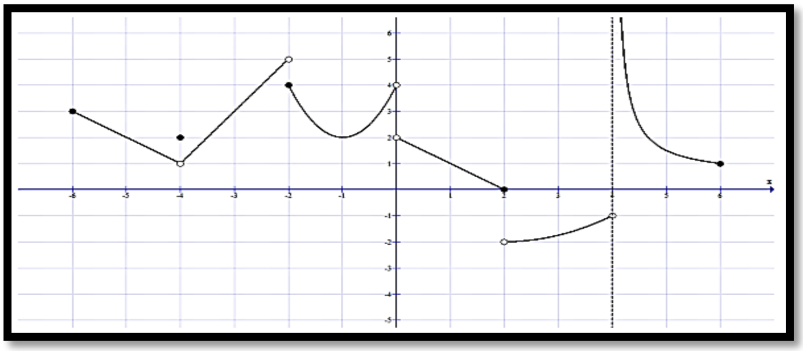
AP Calculus AB - Limits
Fall 2020

Day	Topic / Essential Question	Assignment
Thursday, August 20th	**2.1 Limits from Graphs and Graphs from Limits** *E.Q: How can I estimate limits from graphs and estimate graphs based on limit statements?*	Graphs from Limits and Limits from Graphs worksheet (Packet p. 1 – 4)
Friday, August 21st	**Creative Factoring** **2.2 Algebraic Limits** *E.Q: How can limits of a function be found algebraically or from a table of values?*	**Skills Check 2.1** Algebraic Limits Worksheet #1-9 (Packet p. 7)
Monday, August 24th	**2.2 Algebraic Limits** *E.Q: How can limits of a function be found algebraically or from a table of values?*	Algebraic Limits Worksheet #10-42 (Packet p. 7 – 10)
Tuesday, August 25th	**2.3 Intermediate Value Theorem and Continuity** *E.Q.: What types of functions are continuous? What are the types of discontinuities and what happens in functions to create them?*	**Skills Check 2.2** Continuity and Intermediate Value Theorem Worksheet (Packet p. 11 – 13)
Wednesday, August 26th	**2.4 One-sided Limits** *E.Q.: How do I evaluate limits in piecewise functions and absolute value functions?*	One-sided Limits Graphically & Algebraically Worksheet (Packet p. 1 4– 15)
Thursday, August 27th	**2.5 Limits Involving Infinity** *E.Q.: What happens to functions as x approaches infinity and what causes y to approach infinity?*	**Skills Check 2.3** Vertical and Horizontal Asymptotes Worksheet Infinite Limits Worksheet (Packet p. 16 – 18)
Friday, August 28th	**Review** *E.Q.: How can we put all the limit concepts together?*	**Skills Check 2.5** Released Multiple Choice Questions - Limits Worksheet (Packet p. 19 – 21) Limits Practice Test (Packet p. 22 – 25)
Monday, August 31st	**Unit 2 Test**	This is your opportunity to show what you know!

Graphs from Limit and Limits from Graphs

1. Use the graph to evaluate the limits below



a. $f(-4)$ b. $\lim_{x\to -4^-} f(x)$ c. $\lim_{x\to -4^+} f(x)$ d. $\lim_{x\to -4} f(x)$

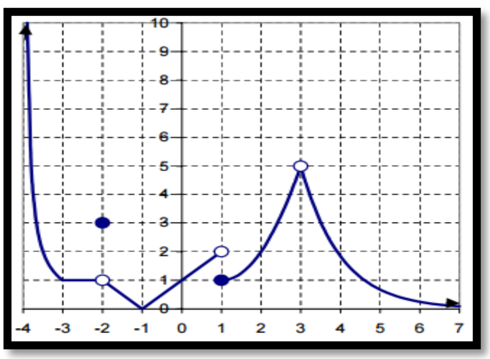
e. $f(-2)$ f. $\lim_{x\to -2^-} f(x)$ g. $\lim_{x\to -2^+} f(x)$ h. $\lim_{x\to -2} f(x)$

i. $f(0)$ j. $\lim_{x\to 0^-} f(x)$ k. $\lim_{x\to 0^+} f(x)$ l. $\lim_{x\to 0} f(x)$

m. $f(2)$ n. $\lim_{x\to 2^-} f(x)$ o. $\lim_{x\to 2^+} f(x)$ p. $\lim_{x\to 2} f(x)$

q. $f(4)$ r. $\lim_{x\to 4^-} f(x)$ s. $\lim_{x\to 4^+} f(x)$ t. $\lim_{x\to 4} f(x)$

2. Use the graph to evaluate the expressions below.



a. $f(-2)$ b. $\lim_{x\to -2^+} f(x)$ c. $\lim_{x\to -2} f(x)$

d. $\lim_{x\to -1^+} f(x)$ e. $\lim_{x\to -1^-} f(x)$ f. $\lim_{x\to -1} f(x)$

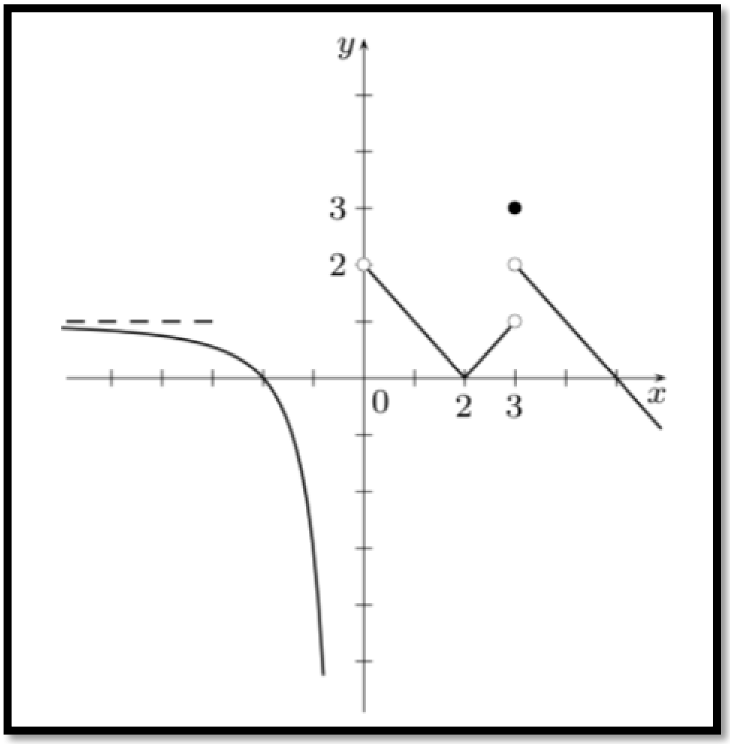
g. $\lim_{x\to 1^+} f(x)$ h. $\lim_{x\to 1^-} f(x)$ i. $\lim_{x\to 1} f(x)$

j. $f(3)$ k. $\lim_{x\to 3^+} f(x)$ l. $\lim_{x\to 3^-} f(x)$

m. $\lim_{x\to 3} f(x)$ n. $\lim_{x\to -4^+} f(x)$ o. $\lim_{x\to \infty} f(x)$

p. $f(1)$ q. $\lim_{x\to -3} f(x)$ r. $f(-4)$

3. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty, or\ DNE$ where appropriate.



a. $f(0) =$

b. $f(2) =$

c. $f(3) =$

d. $\lim_{x\to 0^-} f(x) =$

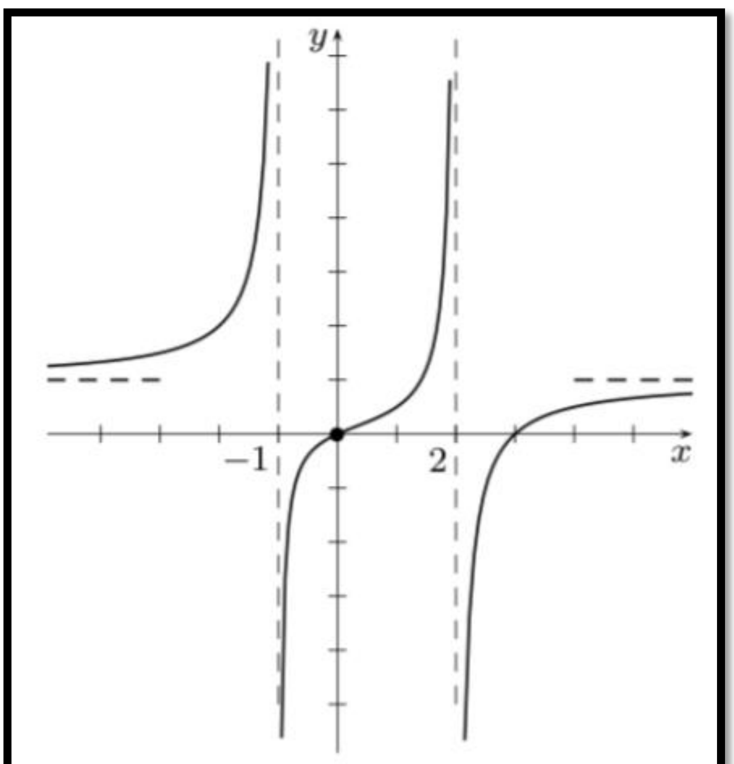
e. $\lim_{x\to 0} f(x) =$

f. $\lim_{x\to 3^+} f(x) =$

g. $\lim_{x\to 3} f(x) =$

h. $\lim_{x\to -\infty} f(x) =$

4. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty, or\ DNE$ where appropriate.



a. $f(0) =$

b. $f(2) =$

c. $f(3) =$

d. $\lim_{x\to -1} f(x) =$

e. $\lim_{x\to 0} f(x) =$

f. $\lim_{x\to 2^+} f(x) =$

g. $\lim_{x\to \infty} f(x) =$

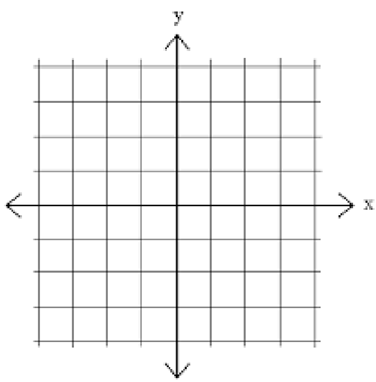
Draw a graph of a function with the give limits.

5. $\lim_{x\to\infty} f(x) = -3$

$\lim_{x\to 1} f(x) = -2$

$\lim_{x\to -\infty} f(x) = 2$

$f(1) = 2$

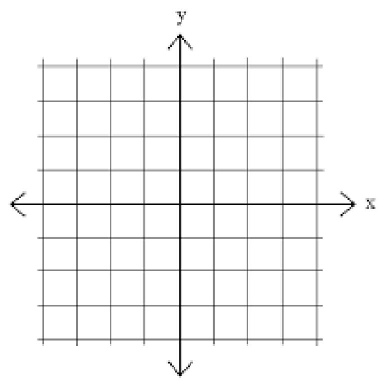


6. $\lim_{x\to\infty} f(x) = 2$

$\lim_{x\to -2^+} f(x) = 3$

$\lim_{x\to -2^-} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = 0$

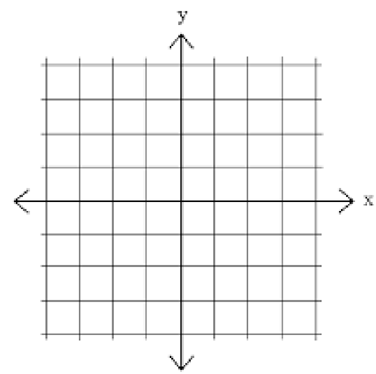


7. $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to 0^+} f(x) = \infty$

$\lim_{x\to 0^-} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = 1$



8. $\lim_{x\to -\infty} f(x) = -\infty$

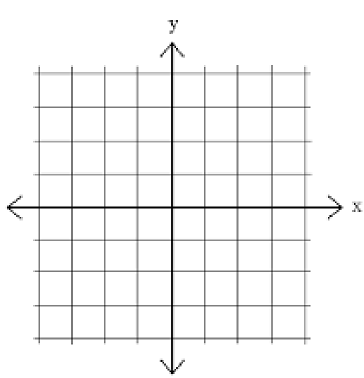
$\lim_{x\to -1} f(x) = 1$

$\lim_{x\to 0} f(x) = \infty$

$\lim_{x\to 2} f(x) = 1$

$f(2) = 2$

$\lim_{x\to\infty} f(x) = \infty$



9. $\lim_{x\to -\infty} f(x) = -\infty$

$\lim_{x\to -2^-} f(x) = \infty$

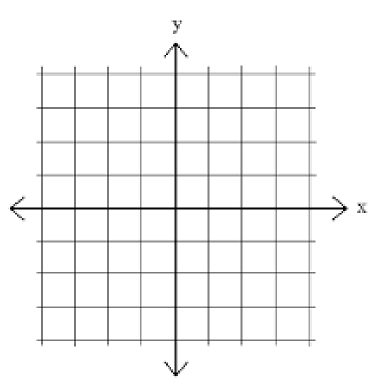
$\lim_{x\to -2^+} f(x) = -\infty$

$\lim_{x\to 0} f(x) = 2$

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = \infty$

$\lim_{x\to\infty} f(x) = -\infty$



10. $\lim_{x\to -\infty} f(x) = -2$

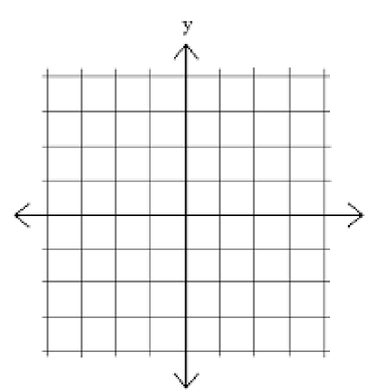
$\lim_{x\to 0^-} f(x) = 3$

$\lim_{x\to 0^+} f(x) = -1$

$\lim_{x\to 2} f(x) = 0$

$\lim_{x\to\infty} f(x) = 2$

$f(2) = -1$



11. Use the table of values to evaluate the limit.

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
f(x)	7.018	7.008	7.002	20	7.002	7.008	7.018
g(x)	4.126	4.789	4.989	8	8.0015	8.1016	8.546
h(x)	4971	8987	9972	undefined	8.997	8.987	8.971

a. $\lim_{x \to 0^+} f(x)$

b. $\lim_{x \to 0^-} f(x)$

c. $\lim_{x \to 0} f(x)$

d. $\lim_{x \to 0^+} g(x)$

e. $\lim_{x \to 0^-} g(x)$

f. $\lim_{x \to 0} g(x)$

g. $\lim_{x \to 0^+} h(x)$

h. $\lim_{x \to 0^-} h(x)$

i. $\lim_{x \to 0} h(x)$

12. Use the table of values to evaluate the limit.

X	2.75	2.9	2.99	2.999	3	3.001	3.01	3.1	3.25
f(x)	5.313	5.710	5.970	5.997	8	6.003	6.030	6.310	6.813
g(x)	1.99499	1.99950	1.99995	1.99999	und	2.00005	2.00050	2.00499	2.01
h(x)	199	540	700	854	2	6.003	6.030	6.310	6.813

a. $\lim_{x \to 3} f(x)$

b. $\lim_{x \to 3} g(x)$

c. $\lim_{x \to 3} h(x)$

Creative Factoring and Other Interesting Algebra

Difference of Squares

Example: $x - 16 = \left(\sqrt{x} + 4\right)\left(\sqrt{x} - 4\right)$

1. $x - 9$
2. $x^2 - 5$
3. $x^{16} - 1$
4. $(x + 5)^2 - 25$
5. $9y - a^4$

Sums or Differences of Cubes **"SOAP"**

Example: $a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$

Example: $a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$

6. $64a^3 + 125b^3$

7. $64a^3x^3 - 125$

8. $(x + 1)^3 + 64$

9. $8c^3 - (a + b)^3$

Factor: $x^6 - y^6$:

10. as a difference of **squares**

11. as a difference of **cubes**

Rationalize the Numerator

12. $\frac{\sqrt{x+2}-\sqrt{2}}{x}$

13. $\frac{\sqrt{x+3}+\sqrt{3}}{x}$

Factor completely. **Use synthetic division to help find all factors.**

14. $x^3+6x^2+5x-12$

15. $x^3+x^2-8x-12$

16. $x^3+6x^2-9x-14$

Simplify:

17. $\frac{2x^3+7x^2+8x+3}{x+1}$

18. $\frac{2x^3+x^2-13x+6}{x+3}$

Algebraic Limits Worksheet

Given $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} g(x) = 0$, and $\lim_{x\to a} h(x) = 8$, find each limit if it exists.

1. $\lim_{x\to a}[f(x) + h(x)]$

2. $\lim_{x\to a}[f(x)]^2$

3. $\lim_{x\to a} \sqrt[3]{h(x)}$

4. $\lim_{x\to a} \frac{1}{f(x)}$

5. $\lim_{x\to a} \frac{g(x)}{h(x)}$

6. $\lim_{x\to a} \frac{h(x)}{g(x)}$

7. $\lim_{x\to a} \frac{2f(x)}{h(x)-f(x)}$

8. $\lim_{x\to a}[f(x)h(x)]$

9. $\lim_{x\to a} \left[\frac{g(x) + h(x)}{f(x)}\right]$

Evaluate the limits:

10. $\lim_{x\to 0} \frac{x^2 + 7x + 6}{x + 3}$

11. $\lim_{x\to 2} \frac{\frac{2}{x^2} - \frac{1}{2}}{x-2}$

12. $\lim_{x\to 2} \frac{(2x+1)^2 - 25}{x-2}$

13. $\lim_{h\to 0} \frac{(2+h)^3 - 8}{h}$

14. $\lim_{x\to 3} \frac{x^2-9}{x+3}$

15. $\lim_{h\to 0} \frac{(1+h)^2 - 1^2}{h}$

16. $\lim_{h\to 0}\frac{(-5+h)^2-25}{h}$

17. $\lim_{t\to 2}\frac{t^2-4}{t^3-8}$

18. $\lim_{u\to 2}\frac{\sqrt{4u+1}-3}{u-2}$

19. $\lim_{x\to 3}\frac{\frac{1}{x}-\frac{1}{3}}{x-3}$

20. $\lim_{h\to 0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}$

21. $\lim_{x\to 2}\frac{x^4-2x^2-8}{x^2-x-6}$

22. $\lim_{x\to -2}\frac{\frac{x}{x+4}+1}{x+2}$

23. $\lim_{x\to 3}\frac{x^2-9}{2x^2+7x+3}$

24. $\lim_{x\to 1}\frac{x^2-x-2}{x-2}$

25. $\lim_{x \to 1} \frac{4x^4 - 5x^2 + 1}{x^2 + 2x - 3}$

26. $\lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$

27. $\lim_{h \to 0} \frac{(3+h)^2 - 9}{h}$

28. $\lim_{h \to 0} \frac{\frac{1}{(h+2)^2} - \frac{1}{4}}{h}$

29. $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^3 - 1}$

30. $\lim_{x \to 16} \frac{4 - \sqrt{x}}{16x - x^2}$

31. $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$

32. $\lim_{x \to 3} \frac{3(x+1)^{-1} - 3(4)^{-1}}{x - 3}$

33. $\lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$

34. $\lim_{x\to 1} \frac{\frac{2x}{x+1}-1}{x-1}$

35. $\lim_{x\to 2} \frac{x^3+x^2-4x-4}{x^2+x-6}$

36. $\lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$

37. $\lim_{t\to -3} \frac{t^2-9}{2t^2+7t+3}$

38. $\lim_{x\to 0} \frac{\frac{3}{x+5}-\frac{3}{5}}{x}$

39. $\lim_{h\to 0} \frac{(3+h)^3-27}{h}$

40. $\lim_{p\to -2} \frac{(p+4)^{-1}-2^{-1}}{p+2}$

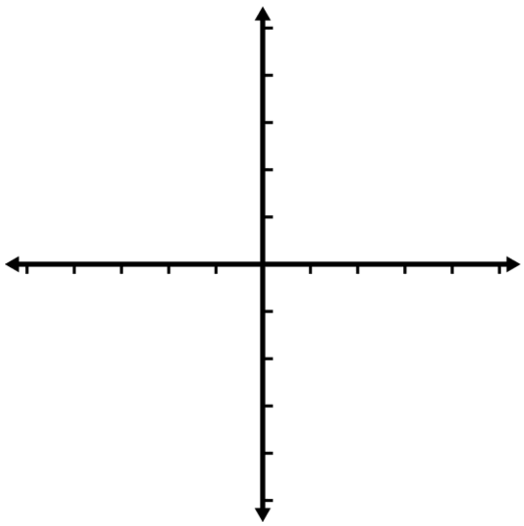
41. $\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$

42. $\lim_{x\to 3} \frac{\sqrt{x+6}-x}{x^3-3x^2}$

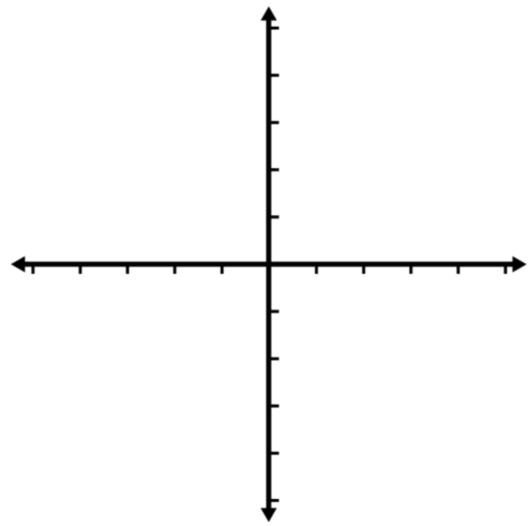
Continuity and Intermediate Value Theorem

Sketch the graph of a function f that satisfies the stated conditions.

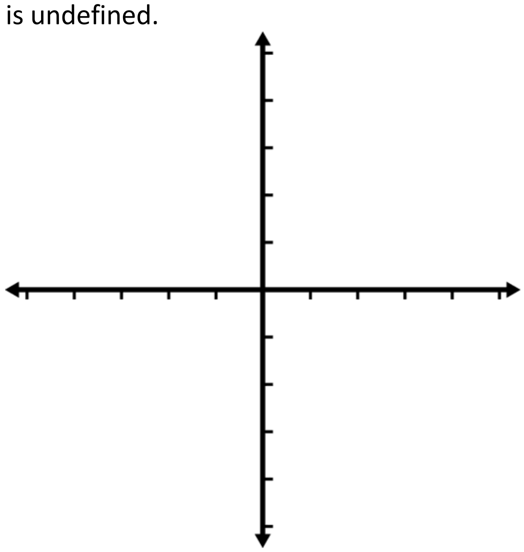
1. f has a limit at $x = 3$, but is not continuous at $x = 3$



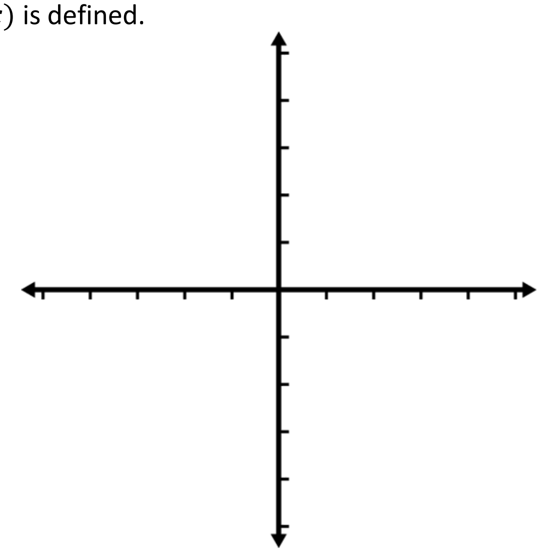
2. f is not continuous at $x = 3$, but if its value at $x = 3$ is changed from $f(3) = 1$ to $f(3) = 0$, f becomes continuous at $x = 3$.



3. f has a removable discontinuity at $x = c$ for which $f(c)$ is undefined.

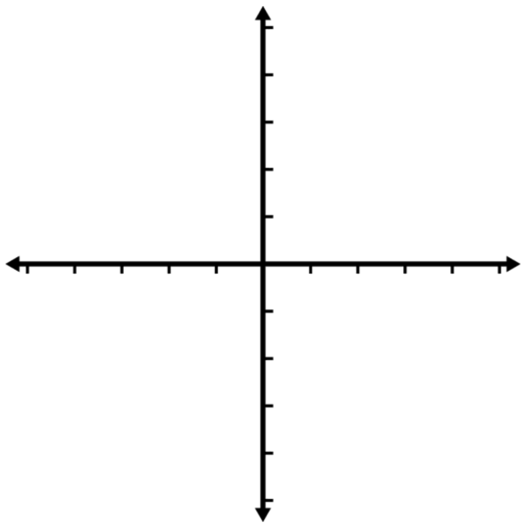


4. f has a removable discontinuity at $x = c$ for which $f(c)$ is defined.

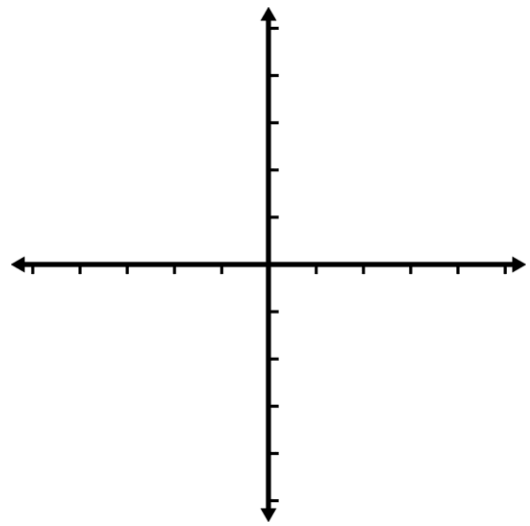


Use the definition of continuity to prove that the function is discontinuous at the given value of a. Sketch the graph of the function.

5. $f(x) = \frac{x^2-5x+4}{x-1}, a = 1$



6. $g(x) = \begin{cases} \frac{x^2-3x}{x^2-9} & if\ x \neq 3 \\ 1 & if\ x = 3 \end{cases},\ a = 3$



Use the definition of continuity to find the values of k and/or m that will make the function continuous everywhere.

7. $f(x) = \begin{cases} kx^2 & x \leq 2 \\ 2x + k & x > 2 \end{cases}$

8. $g(x) = \begin{cases} x^2 + 5,\ x > 2 \\ m(x+3) + k, -1 < x \leq 2 \\ 2x^3 + x + 7,\ x \leq -1 \end{cases}$

A function f and a closed interval $[a, b]$ are given. Show whether the conditions of the Intermediate Value Theorem hold for the given value of k. If the conditions hold, find a number c such that $f(c) = k$. If the theorem does not hold, give the reason.

9. $f(x) = 2 + x - x^2$
$[a, b] = [0,3]$
$k = 1$

10. $f(x) = \sqrt{25 - x^2}$
$[a, b] = [-4.5,3]$
$k = 3$

For Questions 11 and 12: Given the function $f(x) = x^2 + 2x - 5$.

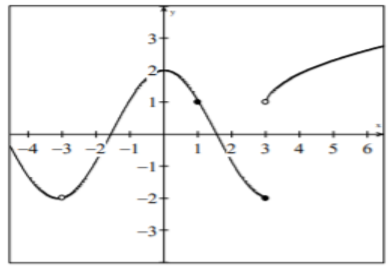
11. Does $f(x) = 7$ somewhere on the interval $[-1,3]$? Use the Intermediate Value Theorem to show why or why not.

12. Must $f(x) = 12$ somewhere on the interval $[-1,3]$? Use the Intermediate Value Theorem to show why or why not.

13. The amount of money raised during a fund-raising campaign is modeled by the function M defined by $M(t) = \frac{240(2^t - 1)}{2^t + 36}$, where $M(t)$ is measured in United States dollars and t is the time in days since that campaign began. According to this model, is there a time t, for $0 \leq t \leq 2$ at which the amount of money raised is 10 dollars? Justify your answer.

One-Sided Limits Graphically and Algebraically

1. Given the graph of $f(x)$, determine the following.



a. $\lim_{x\to -3^-} f(x)$ b. $\lim_{x\to -3^+} f(x)$ c. $\lim_{x\to -3} f(x)$

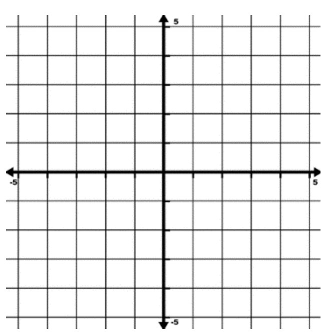
d. $\lim_{x\to 1^-} f(x)$ e. $\lim_{x\to 1^+} f(x)$ f. $\lim_{x\to 1} f(x)$

g. $\lim_{x\to 3^-} f(x)$ h. $\lim_{x\to 3^+} f(x)$ i. $\lim_{x\to 3} f(x)$

j. $f(-3)$ k. $f(1)$ l. $f(3)$

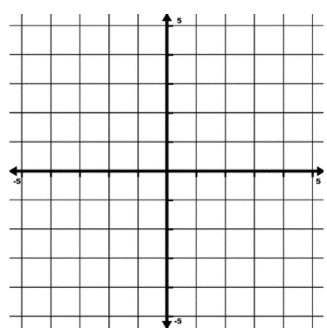
2. Sketch each piecewise function below and determine, if it exists, the given limit. If the limit does not exist, provide an explanation.

a. $f(x) = \begin{cases} 2, & x < 1 \\ 3, & x = 1 \\ x+1, & x > 1 \end{cases}$



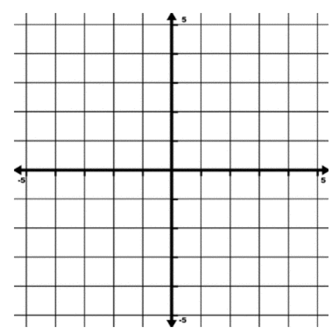
$\lim_{x\to 1} f(x)$

b. $f(x) = \begin{cases} 4 - x^2, & -2 < x \le 2 \\ x - 2, & x > 2 \end{cases}$



$\lim_{x\to 2} f(x)$

c. $f(x) = \begin{cases} |x+2| + 1, & x < -1 \\ -x + 1, & -1 \le x \le 1 \\ x^2 - 2x + 2, & x > 1 \end{cases}$



$\lim_{x\to 1} f(x)$

3. For each function below, determine, if it exists, the given limit. If the limit does not exist, provide an explanation.

a. $f(x) = \begin{cases} 2x - 1, & x \le -2 \\ -x + 2, & x > -2 \end{cases}$

Find $\lim_{x\to -2^+} f(x)$

b. $f(x) = \begin{cases} -x^2 + 4x - 3, & x < 1 \\ x - 7, & x \ge 1 \end{cases}$

Find $\lim_{x\to 1^-} f(x)$

c.

$$f(x) = \begin{cases} x+3, & x \in (-\infty, 0] \\ -x+2, & x \in (0,2) \\ (x-2)^2, & x \in [2, \infty) \end{cases}$$

Find $\lim_{x \to 0} f(x)$ and $\lim_{x \to 2} f(x)$

d.

$$f(x) = \begin{cases} x^2 - 2x + 1, & x < -1 \\ -\frac{x}{2} + \frac{7}{2}, & x \geq -1 \end{cases}$$

Find $\lim_{x \to -1} f(x)$

e.

$$f(x) = \begin{cases} (x+1)^2 - 1, & -2 \leq x < 0 \\ \frac{5}{4}\sin\left(\frac{\pi x}{2}\right), & 0 \leq x < 2 \\ (x-3)^2 - 1, & 2 \leq x \leq 4 \end{cases}$$

Find $\lim_{x \to 2} f(x)$

Evaluate each limit.

4. $\lim_{x \to 2^+} \frac{x}{x-2}$

5. $\lim_{x \to -3^+} \frac{x+1}{x^2-6x+9}$

6. $\lim_{x \to -3^-} \frac{x+2}{x^2+6x+9}$

7. $\lim_{x - 2^+} \frac{x-2}{x^2+4x+4}$

8. $\lim_{x \to -3^-} \frac{x^2}{3x+9}$

9. $\lim_{x \to 2^+} \frac{x^2}{2x-4}$

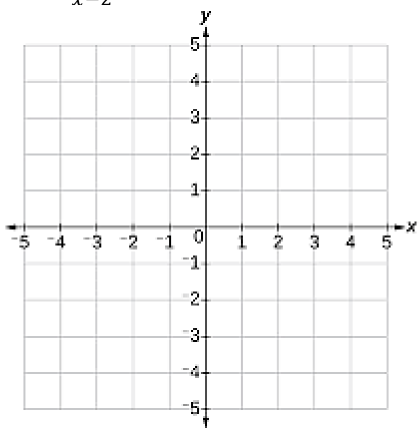
10. $\lim_{x \to -2^+} \frac{1}{x^2-4}$

11. $\lim_{x \to 1^-} -\frac{2}{x^2-1}$

Vertical and Horizontal Asymptotes Worksheet

State the vertical, horizontal, or slant asymptotes for the following (justify using limits). Sketch the graph and find the end behavior.

1. $f(x) = \frac{3}{x-2}$



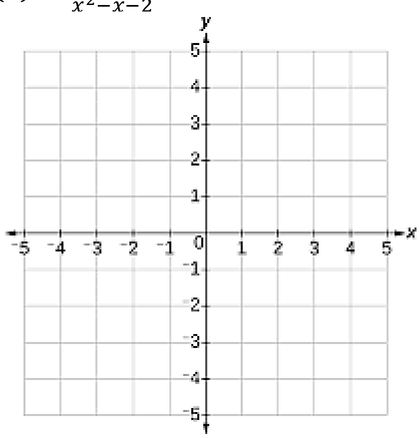
Vertical Asymptote: ____________________________

Horizontal Asymptote: ____________________________

Slant Asymptote: ____________________________

End Behavior: $\lim_{x\to\infty} f(x) =$ ____________
$\lim_{x\to-\infty} f(x) =$ ____________

2. $f(x) = \frac{3x}{x^2-x-2}$



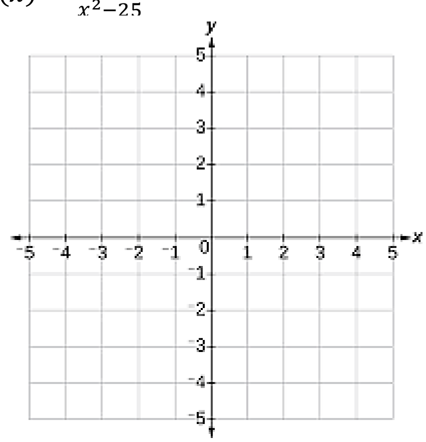
Vertical Asymptote: ____________________________

Horizontal Asymptote: ____________________________

Slant Asymptote: ____________________________

End Behavior: $\lim_{x\to\infty} f(x) =$ ____________
$\lim_{x\to-\infty} f(x) =$ ____________

3. $f(x) = \frac{x^2-5x}{x^2-25}$



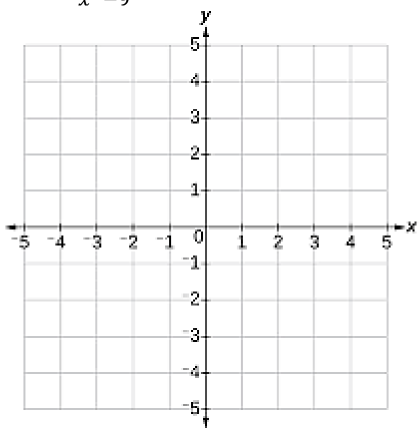
Vertical Asymptote: ____________________________

Horizontal Asymptote: ____________________________

Slant Asymptote: ____________________________

End Behavior: $\lim_{x\to\infty} f(x) =$ ____________
$\lim_{x\to-\infty} f(x) =$ ____________

4. $f(x) = \frac{3x^2-4}{x^2-9}$



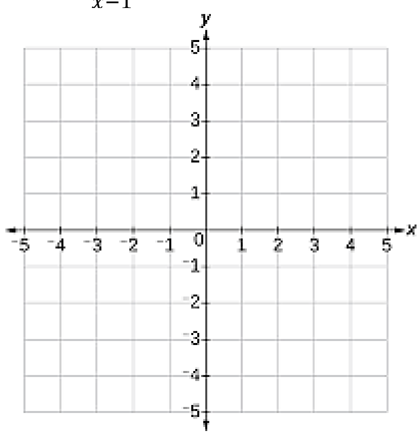
Vertical Asymptote: ____________________________

Horizontal Asymptote: ____________________________

Slant Asymptote: ____________________________

End Behavior: $\lim_{x \to \infty} f(x) =$ ____________
$\lim_{x \to -\infty} f(x) =$ ____________

5. $f(x) = \frac{x^2-x-2}{x-1}$



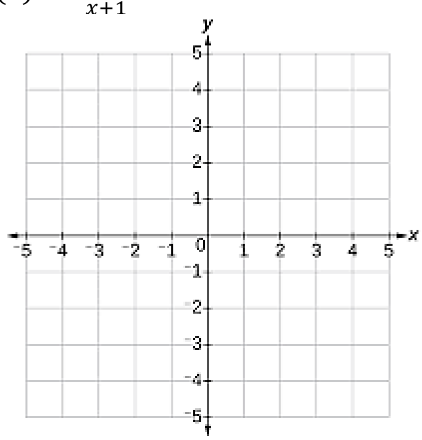
Vertical Asymptote: ____________________________

Horizontal Asymptote: ____________________________

Slant Asymptote: ____________________________

End Behavior: $\lim_{x \to \infty} f(x) =$ ____________
$\lim_{x \to -\infty} f(x) =$ ____________

6. $f(x) = \frac{x^2+3x}{x+1}$



Vertical Asymptote: ____________________________

Horizontal Asymptote: ____________________________

Slant Asymptote: ____________________________

End Behavior: $\lim_{x \to \infty} f(x) =$ ____________
$\lim_{x \to -\infty} f(x) =$ ____________

Infinite Limits Worksheet

Find the Limit.

1. $\lim\limits_{x\to\infty} 3$	2. $\lim\limits_{x\to-\infty} 3$	3. $\lim\limits_{x\to-\infty} (-3)$
4. $\lim\limits_{x\to\infty} (-2x)$	5. $\lim\limits_{x\to\infty} (3-x)$	6. $\lim\limits_{x\to\infty} \sqrt{x}$
7. $\lim\limits_{x\to-\infty} (4-x)$	8. $\lim\limits_{x\to\infty} \frac{8}{5-3x}$	9. $\lim\limits_{x\to\infty} \frac{1}{x-12}$
10. $\lim\limits_{x\to-\infty} \frac{3}{x+4}$	11. $\lim\limits_{x\to\infty} (1+2x-3x^5)$	12. $\lim\limits_{x\to\infty} (2x^3-110x+5)$
13. $\lim\limits_{x\to\infty} \frac{(3+2x^2)}{4+5x}$	14. $\lim\limits_{x\to\infty} \frac{x^2+x}{3-x}$	15. $\lim\limits_{x\to\infty} \frac{x+4}{x^2-2x+5}$
16. $\lim\limits_{x\to-\infty} -\frac{x-2}{x^2+2x+1}$	17. $\lim\limits_{x\to\infty} \frac{7-6x^5}{x+3}$	18. $\lim\limits_{x\to\infty} \frac{6-x^3}{7x^3+3}$
19. $\lim\limits_{x\to\infty} \frac{1}{x^2+1}$	20. $\lim\limits_{x\to\infty} \frac{x^4+x^2}{x^4+1}$	21. $\lim\limits_{x\to\infty} \frac{1+x^2}{2-x^2}$
22. $\lim\limits_{x\to\infty} \frac{2x^2}{x^2+1}$	23. $\lim\limits_{x\to-\infty} \frac{x+4}{3x^2-5}$	24. $\lim\limits_{x\to\infty} \frac{3x^3+25x^2-x+1}{4x^3-7x^2+2x+2}$

Released Multiple Choice Questions - Limits

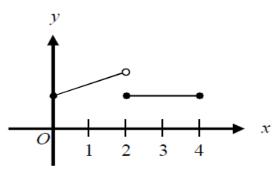
1. $\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

a. -3 b. -2 c. 2 d. 3 e. DNE

2. $\lim_{x\to 0} \frac{5x^4+8x^2}{3x^4-16x^2}$ is

a. $-\frac{1}{2}$ b. 0 c. $\frac{5}{3}$ d. $\frac{7}{6}$ e. None of These

3. The figure below shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?



Graph of f

I. $\lim_{x\to 2^-} f(x)$ *exists.* II. $\lim_{x\to 2^+} f(x)$ *exists.* III. $\lim_{x\to 2} f(x)$ *exists.*

a. I only b. II only c. I and II only d. I and III only e. I, II, and III

4. For $x \ge 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

a. $f(0) = 2$ b. $f(x) \ne 2$ for all $x \ge 0$ c. $f(2)$is undefined d. $\lim_{x\to 2} f(x) = \infty$ e. $\lim_{x\to\infty} f(x) = 2$

5. $\lim_{x\to\infty} \frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1} =$

a. 4 b. 1 c. $\frac{1}{4}$ d. 0 e. -1

6. If $f(x) = \begin{cases} \ln x & for\ 0 < x \leq 2 \\ x^2 \ln 2 & for\ 2 < x \leq 4 \end{cases}$, then $\lim_{x \to 2} f(x)$ is

a. $\ln 2$ b. $\ln 8$ c. $\ln 16$ d. 4 e. DNE

7. The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table. The equation $f(x) = \frac{1}{2}$ must have a least two solutions in the interval $[0,2]$ if $k =$

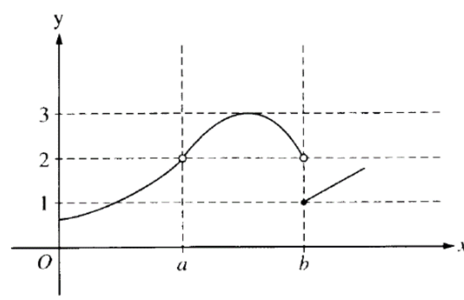
x	0	1	2
$f(x)$	1	k	2

a. 0 b. $\frac{1}{2}$ c. 1 d. 2 e. 3

8. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

a. $\frac{1}{a^2}$ b. $\frac{1}{2a^2}$ c. $\frac{1}{6a^2}$ d. 0 e. DNE

9. The graph of the function f is shown in the figure to the right. Which of the following statements about f is true?



a. $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

b. $\lim_{x \to a} f(x) = 2$

c. $\lim_{x \to b} f(x) = 2$

d. $\lim_{x \to b} f(x) = 1$

e. $\lim_{x \to a} f(x)$ DNE

10. $\lim_{n \to \infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

a. -5 b. -2 c. 1 d. 3 e. DNE

11. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

a. -4 b. -2 c. -1 d. 0 e. 2

12. $\lim_{n\to\infty} \frac{4n^2}{n^2+10,000n}$ is

a. 0 b. $\frac{1}{2500}$ c. 1 d. 4 e. DNE

13. If $\lim_{x\to a} f(x) = L$, where L is a real number which of the following must be true?

a. $f'(a)$ exists

b. $f(x)$ is continuous at $x = a$

c. $f(x)$ is defined at $x = a$

d. $f(a) = L$

e. None of these

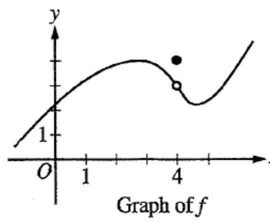
14. If $\begin{cases} f(x) = \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, for\ x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

a. 0 b. $\frac{1}{6}$ c. $\frac{1}{3}$ d. 1 e. $\frac{7}{5}$

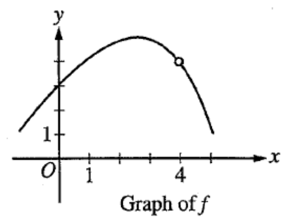
Limits Practice Test

1. For which of the following does $\lim_{x \to 4} f(x)$ exist?

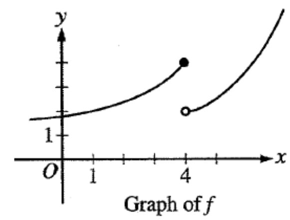
I.



II.



iii.



a. I only b. II only c. III only d. I and II only e. I, II, III

2. Use the table of values to evaluate the limit.

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
f(x)	7.018	7.008	7.002	20	7.002	7.008	7.018
g(x)	4.126	4.789	4.989	8	8.0015	8.1016	8.546
h(x)	4971	8987	9972	undefined	8.997	8.987	8.971

$\lim_{x \to 0^+} f(x)$ $\lim_{x \to 0^-} f(x)$ $\lim_{x \to 0} f(x)$

$\lim_{x \to 0^+} g(x)$ $\lim_{x \to 0^-} g(x)$ $\lim_{x \to 0} g(x)$

$\lim_{x \to 0^+} h(x)$ $\lim_{x \to 0^-} h(x)$ $\lim_{x \to 0} h(x)$

3. Find $\lim_{x \to \infty} \dfrac{-4x + 2x^3}{8x^3 + 4x^2 - 3}$.

a. $\frac{1}{4}$ b. $-\frac{1}{4}$ c. $-\frac{1}{2}$ d. 0 e. ∞

4. Find $\lim_{x \to \infty} \dfrac{e^x + 5}{3 - 2e^x}$.

a. $\frac{5}{3}$ b. $\frac{1}{3}$ c. $\frac{1}{2}$ d. $-\frac{1}{2}$ e. ∞

5. The graph of which function has y = 2 as an asymptote?

a. $y = e^{-x} + 2$ b. $y = \ln(x - 2)$ c. $y = -\frac{2x^2}{4+x^2}$ d. $y = -\frac{2}{1-x}$ e. $y = \frac{4x}{2+x}$

6. State all the vertical and horizontal asymptotes and **justify your answer**.

$$f(x) = \frac{2x^2 - 50}{x^2 + 7x + 10}$$

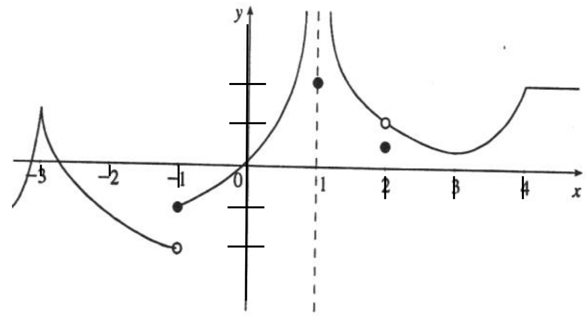
7. Find the indicated limits from the graph below.

a. $\lim_{x \to -1^-} f(x) =$ b. $\lim_{x \to -1^+} f(x) =$

c. $\lim_{x \to -1} f(x) =$ d. $\lim_{x \to 0} f(x) =$

e. $\lim_{x \to 1} f(x) =$ f. $\lim_{x \to 2} f(x) =$

g. $\lim_{x \to 4} f(x) =$ h. $\lim_{x \to -3^-} f(x) =$



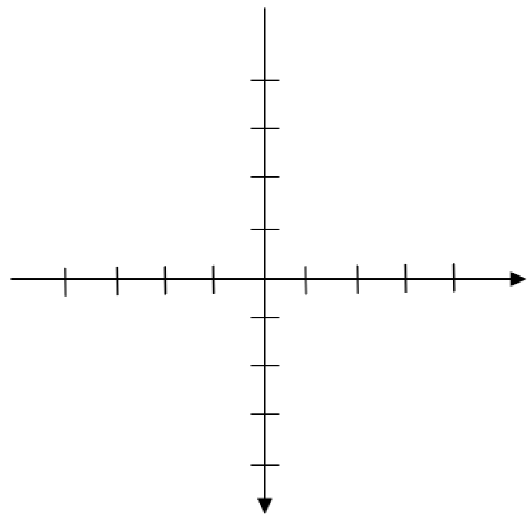
8. Draw a graph of g(x) that has the following conditions.

$\lim_{x \to \infty} g(x) = -\infty$ $\lim_{x \to -1} g(x) = -\infty$

$\lim_{x \to 1^+} g(x) = 1$ $\lim_{x \to -2} g(x) = 3$

$\lim_{x \to 1^-} g(x) = 0$ $\lim_{x \to -\infty} g(x) = 1$

$g(1) = 0$



9. Draw an example of a function where a limit exists at a point but the function is still discontinuous at that point. Explain.

10. Draw the graph of an example of each of the following discontinuities. (Bonus for giving the equation of such a graph. The equation does not have to match the graph you draw.)

a. An infinite discontinuity b. A jump discontinuity

Find the limit if it exists.

11. $\lim_{x \to 5^-} \dfrac{4x-20}{|x-5|}$

12. $\lim_{t \to 2^+} \dfrac{1-\sqrt{3-t}}{t-2}$

13. $\lim_{x \to -\infty} e^{-x}$

14. $\lim_{x \to -\infty} \dfrac{5-2x^2}{x+2}$

15. $\lim_{x \to -2^-} \dfrac{-3}{2+x}$

16. $\lim_{x \to 0} (\sec x)$

17. $\lim_{x \to 0^+} \ln x$

18. $\lim_{x \to 0} \dfrac{\frac{2}{x+3} - \frac{2}{3}}{x}$

19. $\lim_{x \to -\infty} \dfrac{\sqrt{4x^2-4x+2}}{5x-3}$

20. $\lim_{x \to \infty} \dfrac{8x^3-2x}{3x^2-5x^3}$

21. Is the function continuous? **Justify your answer**.

$$f(x)=\begin{cases} x^2-1 & x>2 \\ 3 & x=2 \\ 4x-3 & x<2 \end{cases}$$

22. Find the value of a and b that make the function continuous

$$f(x)=\begin{cases} ax^2-b & \text{if } x\le -1 \\ 2bx+5 & \text{if } -1<x<2 \\ bx^2+ax+1 & \text{if } x\ge 2 \end{cases}$$

23. Given $f(x)=\dfrac{2x^2+5x-3}{x^2-x-12}$, complete the chart below.

f(x) is discontinuous at x =	Type of discontinuity

Can f(x) be made continuous at any of the x values above? If so, at which x value and what point would you use to "repair" the discontinuity?

24. Verify the conditions of the Intermediate Value Theorem and find the guaranteed value of c in $(-3, 3)$ when $f(x)=x^2-3x-4$, and $f(c) = 6$.