

Unit 1 Prerequisite Topics

Homework will be assigned daily. Be prepared for potential HW pop quizzes every day by doing homework nightly.

Topics Covered:

- ❖ Factoring Polynomials
- ❖ Rates of Change & Linear Equations
- ❖ Functions & Their Graphs (Domain, Piecewise Functions, Compositions, & Transformations)
- ❖ Exponential & Logarithmic Functions
- ❖ The Unit Circle & Evaluating Trig Functions

Quiz is _____

Test is _____

Name: Bonanni

Factoring

Factoring Trinomials

1. $x^2 + 6x + 8$

$(x+4)(x+2)$

2. $x^2 - x - 6$

$(x-3)(x+2)$

3. $c^2 + 5c + 6$

$(c+3)(c+2)$

4. $y^2 + 3y - 18$

$(y+6)(y-3)$

5. $3y^2 + 14y - 49$

$(3y-7)(y+7)$

6. $3b^2 + 21b - 54$

$3(b^2+7b-18)$

$3(b+9)(b-2)$

7. $6x^2 - 42x + 72$

$6(x^2-7x+12)$

$6(x-3)(x-4)$

8. $7a^2 - 38a - 24$

$(7a+4)(a-6)$

9. $15a^2 + 4a - 4$

$(5a-2)(3a+2)$

10. $8c^2 = 30 + 43c$

$8c^2 - 43c - 30 = 0$
 $(8c+5)(c-6) = 0$

11. $x^6 - 14x^3 + 24$

$(x^3-12)(x^3-2)$

12. $15x^3 + 20x^2 - 25x$

$5x(3x^2+4x-5)$

Factoring by GCF

13. $5c^3 - 2c^2$

$c^2(5c-2)$

14. $81r + 48rs$

$3(27r+16s)$

15. $15cd + 30c^2d^2$

$15cd(1+2cd)$

16. $a^2b^2 + a$

$a(ab^2+1)$

Factoring Special Binomials

17. $5a^2 - 20$

$5(a^2-4)$
 $5(a+2)(a-2)$

18. $y^3 - 1$

$(y-1)(y^2+y+1)$

19. $8x^3 + 64$

$8(x^3+8)$
 $8(x+2)(x^2-2x+4)$

20. $1 - 49c^2$

$(1+7c)(1-7c)$

21. $-27 + p^3 = p^3 - 27$

$(p-3)(p^2+3p+9)$

22. $200r^2 - 18$

$2(100r^2-9)$
 $2(10r+3)(10r-3)$

23. $36 - n^4$

$(6-n^2)(6+n^2)$
or
 $-1(n^2-6)(n^2+6)$

24. $16 - 81f^8$

$(4+9f^4)(4-9f^4)$
 $(4+9f^4)(2+3f^2)(2-3f^2)$

Factoring by Grouping

25. $6mn - 9m - 4n + 6$

$3m(2n-3) - 2(2n-3)$
 $(3m-2)(2n-3)$

26. $2x^2y + 6xy - x - 3$

$2xy(x+3) - 1(x+3)$
 $(2xy-1)(x+3)$

27. $6x^3 - 3x^2 + 8x - 4$

$3x^2(2x-1) + 4(2x-1)$
 $(3x^2+4)(2x-1)$

28. $4r^3 - 8r^2 - 3r + 6$

$4r^2(r-2) - 3(r-2)$
 $(4r^2-3)(r-2)$

Average Rate of Change

Use the table of values to find the average rate of change over the given interval.

x	1	2	3	3.5	3.7	6
y	40	25	18	15	18	38

1. [1,3]

$$f(1) = 40 \quad f(3) = 18$$

$$\frac{18-40}{3-1} = \frac{-22}{2} = -11$$

2. [2,6]

$$f(2) = 25 \quad f(6) = 38$$

$$\frac{38-25}{6-2} = \frac{13}{4} \text{ or } 3.25$$

3. [2,3.7]

$$f(2) = 25 \quad f(3.7) = 18$$

$$\frac{18-25}{3.7-2} = \frac{-7}{1.7} = \frac{-70}{17}$$

4. [3.5,6]

$$f(3.5) = 15 \quad f(6) = 38$$

$$\frac{38-15}{6-3.5} = \frac{23}{2.5} = \frac{46}{5} \text{ or } 9.2$$

Find the average rate of change for each function on the given interval.

5. $f(x) = x^2 - 4x - 12$ on $[0,6]$

$$f(0) = -12 \quad f(6) = 0$$

$$\frac{0 - (-12)}{6 - 0} = \frac{12}{6} = 2$$

6. $f(x) = x^2 - 4x - 12$ on $[-1,7]$

$$f(-1) = -7 \quad f(7) = 9$$

$$\frac{9 - (-7)}{7 - (-1)} = \frac{16}{8} = 2$$

7. $f(x) = 3x^2 - x - 2$ on $[-1,4]$

$$f(-1) = 2 \quad f(4) = 42$$

$$\frac{42-2}{4-(-1)} = \frac{40}{5} = 8$$

8. $f(x) = 0.02x^2 - 1.6x + 20.5$ on $[25,33]$

$$f(25) = 7 \quad f(33) = -10.52$$

$$\frac{-10.52-7}{33-25} = \frac{-17.52}{8} = \frac{-219}{100} \text{ or } -2.19$$

9. $f(x) = 3x^5 - 4x^4 + 3x^2 - 4x + 1$ on $[-6,1]$

$$f(-6) = -28,379 \quad f(1) = -1$$

$$\frac{-1 - (-28,379)}{1 - (-6)} = \frac{28,378}{7}$$

$$= 4054$$

10. $f(x) = \frac{x^2-6}{x+3}$ on $[-5,-2]$

$$f(-5) = -\frac{19}{2} \quad f(-2) = -2$$

$$\frac{-2 - (-\frac{19}{2})}{-2 - (-5)} = \frac{7.5}{3} = 2.5 \text{ or } \frac{5}{2}$$

Equations of Lines

1. Write the slope-intercept form of the equation:

$$11x - 8y = -48$$

$$-8y = -11x - 48$$

$$y = \frac{11}{8}x + 6$$

2. Write the point-slope form of the equation of the line through the given point with the given slope:

$$(3,5), m = \frac{5}{3}$$

$$y - 5 = \frac{5}{3}(x - 3)$$

3. Write the point-slope form of the line through the point (4,2) & parallel to $y = -5x + 2$

$$m = -5$$

$$y - 2 = -5(x - 4)$$

4. Write the point-slope form of the line through the point (-2,4) & perpendicular to

$$y = -\frac{5}{2}x + 5$$

$$m = \frac{2}{5}$$

$$y - 4 = \frac{2}{5}(x + 2)$$

5. Write the slope-intercept form of the equation:

$$-4x - 7y + 3 = -48$$

$$-4x - 7y = -51$$

$$-7y = 4x - 51$$

$$y = -\frac{4}{7}x + \frac{51}{7}$$

6. Write the slope-intercept form of the equation of the line through the given point with the given slope:

$$(1, -3), m = -\frac{2}{3}$$

$$y + 3 = -\frac{2}{3}(x - 1)$$

$$y + 3 = -\frac{2}{3}x + \frac{2}{3}$$

$$y = -\frac{2}{3}x - \frac{7}{3}$$

7. Write the point-slope form of the equation of the line through (5, -3), parallel to $4y = -3x + 6$.

$$y = -\frac{3}{4}x + \frac{3}{2} \quad m = -\frac{3}{4}$$

$$y + 3 = -\frac{3}{4}(x - 5)$$

8. Write the slope-intercept form of the equation of the line through the point (5,7) & slope of -2.

$$y - 7 = -2(x - 5)$$

$$y - 7 = -2x + 10$$

$$y = -2x + 17$$

Piecewise Functions

Evaluate for the following piecewise functions:

1. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

a. $f(-2)$

6

b. $f(0)$

2

c. $f(1)$

3

d. $f(s^2 + 2)$

$2(s^2 + 2)^2 + 2$

2. $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases}$

a. $f(-2)$

1

b. $f(0)$

0

c. $f(1)$

1

d. $f(s^2)$

$\sqrt{s^2} = s$

3. $f(x) = \begin{cases} x + 2, & -2 \leq x \leq -1 \\ 1 - \sqrt{1 - x^2}, & -1 < x < 1 \\ -x + 2, & 1 \leq x \leq 2 \end{cases}$

a. $f(-1.5)$

0.5

b. $f(0)$

0

c. $f(3)$

DNE

d. $f(1.5)$

0.5

4. $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ \frac{3}{2}x + \frac{3}{2}, & 1 \leq x \leq 3 \\ -x + 2, & x > 3 \end{cases}$

a. $f(5)$

-3

b. $f(0)$

4

c. $f(3)$

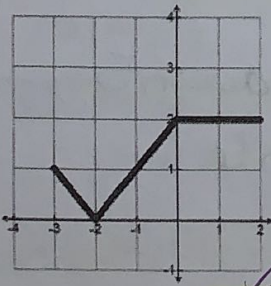
6

d. $f(2)$

9/2

Find the formula for the following:

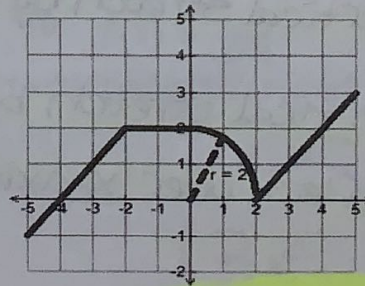
5.



of 2 linear equations

$f(x) = \begin{cases} |x+2|, & x \leq 0 \\ 2, & x > 0 \end{cases}$

6.

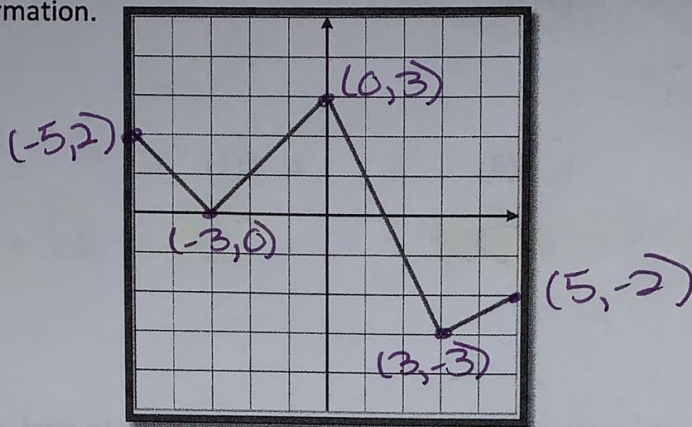


circle eq.
 $x^2 + y^2 = 4$
 $y^2 = 4 - x^2$
 $y = \pm\sqrt{4 - x^2}$

$f(x) = \begin{cases} x + 4, & x < -2 \\ 2, & -2 \leq x \leq 2 \\ \sqrt{4 - x^2}, & 0 < x < 2 \\ x - 2, & x \geq 2 \end{cases}$

Transformations

Draw a graph of each transformation. Let $f(x)$ be the function below. Label the critical points. Finally write a description of the transformation.

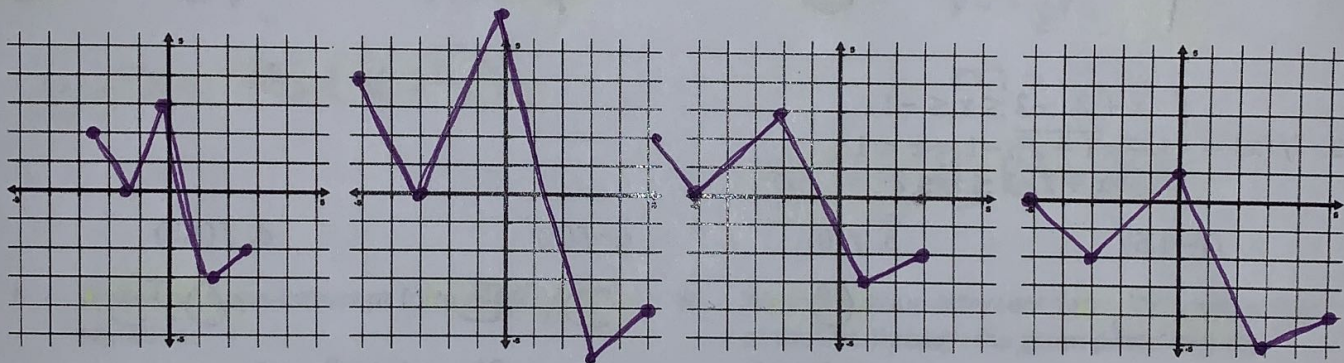


1. $f(2x)$

2. $2f(x)$

3. $f(x+2)$

4. $f(x) - 2$



Describe the transformations from the parent graph for the following functions:

5. $g(x) = \frac{1}{2}x^3 - 5$ vertical shrink by $\frac{1}{2}$ + down 5

6. $g(x) = \sqrt{x-7} + 4.8$ right 7 + up 4.8

7. $g(h) = -5(h+1)$ reflect over x-axis, vert. stretch by 5, + left 1

8. $j(k) = \frac{5}{3}|k| - 10$ vertical stretch by $\frac{5}{3}$ + down 10

9. $g(x) = 3e^{x-3} - 5$ vertical stretch by 3, right 3, + down 5

10. $m(x) = \frac{-1}{x+7} + 6$ reflect over x-axis, left 7, + up 6

Domain

State the domain of the following. Show your work.

1. $f(x) = \frac{3x-5}{x+1}$

$x+1 \neq 0$

$x \neq -1$

$(-\infty, -1) \cup (-1, \infty)$

2. $f(x) = \sqrt{\frac{3x-5}{x+1}}$

$x \neq -1$

$\frac{3x-5}{x+1} \geq 0$

$3x-5 \geq 0$

$x \geq 5/3$

$[5/3, \infty)$

3. $f(x) = \ln(x-7)$

$x-7 > 0$

$x > 7$

$(7, \infty)$

4. $f(x) = \ln\left(\frac{x}{x-1}\right)$

$x \neq 1$

$\frac{x}{x-1} > 0$

$x > 0$

$(0, 1) \cup (1, \infty)$

5. $f(x) = \sqrt[3]{3x+2}$
 \mathbb{R}

$(-\infty, \infty)$

6. $f(x) = \frac{x+1}{x^2+5x+4}$

$(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$

$(-4, \infty)$

$x^2+5x+4 \neq 0$

$(x+4)(x+1) \neq 0$

$x+4 \neq 0$ $x+1 \neq 0$

$x \neq -4$ $x \neq -1$

7. $f(x) = \ln\left(\frac{x+1}{x-3}\right)$

$x \neq 3$

$\frac{x+1}{x-3} > 0$

$x+1 > 0$

$x > -1$

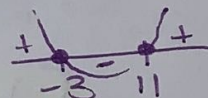
$(-1, 3) \cup (3, \infty)$

8. $f(x) = \sqrt[4]{x^2-8x-33}$

$x^2-8x-33 \geq 0$

$(x-11)(x+3) \geq 0$

$x=11$ $x=-3$



$(-\infty, -3] \cup [11, \infty)$

9. $f(x) = \frac{x}{x^2-9}$

$x^2-9 \neq 0$

$x^2 \neq \pm 3$

$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

10. $f(x) = \frac{1}{\sqrt{x-2}}$

$x-2 > 0$

$x > 2$

$(2, \infty)$

$$11. f(x) = \sqrt{x+1}$$

$$x+1 \geq 0$$

$$x \geq -1$$

$$[-1, \infty)$$

$$12. f(x) = \frac{1}{x}$$

$$x \neq 0$$

$$(-\infty, 0) \cup (0, \infty)$$

$$13. f(x) = \frac{x+2}{2x-1}$$

$$2x-1 \neq 0$$

$$x \neq \frac{1}{2}$$

$$(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$$

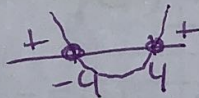
$$14. f(x) = |x-2|$$

$$(-\infty, \infty)$$

$$15. f(x) = \sqrt{x^2-16}$$

$$x^2-16 \geq 0$$

$$x = \pm 4$$



$$(-\infty, -4] \cup [4, \infty)$$

$$16. f(x) = \log_2(4x-8) + 3$$

$$4x-8 > 0$$

$$x > 2$$

$$(2, \infty)$$

$$17. f(x) = \sqrt{2x-1}$$

$$2x-1 \geq 0$$

$$x \geq \frac{1}{2}$$

$$[\frac{1}{2}, \infty)$$

$$18. f(x) = \frac{2}{x^2-1}$$

$$x^2-1 \neq 0$$

$$x \neq \pm 1$$

$$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$$

$$19. f(x) = \ln(x^2-9)$$

$$x^2-9 > 0$$

$$x = \pm 3$$



$$(-\infty, -3) \cup (3, \infty)$$

$$20. f(x) = \frac{3x+2}{(3x+2)(x+1)} = \frac{1}{x+1}$$

$$3x+2 \neq 0 \quad x+1 \neq 0$$

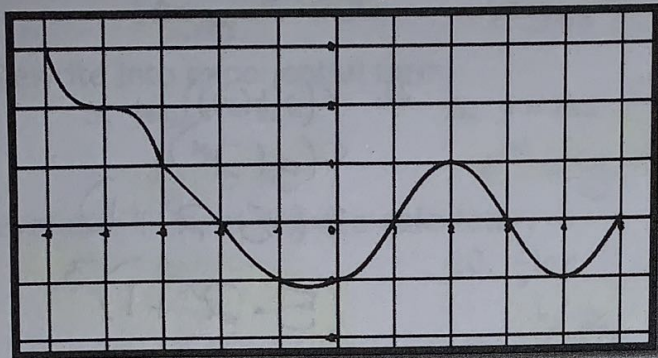
$$x \neq -\frac{2}{3} \quad x \neq -1$$

$$(-\infty, -1) \cup (-1, -\frac{2}{3}) \cup (-\frac{2}{3}, \infty)$$

Composition of Functions

Given the following functions $f(x)$ and $g(x)$, find the following.

Graph of f



Graph of g



- $f(g(-2))$
 $f(3) = 0$
- $f(g(2))$
 $f(-1) = -1$
- $g(f(-1))$
 $g(3) = 3$
- $f(f(5))$
 $f(0) = -1$
- $g(g(-2))$
 $g(3) = -1$
- All inputs for x for which $g(g(x)) = -1$
 $x = -3, -2, -1, 0, 5$

Given the tables for functions f and g , find the following.

x	$f(x)$
-1	2
0	4
1	3
2	0
3	1
4	-1

x	$g(x)$
-1	3
0	4
1	2
2	6
3	2
4	-1

- $f(g(3))$
 $f(2) = 0$
- $g(f(2))$
 $g(4) = 4$
- $f(f(4))$
 $f(-1) = 2$
- $g(g(4))$
 $g(-1) = 3$
- All inputs x such that $f(g(x)) = 2$
 $x = 4$
 $g(4) = -1$
 $f(-1) = 2$

Given the tables for functions f and g , find the following.

x	$f(x)$
-2	0
0	1
2	-1
4	2

x	$g(x)$
-1	-2
0	2
1	3
2	1

- $f(f(-2))$
 $f(0) = 1$
- $f(g(0))$
 $f(2) = -1$
- $g(f(1))$
DNE
- Domain of $f(g(x))$
 $f(g(-1)) = 0$
 $f(g(0)) = 1$
 $f(g(1)) = \text{DNE}$
 $f(g(2)) = \text{DNE}$
 $\{-1, 0\}$

Let $f(x) = 2x - 3$, $g(x) = e^x$, and $h(x) = \ln x$. Find a formula for each function.

- $f(f(x))$
 $2(2x-3)-3$
 $4x-6-3$
 $4x-9$
- $f(g(x))$
 $2e^x-3$
- $g(h(x))$
 $e^{\ln x}$
 x
- $h(g(x))$
 $\ln e^x$
 x

Let $f(x) = x^3$, $g(x) = 5x + 1$, and $h(x) = 2^x$, find the following.

20. $f(g(x))$

$(5x+1)^3$
or $125x^3 + 75x^2 + 15x + 1$

21. $h(f(x))$

2^{x^3}

22. $h(g(x))$

2^{5x+1}

23. $g(h(x))$

$5 \cdot 2^x + 1$

24. $g(f(x))$

$5x^3 + 1$

25. $f(g(h(x)))$

$f(g(2^x))$
 $f(5 \cdot 2^x + 1)$

$(5 \cdot 2^x + 1)^3$

In the composition of functions $f(g(x))$, g is known as the inner function, and f is known as the outer function. For each problem below, specify the inner and outer functions.

26. $y = (7x - 3)^2$

$f(x) = x^2$
 $g(x) = 7x - 3$

27. $y = \ln(x^2 + 4)$

$f(x) = \ln x$
 $g(x) = x^2 + 4$

28. $y = 2^{3x-5}$

$f(x) = 2^x$
 $g(x) = 3x - 5$

29. $y = e^{\ln x}$

$f(x) = e^x$
 $g(x) = \ln x$

30. $y = \sqrt{x}$

$f(x) = \sqrt{x}$
 $g(x) = x$

31. $y = (\ln x)^2 + 1$

$f(x) = x^2 + 1$
 $g(x) = \ln x$

32. $y = (x^2 - 4)^3$

$f(x) = x^3$
 $g(x) = x^2 - 4$

33. $y = \frac{2}{(x-7)^2}$

$f(x) = \frac{2}{x^2}$
 $g(x) = x - 7$

34. $y = \sqrt[5]{x+3}$

$f(x) = \sqrt[5]{x}$
 $g(x) = x + 3$

Answer each of the following.

35. If $f(g(x)) = \ln(x^2 + 1)$ and $f(x) = \ln x$, what is $g(x)$?

$g(x) = x^2$

36. If $f(g(x)) = \sqrt[3]{x-3}$ and $f(x) = \sqrt[3]{x}$, what is $g(x)$?

$g(x) = x - 3$

37. If $f(g(x)) = \sin(3x)$ and $f(x) = \sin x$, what is $g(x)$?

$g(x) = 3x$

Exponentials and Logarithms

Rewrite into logarithmic form.

1. $25^x = 5$

$\log_5 25 = x$

2. $\frac{1}{7} = 49^x$

$\log_{49} \frac{1}{7} = x$

3. $e^x = 54.6$

$\ln 54.6 = x$

4. $y = e^x$

$\ln y = x$

Rewrite into exponential form.

5. $\log_6 x = 2$

$6^2 = x$

6. $y = \ln x$

$e^x = y$

7. $\log_a b = c$

$a^c = b$

8. $\ln x = 1.946$

$e^{1.946} = x$

Evaluate without using a calculator.

9. $\log_6 6^2$

2

10. $\frac{1}{3} \ln e$

$\frac{1}{3}$

11. $4 \log_8 100$

$\frac{4(2)}{8}$

12. $\log_5 \frac{1}{25}$ $5^x = \frac{1}{25}$

-2

Find the value of the following without using your calculator. Show work.

13. $3 \ln e + \ln \left(\frac{1}{e}\right)$

$3(1) + \ln(e^{-1})$

$3 - 1$

2

14. $\ln e^2 + e^{-\ln e}$

$2(1) + e^{-1}$

$2 + \frac{1}{e}$

15. $e^{5 \ln 2}$

$e^{\ln 5^2}$

25

16. $4 \ln e^{7x}$

$\ln e^{28x}$

28x

Simplify to a single ln or e expression or a single number.

17. $2 \ln a - 3 \ln b + \ln(ab)$

$\ln(a^2) - \ln(b^3) + \ln(ab)$

$\ln \frac{a^2(ab)}{b^3} = \ln \left(\frac{a^3}{b^2}\right)$

18. $\ln(e^2 \ln(e \ln e))$

$\ln(e^2 \ln(e))$

$\ln(e^2)$

2 \ln e = 2

Express each of the following as the logarithm of a single expression. (Condense)

19. $2 \ln x + 4 \ln y - \ln 13$

$\ln \left(\frac{x^2 y^4}{13}\right)$

20. $\ln 7 + 5 \ln y - \frac{1}{2} \ln x$

$\ln \left(\frac{7y^5}{x^{1/2}}\right)$ or $\ln \left(\frac{7y^5}{\sqrt{x}}\right)$

Use the properties of logarithms to express the following as an algebraic expression involving $\log x$, $\log y$, and/or $\log z$. (Expand)

21. $\log \frac{x^2 y^3}{z}$

$2 \log x + 3 \log y - \log z$

22. $\log \frac{z}{\sqrt{xy}} = \log \frac{z}{x^{1/2} y^{1/2}}$

$\log z - \frac{1}{2} \log x - \frac{1}{2} \log y$ 10 | Page

Trig Exact Values and Inverse Trig

Evaluate each expression.

1. $\sin\left(\frac{\pi}{4}\right)$ $\frac{\sqrt{2}}{2}$	2. $\cos(210^\circ)$ $-\frac{\sqrt{3}}{2}$	3. $\sin\left(\frac{3\pi}{4}\right)$ $\frac{\sqrt{2}}{2}$
4. $\csc 270^\circ$ -1	5. $\tan\left(\frac{3\pi}{2}\right)$ $\frac{1}{0}$ DNE	6. $\tan\left(\frac{5\pi}{4}\right)$ 1
7. $\csc(2\pi)$ $\sin(2\pi) = 0$ DNE	8. $\sec 150^\circ$ $\cos(150^\circ) = \frac{\sqrt{3}}{2}$ $\frac{2\sqrt{3}}{3}$	9. $\sin 2\pi$ 0
10. $\cot 600^\circ$ $\cot 240^\circ$ $\frac{\sqrt{3}}{3}$	11. $\cot\left(\frac{17\pi}{6}\right)$ $\cot\left(\frac{7\pi}{6}\right)$ $\sqrt{3}$	12. $\csc(-510^\circ)$ $\csc(210^\circ)$ $\sin(210^\circ) = -\frac{1}{2}$ -2
13. $\cos\frac{11\pi}{4}$ $-\frac{\sqrt{2}}{2}$	14. $\cot(-150^\circ)$ $\cot(210^\circ)$ $\sqrt{3}$	15. $\cos(30^\circ)$ $\frac{\sqrt{3}}{2}$
16. $\cos\left(-\frac{\pi}{2}\right) = \cos\frac{3\pi}{2}$ 0	17. $\tan(-225^\circ)$ $\tan(135^\circ)$ -1	18. $\cot(-3\pi)$ $\cot(\pi)$ $-\frac{1}{0}$ DNE

19. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$ $\frac{\pi}{3}$	20. $\arccos(-1)$ π	21. $\tan^{-1}(-1)$ $-\frac{\pi}{4}$
22. $\sin^{-1}(-1)$ $-\frac{\pi}{2}$	23. $\arcsin\left(-\frac{1}{2}\right)$ $-\frac{\pi}{6}$	24. $\arctan(-\sqrt{3})$ $-\frac{\pi}{3}$
25. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $\frac{3\pi}{4}$	26. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $-\frac{\pi}{4}$	27. $\tan^{-1}(0)$ 0
28. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$ $-\frac{\pi}{3}$	29. $\cos^{-1}\left(-\frac{1}{2}\right)$ $\frac{2\pi}{3}$	30. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ $\frac{5\pi}{6}$

Simplify using trig identities. Write your answer as a single trig expression or a number.

31. $\cot x \cdot \sec x$
 $\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} = \frac{1}{\sin x}$
 $\csc x$

32. $\frac{\sin x}{\csc^2 x}$
 $\sin x \cdot \frac{1}{\csc^2 x}$
 $\sin x \cdot \sin^2 x$
 $\sin^3 x$

33. $\frac{1 - \cos^2 x}{\tan^2 x}$
 $\sin^2 x \div \frac{\sin^2 x}{\cos^2 x}$
 $\sin^2 x \cdot \frac{\cos^2 x}{\sin^2 x} = \cos^2 x$

34. $\sin x \cdot \csc x$
 $\sin x \cdot \frac{1}{\sin x}$
 1

35. $(1 + \tan^2 x)$
 $\sec^2 x$

36. $\frac{\sin^2 x + \cos^2 x}{\tan x}$
 $\frac{1}{\tan x}$
 $\cot x$

Unit 1: Pre-Calculus Practice Test

Answer the questions below. Please be sure to highlight your final answer. To earn full credit you must show all of your work.

1. Factor Completely: $x^2 - x - 6$

$$(x-3)(x+2)$$

2. Factor Completely: $x^4 - 17x^2 + 16$

$$(x^2 - 16)(x^2 - 1)$$

$$(x+4)(x-4)(x+1)(x-1)$$

3. Factor Completely: $6x^3 + 6$

$$6(x^3 + 1)$$

$$6(x+1)(x^2 - x + 1)$$

4. Factor Completely: $6x^2 - 7x - 3$

$$(3x+1)(2x-3)$$

5. Find the point-slope form of the line that passes through $(2, -3)$ and is parallel to the line $y = -\frac{4}{3}x + 18$

$$m = -\frac{4}{3}$$

$$y + 3 = -\frac{4}{3}(x - 2)$$

6. Find the average rate of change of $\frac{2x+3}{x^2+2}$ on the interval $[-1, 1]$

$$f(-1) = \frac{1}{3} \quad f(1) = \frac{5}{3}$$

$$\frac{\frac{5}{3} - \frac{1}{3}}{1 - (-1)} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

7. State the domain: $f(x) = \ln(x^2 - 8x + 12)$

$$x^2 - 8x + 12 > 0$$

$$(x-6)(x-2) = 0$$

$$x = 6, 2$$

$$(-\infty, 2) \cup (6, \infty)$$

8. State the domain: $f(x) = \frac{x+5}{x^2-6x+8}$

$$x^2 - 6x + 8 \neq 0$$

$$(x-4)(x-2) = 0$$

$$x \neq 4, 2$$

$$(-\infty, 2) \cup (2, 4) \cup (4, \infty)$$

9. State the domain: $f(x) = \sqrt{x^2 - 4x - 12}$

$$x^2 - 4x - 12 \geq 0$$

$$(-\infty, -2] \cup [6, \infty)$$

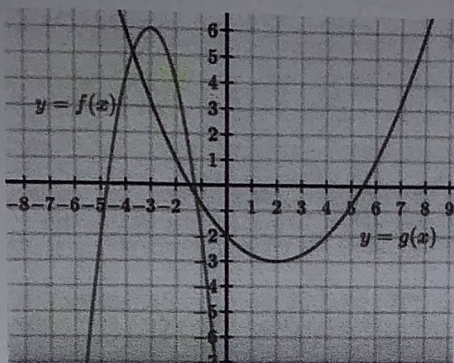
10. Describe the transformations from the parent function:

a. $f(x) = -3(x-5)^3$ reflect over x-axis, stretch by 3, right 5

b. $f(x) = -\frac{1}{2}x^2 + 3$ reflect over x-axis, shrink vert. by $\frac{1}{2}$, up 3

c. $f(x) = \sqrt{x+4} - 9$ left 4 + down 9

11.



Evaluate:

a. $f(g(-3))$

$f(3)$

DNE

b. $g(f(-4))$

$g(4)$

-2

13. Evaluate the following:

$$f(x) = \begin{cases} 2 \cdot 3^x + 1, & x \leq -2 \\ \sqrt{8x} + 1, & -2 < x \leq 2 \\ -|x+1|, & x > 2 \end{cases}$$

a. $f(-2) = 2 \cdot 3^{-2} + 1 = \frac{2}{9} + 1 = \frac{11}{9}$

b. $f(2) = \sqrt{8 \cdot 2} + 1 = 5$

c. $f(3)$ DNE

d. $f(6) = -|6+1| = -7$

15. Evaluate:

a. $5e^{2 \ln 3n} = 5e^{\ln(3n)^2} = 5e^{\ln(9n^2)} = 5(9n^2) = 45n^2$

b. $e^{\ln \sqrt{y}} = \sqrt{y}$

c. $\frac{1}{2} \ln e^{5x} = \frac{1}{2} (5x) = \frac{5}{2}x$

12. Given that $f(x) = 3(2x-1)^2 + 1$, $g(x) = x+2$, and $h(x) = e^{2x-1} + 3$. Perform the indicated operation:a. Find $(g \circ f)(x)$

$$3(2x-1)^2 + 1 + 2$$

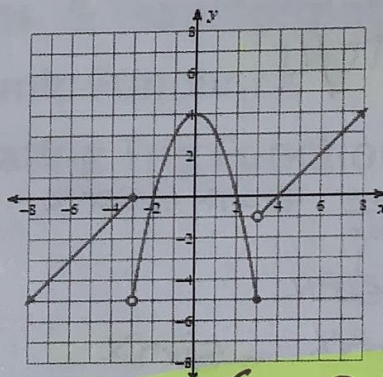
$$3(2x-1)^2 + 3$$

b. Find $(h \circ g)(x)$

$$e^{2(x+2)-1} + 3$$

$$e^{2x+3} + 3$$

14. Write the equation for the graph:



$$f(x) = \begin{cases} x+3, & x \leq -3 \\ -x^2+4, & -3 < x \leq 3 \\ x-4, & x > 3 \end{cases}$$

16. Simplify to a single expression:

a. $\ln(\ln e(e^7) 5 \ln e^{-x})$

$\ln(\ln e(e^7(5 \cdot -x)))$

$\ln e^7 \cdot -5x$

$7 \cdot -5x = -35x$

b. $\log(5x) - 2 \log 3 + \frac{1}{2} \log(x-3)$

$$\log \frac{5x \sqrt{x-3}}{3^2}$$

$$\log \left(\frac{5x \sqrt{x-3}}{9} \right)$$

$$17. \text{ Evaluate } \sec\left(\frac{41\pi}{6}\right)$$

$$\frac{41\pi}{6} - 2\pi = \frac{29\pi}{6} - 2\pi = \frac{17\pi}{6} - 2\pi = \frac{5\pi}{6}$$

$$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2} \quad \sec\left(\frac{5\pi}{6}\right)$$

$$-\frac{2\sqrt{3}}{3}$$

$$19. \text{ Evaluate } \cot\left(\frac{11\pi}{6}\right) \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$$-\sqrt{3}$$

$$21. \text{ Evaluate } \arcsin\left(\frac{1}{2}\right)$$

$$\pi/6$$

$$23. \text{ Evaluate } \arctan\left(-\frac{\sqrt{2}}{2}\right)$$

$$-\pi/4$$

$$25. \text{ Simplify } \frac{\tan x}{\sec x} \quad \tan x \div \sec x$$

$$\frac{\sin x}{\cos x} \div \frac{1}{\cos x}$$

$$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = \sin x$$

$$27. \text{ Simplify } \frac{\sin^2 x + \cos^2 x}{\cot^2 x}$$

$$\frac{1}{\cot^2 x} = \tan^2 x$$

$$18. \text{ Evaluate } \sin\left(-\frac{14\pi}{3}\right)$$

$$-\frac{14\pi}{3} + 2\pi = -\frac{8\pi}{3} + 2\pi = -\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}$$

$$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$20. \text{ Evaluate } \csc(-3\pi)$$

$$-3\pi + 2\pi = -\pi + 2\pi = \pi$$

$$\sin \pi = 0$$

$$\csc \pi = \frac{1}{0}$$

DNE

$$22. \text{ Evaluate } \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\pi/6$$

$$24. \text{ Simplify } \sin x \cdot \cot x$$

$$\sin x \cdot \frac{\cos x}{\sin x}$$

$$\cos x$$

$$26. \text{ Simplify } \sin^3 x \cdot \csc^3 x$$

$$\sin^3 x \cdot \frac{1}{\sin^3 x}$$

$$1$$

$$28. \text{ Simplify } \cos^2 x (1 + \tan^2 x)$$

$$\cos^2 x \cdot \sec^2 x$$

$$\cos^2 x \cdot \frac{1}{\cos^2 x}$$

$$1$$