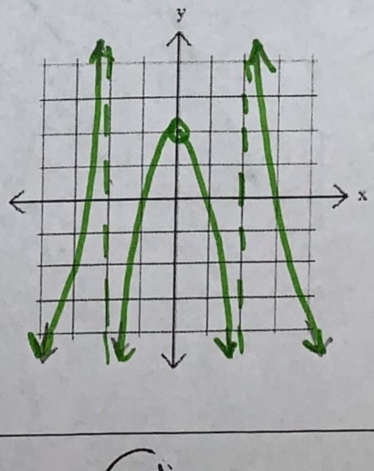
$\lim_{x\to -2^+} f(x) = -\infty$ VA

$\lim_{x\to 0} f(x) = 2$ (0,2) both sides

$\lim_{x\to 2^-} f(x) = -\infty$ VA

$\lim_{x\to 2^+} f(x) = \infty$ VA

$\lim_{x\to\infty} f(x) = -\infty$ EB



6.

$\lim_{x\to -\infty} f(x) = -2$ HA

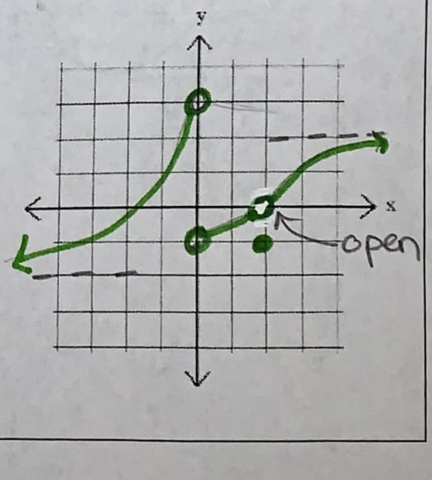
$\lim_{x\to 0^-} f(x) = 3$ (0,3) from left

$\lim_{x\to 0^+} f(x) = -1$ (0,-1) from right

$\lim_{x\to 2} f(x) = 0$ (2,0) both sides

$\lim_{x\to\infty} f(x) = 2$ HA

$f(2) = -1$ (2,-1) pt.



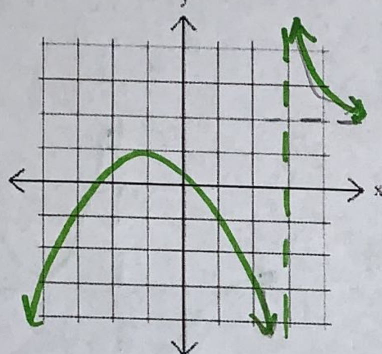
(Ui)

7. $\lim_{x\to\infty} f(x) = 2$ HA

$\lim_{x\to 3^+} f(x) = \infty$ VA ↑

$\lim_{x\to 3^-} f(x) = -\infty$ VA ↓

$\lim_{x\to -\infty} f(x) = -\infty$ E.B.

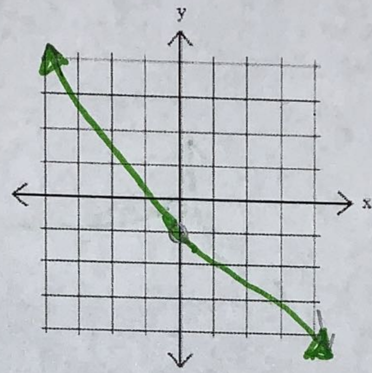


8. $\lim_{x\to\infty} f(x) = -\infty$ E.B. ↓

$\lim_{x\to -\infty} f(x) = \infty$ E.B. ↑

$\lim_{x\to 0^-} f(x) = -1$ (0,-1) from left

$\lim_{x\to 0^+} f(x) = -1$ (0,-1) from right

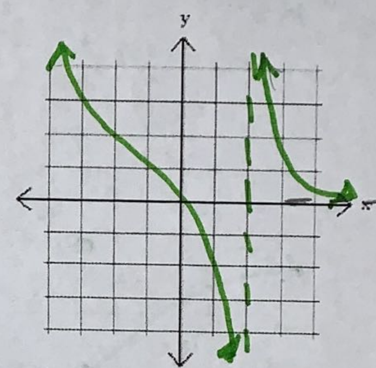


9. $\lim_{x\to\infty} f(x) = 0$ HA

$\lim_{x\to 2^+} f(x) = \infty$ VA

$\lim_{x\to 2^-} f(x) = -\infty$ VA

$\lim_{x\to -\infty} f(x) = \infty$ EB



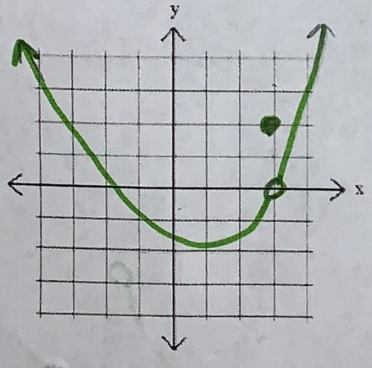
10. $\lim_{x\to\infty} f(x) = \infty$ E.B.

$\lim_{x\to 3^+} f(x) = 0$ (3,0) right

$\lim_{x\to 3^-} f(x) = 0$ (3,0) left

$\lim_{x\to -\infty} f(x) = \infty$ EB

$f(3) = 2$ (3,2) point

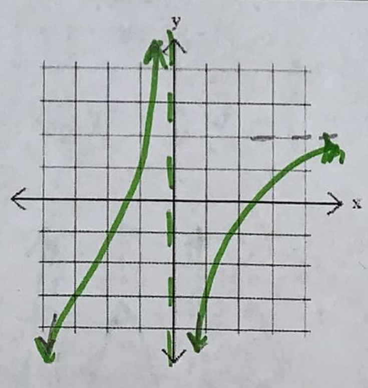


11. $\lim_{x\to\infty} f(x) = 2$ HA

$\lim_{x\to 0^+} f(x) = -\infty$ VA

$\lim_{x\to 0^-} f(x) = \infty$ VA

$\lim_{x\to -\infty} f(x) = -\infty$ EB

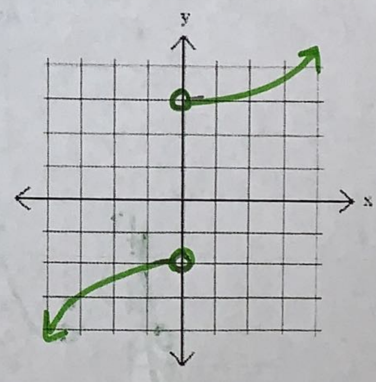


12. $\lim_{x\to\infty} f(x) = \infty$ E.B.

$\lim_{x\to 0^+} f(x) = 3$ (0,3) from right

$\lim_{x\to 0^-} f(x) = -2$ (0,-2) from left

$\lim_{x\to -\infty} f(x) = -\infty$ EB



One-sided Limits Worksheet

Evaluate each limit.

1. $\lim_{x\to 2^+} \frac{x}{x-2}$ $\frac{2.001}{2.001-2} = \infty$

2. $\lim_{x\to 3^+} \frac{x+1}{x^2-6x+9}$ $\frac{3.001+1}{(3.001)^2-6(3.001)+9}$

∞

3. $\lim_{x\to -3^-} \frac{x+2}{x^2+6x+9}$

$\frac{-3.001+2}{(-3.001)^2+6(-3.001)+9} = \frac{-}{+} = -\infty$

4. $\lim_{x\to -2^+} \frac{x-2}{x^2+4x+4}$

$\frac{-1.99-2}{(-1.99)^2+4(-1.99)+4} = \frac{-}{+} = -\infty$

5. $\lim_{x\to -3^-} \frac{x^2}{3x+9}$

$\frac{(-3.01)^2}{3(-3.01)+9} = \frac{+}{-} = -\infty$

6. $\lim_{x\to 2^+} \frac{x^2}{2x-4}$

$\frac{+}{+} = \infty$

7. $\lim_{x\to -2^+} \frac{1}{x^2-4}$

$\frac{1}{(-1.99)^2-4} = \frac{+}{-} = -\infty$

8. $\lim_{x\to 1^-} -\frac{2}{x^2-1}$

$\frac{-2}{(.999)^2-1} = \frac{-2}{-} = \infty$

9. $\lim_{x\to 3^-} f(x), f(x) = \begin{cases} -x+4, & x<3 \\ \frac{x}{2}+1, & x\geq 3 \end{cases}$

$x<3$

$-3+4 = 1$

10. $\lim_{x\to -1^+} f(x), f(x) = \begin{cases} x+3, & x\leq -1 \\ -x-1, & x>-1 \end{cases}$

$x>-1$

$-(-1)-1 = 0$

11. $\lim_{x\to -2^-} f(x), f(x) = \begin{cases} -x^2-8x-17, & x\leq -2 \\ 2x-1, & x>-2 \end{cases}$

$x<-2$

$-(-2)^2-8(-2)-17 = -5$

12. $\lim_{x\to 1^-} (|x-1|-2)$

$|1-1|-2 = 0-2 = -2$

13. $\lim_{x\to 0^+} \frac{2x}{|x|}$ $= 2$

x	y
.1	2
1	2
2	2

*absolute value

$x\to 0^-$ would $= -2$

14. $\lim_{x\to 1^-} f(x), f(x) = \begin{cases} -\frac{x}{2}-\frac{3}{2}, & x\leq 1 \\ -x^2+4x-5, & x>1 \end{cases}$

$x<1$

$-\frac{1}{2}-\frac{3}{2} = -\frac{4}{2} = -2$

15. $\lim_{x\to -3^-} f(x), f(x) = \begin{cases} x+6, & x<-3 \\ 3, & x\geq -3 \end{cases}$

$x<-3$

$-3+6 = 3$

16. $\lim_{x\to 0^-} f(x), f(x) = \begin{cases} -2x+3, & x\leq 0 \\ -\frac{x}{2}+3, & x>0 \end{cases}$

$x<0$

$-2(0)+3 = 3$

Continuity Worksheet

Determine if each function is continuous. If the function is not continuous, find the x-axis location of and classify each discontinuity.

1. $f(x) = -\frac{x}{2x^2+2x+1}$

$2x^2+2x+1 \leftarrow$ not factorable imaginary solutions

Continuous

2. $f(x) = \frac{x}{x^2+6x+9}$

$(x+3)(x+3)$ $x+3=0$ $x=-3$

Infinite discontinuity at $x=-3$

3. $f(x) = \frac{x^2+4x+3}{x+3} = \frac{\cancel{(x+3)}(x+1)}{\cancel{(x+3)}}$ hole

Removable discontinuity at $x=-3$

4. $f(x) = \frac{x}{x^2-4x} = \frac{\cancel{x}}{\cancel{x}(x-4)}$ hole $x-4=0$ $x\neq 4$

Removable at $x=0$
Infinite disc. at $x=4$

5. $f(x) = \begin{cases} x+4, & x \leq -2 \\ -2x-11, & x > -2 \end{cases}$ $-2+4=2$ $-2(-2)-11=-7$

Jump at $x=-2$

6. $f(x) = \frac{x+7}{x^2+3x} = \frac{x+7}{x(x+3)}$ $x\neq 0$ $x\neq -3$

Infinite discontinuity at $x=0$ + $x=-3$

Find the intervals on which each function is continuous.

7. $f(x) = \begin{cases} x, & x \neq 4 \\ 2, & x = 4 \end{cases}$

$(-\infty,4)\cup(4,\infty)$

8. $f(x) = \begin{cases} -2, & x < 3 \\ -2x+6, & x \geq 3 \end{cases}$ $-2(3)+6=0$

$(-\infty,3)\cup[3,\infty)$

9. $f(x) = \frac{(x-1)}{x^2-4x+3} = \frac{\cancel{x-1}}{\cancel{(x-1)}(x-3)}$ hole at $x=1$ VA at $x=3$

$(-\infty,1)\cup(1,3)\cup(3,\infty)$

10. $f(x) = \frac{x^2}{2} + 4x + 10$

$(-\infty,\infty)$

11. $f(x) = -x^2 - 4x + 2$

$(-\infty,\infty)$

12. $f(x) = -\frac{x-2}{x^2-3x+2} = -\frac{\cancel{x-2}}{\cancel{(x-2)}(x-1)} = \frac{-1}{x-1}$

$(-\infty,1)\cup(1,2)\cup(2,\infty)$ hole at $x=2$ VA $x=1$

13. $f(x) = -\frac{x-1}{x^2-x} = -\frac{\cancel{x-1}}{x\cancel{(x-1)}} = \frac{-1}{x}$

$(-\infty,0)\cup(0,1)\cup(1,\infty)$ hole at $x=1$ V.A. at $x=0$

14. $f(x) = \frac{x}{x^2-6x+9} = \frac{x}{(x-3)(x-3)}$ VA at $x=3$

$(-\infty,3)\cup(3,\infty)$

15. Critical Thinking: Write a function that has an infinite discontinuity at $x = 100$

$f(x)=\frac{1}{x-100}$

16 Critical Thinking: Write a function that is continuous over $(-\infty, 0), (0,1), and\ (1, \infty)$ and discontinuous everywhere else.

$\frac{1}{x(x-1)}$

$f(x)=\frac{1}{x^2-x}$

AP Calculus AB: Limits and Continuity

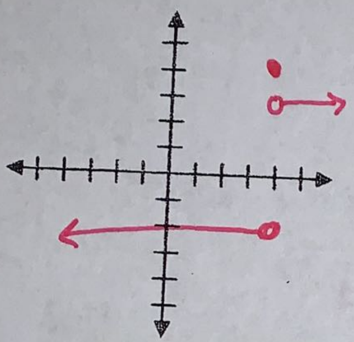
Name ____________________

One Sided Limits

Date ____________ Period ______

Draw a sketch. Find the indicated limit if it exists. If the limit does not exist, explain why.

1. $G(x)=\begin{cases}3, & \text{if } x>4\\ 5, & \text{if } x=4\\ -2, & \text{if } x<4\end{cases}$



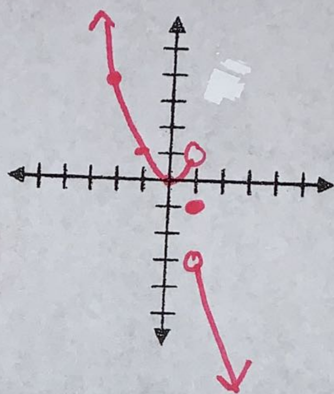
a. $\lim_{x\to 4^+} G(x)$ 3

b. $\lim_{x\to 4^-} G(x)$ -2

c. $\lim_{x\to 4} G(x)$ DNE

d. $G(4)$ 5

2. $T(x)=\begin{cases}3-6x, & \text{if } x>1\\ -1, & \text{if } x=1\\ x^2, & \text{if } x<1\end{cases}$



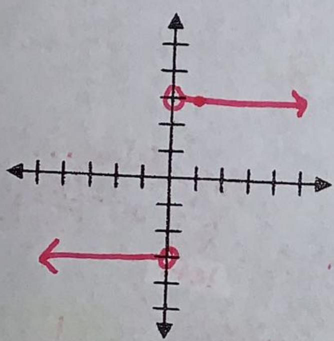
a. $\lim_{x\to 1^-} T(x)$ 1

b. $\lim_{x\to 1^+} T(x)$ -3

c. $\lim_{x\to 1} T(x)$ DNE

d. $T(1)$ -1

3. $G(x)=\dfrac{|3x|}{x}$



a. $\lim_{x\to 0^+} G(x)$ 3

b. $\lim_{x\to 0^-} G(x)$ -3

c. $\lim_{x\to 0} G(x)$ DNE

d. $G(0)$ DNE

4. Find the limit without sketching the graph

$F(x)=\begin{cases}x^2-16, & \text{if } x<3\\ 5, & \text{if } x=3\\ 14-x^2, & \text{if } x>3\end{cases}$

a. $\lim_{x\to 3^+} F(x)$ $14-(3)^2=5$

b. $\lim_{x\to 3^-} F(x)$ $(3)^2-16=-7$

c. $\lim_{x\to 3} F(x)$ DNE

d. $F(3)$ 5

AP Calculus AB: Limits and Continuity Name ____________

One Sided Limits Date ____________ Period ______

5. $F(x)=\begin{cases} 2x-5, & \text{if } x>\frac{1}{2} \\ 3kx-1, & \text{if } x<\frac{1}{2} \end{cases}$

Find the value of k such that $\lim_{x\to\frac{1}{2}} F(x)$ exists.

$2x-5=3kx-1$
$2(1/2)-5=3k(1/2)-1$
$-4=3/2k-1$
$\frac{2}{3}\cdot -3=3/2k$
$k=-2$

6. $G(x)=\begin{cases} 4x-7k, & \text{if } x\geq -3 \\ 2k+x, & \text{if } x<-3 \end{cases}$

Find the value of k such that $\lim_{x\to -3} G(x)$ exists.

$4x-7k=2k+x$
$4(-3)-7k=2(\cancel{k})-3$
$-12-7k=2k-3$
$-9k=9$
$k=-1$

7. Find the values of m and k such that $\lim_{x\to 1} G(x)$ and $\lim_{x\to -1} G(x)$ both exist

$G(x)=\begin{cases} 3x^2-kx+m, & \text{if } x\geq 1 \\ mx-2k, & \text{if } -1<x<1 \\ -3m+4x^2k, & \text{if } x\leq -1 \end{cases}$

$3x^2-kx+m=mx-2k$
$3(+1)^2-k(+1)+m=m(+1)-2k$
$3-k+m=+m-2k$
$k=-3$

$mx-2k=-3m+4x^2k$
$m(-1)-2(-3)=-3m+4(-1)^2(-3)$
$-m+6=-3m-12$
$2m=-18$
$m=-9$

Find the one- sided limits.

8. $\lim_{x\to 2^-}\frac{3}{x-2}$

$\frac{3}{1.99-2}=\frac{3}{-.01}$

$-\infty$

9. $\lim_{x\to -3^+}\frac{5}{x+3}$

$\frac{5}{-2.99+3}=\frac{5}{+}$

∞

10. $\lim_{x\to 2}\frac{-7}{2-x}$ DNE

$\frac{-7}{2-1.999}=\frac{-7}{+}=-\infty$

$\frac{-7}{2-2.001}=\frac{-7}{-}=+\infty$

11. $\lim_{x\to 2^-}\frac{|x-2|}{x-2}$

Abs Value...
$\frac{x-2}{x-2}$ or $-\frac{(x-2)}{x-2}$
c^+ c^-

$-\frac{(\cancel{x-2})}{\cancel{x-2}}=-1$

12. $\lim_{x\to 5^+}\frac{3x-15}{|4x-20|}$

$\frac{3(\cancel{x-5})}{4|\cancel{x-5}|}$

$3/4$

13. $\lim_{x\to 5^-}\frac{3x-15}{|4x-20|}$

$-3/4$

14. $\lim_{x\to 5}\frac{3x-15}{|4x-20|}$

DNE

15. $\lim_{x\to 5^+}\frac{|x-4|}{x^2-3x+2}=\frac{|x-4|}{(x-2)(x-1)}$

$\frac{|5.01-4|}{(5.01)^2-3(5.01)+2}=\frac{+}{+}$

∞

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