Unit 1 – Pre-Calculus Assignments
Revised Homework Packet COVID19 – Fall 2020

Essential question: *What pre-calculus concepts are necessary for success in AP Calculus?*

Day	Topic	Assignment
Day 1	EQ: How can we write the equation of functions that are made from known graphs? **Take Notes** **Keeper 1.1 – Equations of Lines, Piecewise and Transformations**	- Read syllabus - Show Parents Parent Letter - Read Keeper (Notes) Requirements - **Take Notes – Keeper 1.1** Assignment(s): - Lines, Piecewise Functions, and Transformations Worksheet
Day 2	EQ: How do I determine the domain of functions? How do I transform e and ln graphs? **Take Notes** **Keeper 1.2 – Domain Rules and e ln Transformations**	Take Notes – Keeper 1.2 Assignment(s): - Domain and Range, e-ln transformation Worksheet
Day 3	EQ: What trigonometry do I need to know? **Take Notes** **Keeper 1.3 – Trig, Inverse Trig, and Solving Trig Equations**	Take Notes – Keeper 1.3 Assignment(s): - Inverse Trig and Solving Trig Equations Worksheet
Day 4	EQ: How do I put two functions together **Take Notes** **Keeper 1.4 – Composition, Exponentials and Logarithms**	Take Notes – Keeper 1.4 Assignment(s): - Exponentials and Logarithms Worksheet - Composition of Functions Worksheet
Day 5	EQ: How do I solve inequalities? How do I find inverses? **Take Notes** **Keeper 1.5 – Solving Inequalities and Absolute Value**	Take Notes – Keeper 1.5 Assignment(s): - Inequalities Worksheet - Absolute Value and Piecewise Worksheet

***You must show all work and your work must be neat and organized. Sloppy work and unorganized work WILL NOT be accepted!!

Equations of Lines, Piecewise Functions, and Transformations

Equations of Lines

1. Write the slope-intercept form of the equation:
$11x - 8y = -48$

$-8y = -11x - 48$

$y = \frac{11}{8}x + 6$

2. Write the <u>standard form</u> of the equation of the line through the given point with the given slope: $(3,5), m = \frac{5}{3}$

$y - 5 = \frac{5}{3}(x-3)$

$3y - 15 = 5x - 15$

$-5x + 3y = 0$

3. Write the equation of the line described: through (4,2), parallel to $y = -5x + 2$

$M = -5$

$y - 2 = -5(x-4)$

4. Write the equation of the line described: through $(-2,4)$, perpendicular to $y = -\frac{5}{2}x + 5$

$M = 2/5$

$y - 4 = 2/5(x+2)$

Piecewise functions

5. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

Domain:
$\mathbb{R}$

Range:
$[2, \infty)$

Find:

$f(-2) = (-2)^2 + 2$ $= 6$	$f(0) = 0^2 + 2$ $= 2$
$f(1) = 1^2 + 2$ $= 3$	$f(s^2 + 2) = 2(s^2+2)^2 + 2$ ↓ Must be greater than 1

1
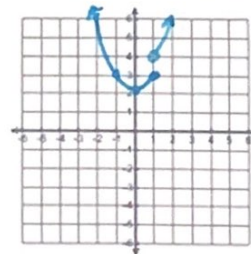
6. $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases}$

Domain:	$\mathbb{R}$
Range:	$[0, \infty)$

Find:

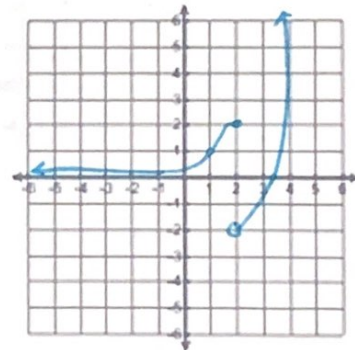| $f(-2) = 1$ | $f(0) = \sqrt{0}$ |
	$= 0$
$f(1) = \sqrt{1}$	$f(s^2) = \sqrt{s^2}$
$= 1$	$= |s|$
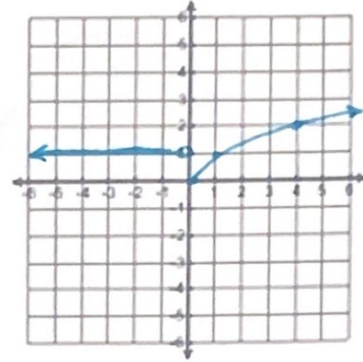
7. $f(x) = \begin{cases} \dfrac{2}{x^2 - 4x + 5}, & x \leq 2 \\ x^3 - 6x^2 + 12x - 10, & x > 2 \end{cases}$
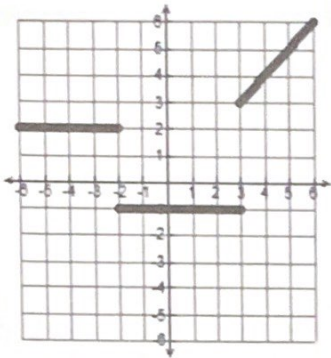
Domain:	$\mathbb{R}$
Range:	$(-2, \infty)$

Find:

| $f(-2) = \dfrac{2}{4 + 8 + 5}$ | $f(0) = \dfrac{2}{0 - 0 + 5}$ |
$= 2/17$	$= 2/5$
$f(1) = \dfrac{2}{1 - 4 + 5}$	$f(2) = \dfrac{2}{4 - 8 + 5}$
$= 1$	$= 2$

Find the formula for the following:

8.

$$f(x) = \begin{cases} 2, & x \le -2 \\ -1, & -2 < x < 3 \\ x, & x \ge 3 \end{cases}$$
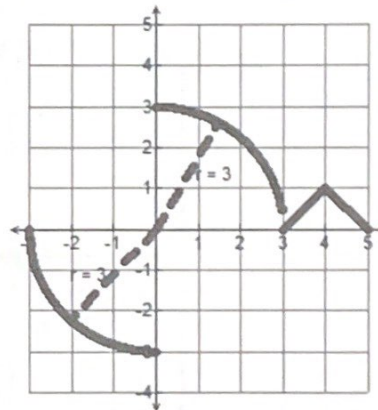
9.
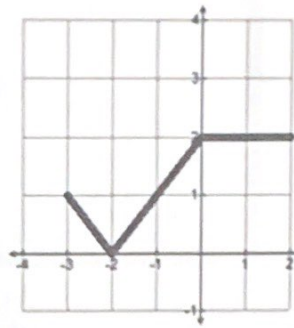
$$f(x) = \begin{cases} |x+2|, & -3 \le x \le 0 \\ 2, & x > 0 \end{cases}$$

10.
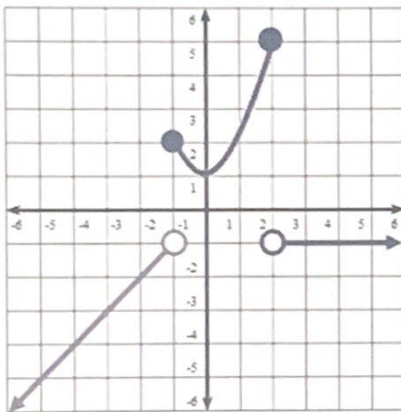
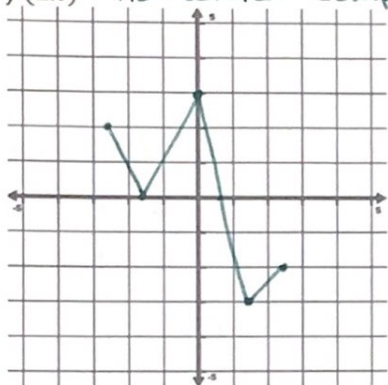$$f(x) = \begin{cases} x, & x < -1 \\ x^2+1, & -1 \le x \le 2 \\ -1, & x > 2 \end{cases}$$

11.

$$x^2 + y^2 = 3^2$$
$$y^2 = 9 - x^2$$
$$y = \pm\sqrt{9-x^2}$$

$$f(x) = \begin{cases} -\sqrt{9-x^2}, & -3 \le x < 0 \\ \sqrt{9-x^2}, & 0 \le x \le 3 \\ -|x-4|+1, & x > 3 \end{cases}$$
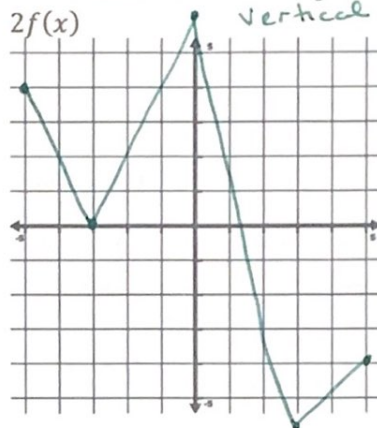
3

Draw a graph of each transformation. Let $f(x)$ be the function below. Label the critical points. Finally write a description of the transformation.
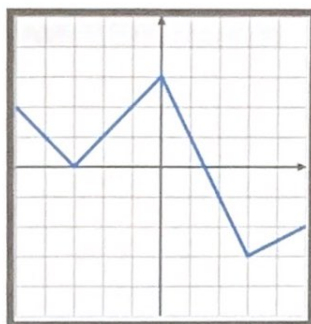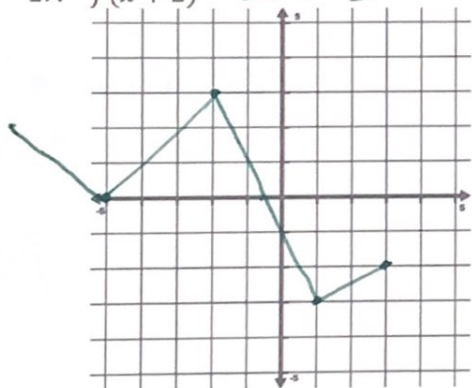

15. $f(2x)$ Horizontal Compression
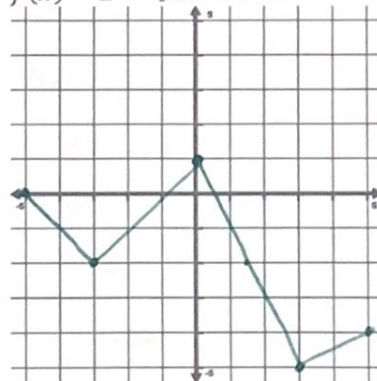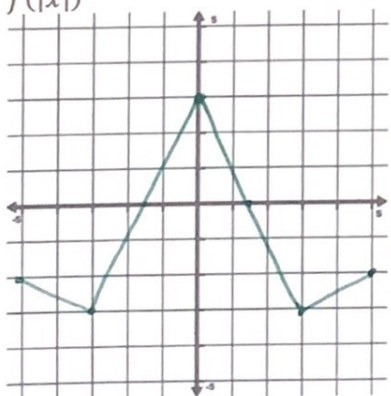

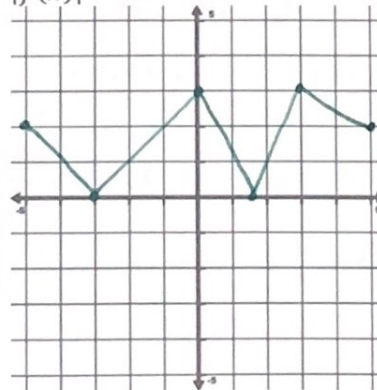16. $2f(x)$ Vertical Stretch

17. $f(x+2)$ Left 2

18. $f(x)-2$ Down 2

19. $f(|x|)$ ← neg x values Mimic pos x values

20. $|f(x)|$ ← All y values Are positive

Domain and $e/\ln$ Transformations

Find the domain of the following. Show your work.

1. $f(x) = \frac{3x-5}{x+1}$

   $x+1 \neq 0$

   $x \neq -1$

   $(-\infty, -1) \cup (-1, \infty)$

2. $f(x) = \sqrt{\frac{3x-5}{x+1}}$

   $x+1 \neq 0$ and $\frac{3x-5}{x+1} \geq 0$

   $x \neq -1$

   $\begin{array}{ccc} + & - & + \\ \hline & -1 & 5/3 \end{array}$

   $(-\infty, -1) \cup [5/3, \infty)$

3. $f(x) = \ln(x - 7)$

   $x - 7 > 0$

   $x > 7$

   $(7, \infty)$

4. $f(x) = \ln\left(\frac{x}{x-1}\right)$

   $x-1 \neq 0$ and $\frac{x}{x-1} > 0$

   $x \neq 1$

   $\begin{array}{ccc} + & - & + \\ \hline & 0 & 1 \end{array}$

   $(-\infty, 0) \cup (1, \infty)$

5. $f(x) = \sqrt[3]{3x + 2}$

   $\mathbb{R}$

6. $f(x) = \frac{x+1}{x^2+5x+4}$

   $x^2 + 5x + 4 \neq 0$

   $(x+4)(x+1) \neq 0$

   $x \neq -4 \quad\quad x \neq -1$

   $(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$

7. $f(x) = \ln\left(\frac{x+1}{x-3}\right)$

   $x - 3 \neq 0$ and $\frac{x+1}{x-3} > 0$

   $x \neq 3$

   $\begin{array}{ccc} + & - & + \\ \hline & -1 & 3 \end{array}$

   $(-\infty, -1) \cup (3, \infty)$

8. $f(x) = \sqrt[4]{x^2 - 8x - 33}$

   $x^2 - 8x - 33 \geq 0$

   $(x-11)(x+3) \geq 0$

   $\begin{array}{ccc} + & - & + \\ \hline & -3 & 11 \end{array}$

   $(-\infty, -3] \cup [11, \infty)$

Graph the Following and describe the transformations:

9. $y = e^x$ Parent graph
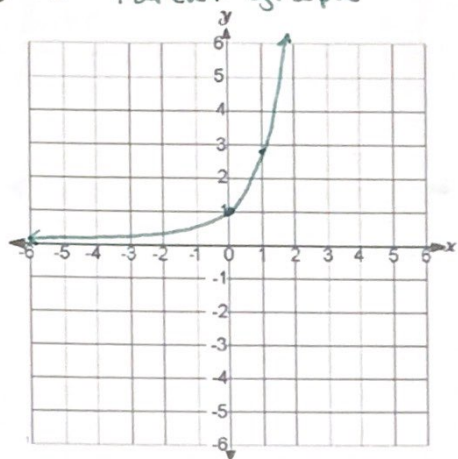
10. $y = 2(e)^x$ Vertical Stretch
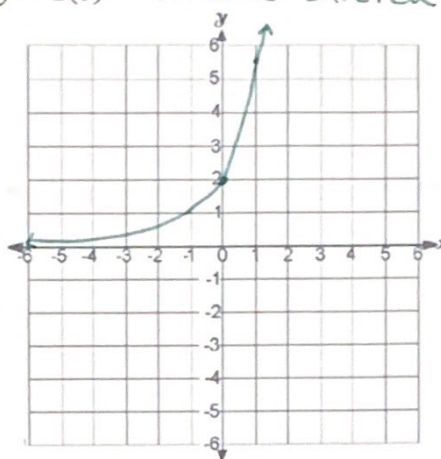
11. $y = e^{x+1}$ Left 1

12. $y = \ln(x) - 2$ Down 2

13. $y = |e^x|$ All y values positive
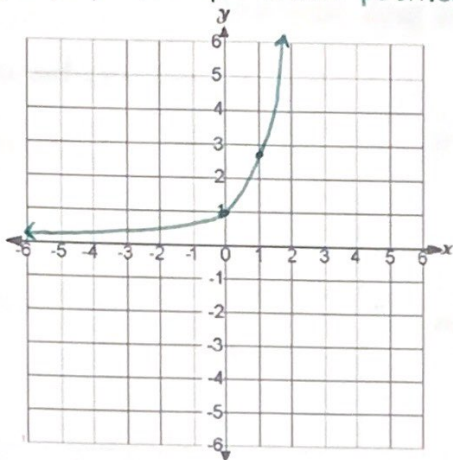
14. $y = -|\ln x|$ All y values positive then reflect x axis
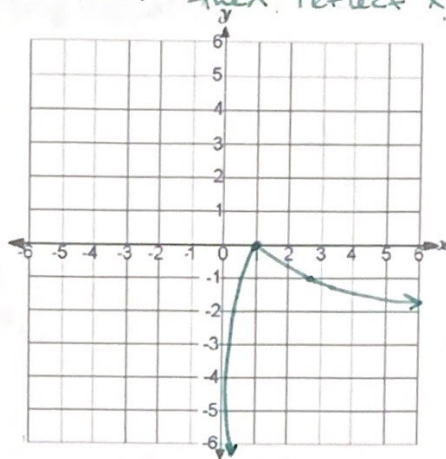
15. $y = -|e^{x+2}|$ Left 2
All y values positive
reflect x axis
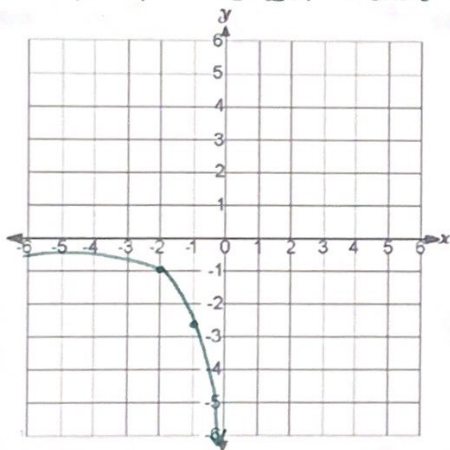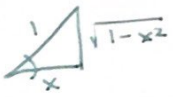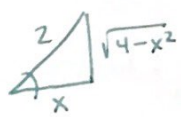
16. $y = -|e^x - 3| + 2$ down 3
all y values pos
x axis ref
up 2
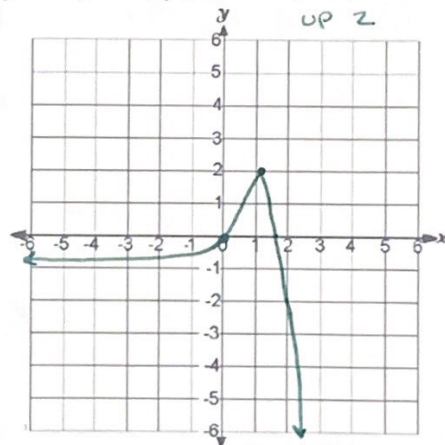
Inverse Trig and Solving Trig Equations

Evaluate each expression.

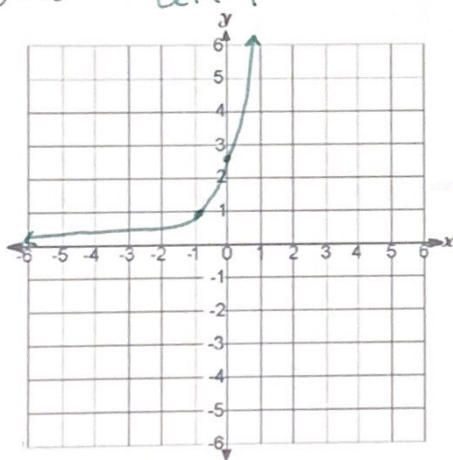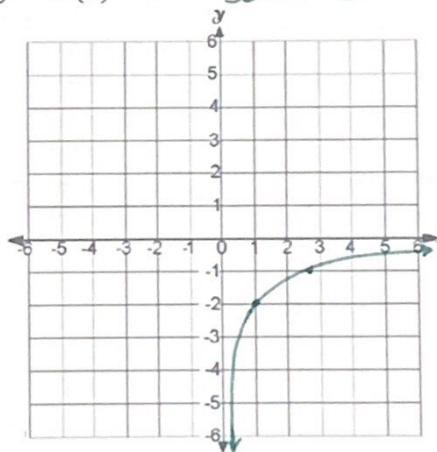|		
1. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$ $\pi/3$	2. $\arccos(-1)$ π	3. $\tan^{-1}(-1)$ $-\pi/4$
4. $\sin^{-1}(-1)$ $-\pi/2$	5. $\arcsin\left(-\frac{1}{2}\right)$ $-\pi/6$	6. $\arctan(-\sqrt{3})$ $-\pi/3$
7. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $\frac{3\pi}{4}$	8. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $-\frac{\pi}{4}$	9. $\tan^{-1}(0)$ 0
10. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$ $-\pi/3$	11. $\cos^{-1}\left(-\frac{1}{2}\right)$ $\frac{2\pi}{3}$	12. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ $\frac{5\pi}{6}$
13. $\sin^{-1}\left(\sin\frac{3\pi}{2}\right)$ $-\frac{\pi}{2}$	14. $\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$ $-\frac{\pi}{4}$	15. $\cos^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$ $\frac{2\pi}{3}$
16. $\cos\left(\sin^{-1}\frac{12}{13}\right)$ $\frac{5}{13}$	17. $\tan\left(\sin^{-1}\left(-\frac{8}{17}\right)\right)$ $-\frac{8}{15}$	18. $\sin\left(\cos^{-1}\left(-\frac{4}{5}\right)\right)$ $\frac{3}{5}$
19. $\sin(\cos^{-1}x)$ $\sqrt{1-x^2}$	20. $\sin\left(\cos^{-1}\frac{x}{2}\right)$ $\frac{\sqrt{4-x^2}}{2}$	21. $\tan(\sin^{-1}2x)$ $\frac{2x}{\sqrt{1-4x^2}}$

Solve for x, where $0 \leq x < 2\pi$

22. $2\sin x - 1 = 0$ $\sin x = \frac{1}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$	23. $2\cos x + 1 = 0$ $\cos x = -\frac{1}{2}$ $x = \frac{2\pi}{3}, \frac{4\pi}{3}$
24. $4\sin x + 2\sqrt{3} = 0$ $\sin x = -\frac{\sqrt{3}}{2}$ $x = \frac{4\pi}{3}, \frac{5\pi}{3}$	25. $2\tan x + \sqrt{3} = -\tan x$ $3\tan x = -\sqrt{3}$ $\tan x = -\frac{\sqrt{3}}{3}$ $x = \frac{5\pi}{6}, \frac{11\pi}{6}$
26. $2\cos^2 x = 3\cos x + 2$ $2\cos^2 x - 3\cos x - 2 = 0$ $(2\cos x + 1)(\cos x - 2) = 0$ $\cos x = -\frac{1}{2} \qquad \cos x = 2$ $\qquad\qquad\qquad\quad$ DNE $x = \frac{2\pi}{3}, \frac{4\pi}{3}$	27. $2\cos^2 x = \sin x + 1$ $2(1 - \sin^2 x) = \sin x + 1 \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ $2 - 2\sin^2 x = \sin x + 1$ $-2\sin^2 x - \sin x + 1 = 0$ $2\sin^2 x + \sin x - 1 = 0$ $(2\sin x - 1)(\sin x + 1) = 0$ $\sin x = \frac{1}{2} \qquad \sin x = -1$
28. $\sin 2x = \sin x$ $2\sin x \cos x - \sin x = 0$ $\sin x (2\cos x - 1) = 0$ $\sin x = 0 \qquad \cos x = \frac{1}{2}$ $x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$	29. $\tan 2x = -\sqrt{3}$ $2x = \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}, \frac{11\pi}{3}$ $x = \frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$ * $2x$ = twice around circle
30. $\sin 2x = -1$ $2x = \frac{3\pi}{2}, \frac{7\pi}{2}$ $x = \frac{3\pi}{4}, \frac{7\pi}{4}$ * $2x$ = twice around	31. $\cos 2x = -\frac{1}{2}$ $2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
32. $\tan 3x = 1$ $3x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4}$ $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}$ * $3x$ = 3 times around	33. $2\sin(2x) + 1 = 0$ $\sin(2x) = -\frac{1}{2}$ $2x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$ $x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

Exponentials and Logarithms

Find the value of the following without using your calculator.

1. $e^{\ln(5)} + e^x e^{-x}$

$5 + e^0$

$5 + 1$

6

2. $\ln\left(\frac{1}{e}\right) - \ln 1 + e^{3\ln 2}$

$\ln e^{-1} - 0 + e^{\ln 2^3}$

$-1 + 2^3$

$-1 + 8$

7

3. $e^{-2\ln 3 + 3\ln 2}$

$e^{\ln 3^{-2}} \cdot e^{\ln 2^3}$

$3^{-2} \cdot 2^3$

$\frac{1}{9} \cdot 8 = \frac{8}{9}$

Express each of the following as the logarithm of a single expression.

4. $\dfrac{1}{3}\ln x + 2\ln(3x-5)$

$\ln x^{1/3} + \ln(3x-5)^2$

$\ln\left(x^{1/3}(3x-5)^2\right)$

5. $2\ln x - \dfrac{1}{2}\ln(x^2-1) + 3\ln(x^2+1)$

$\ln x^2 - \ln\sqrt{x^2-1} + \ln(x^2+1)^3$

$\ln\left(\dfrac{x^2(x^2+1)^3}{\sqrt{x^2-1}}\right)$

6. $2\ln x + 3\ln(1+x) - 4\ln(2+x)$

$\ln x^2 + \ln(1+x)^3 - \ln(2+x)^4$

$\ln\left(\dfrac{x^2(1+x)^3}{(2+x)^4}\right)$

7. $\dfrac{1}{2}\ln x - 2\ln(x^2+x+1)$

$\ln\sqrt{x} - \ln(x^2+x+1)^2$

$\ln\left(\dfrac{\sqrt{x}}{(x^2+x+1)^2}\right)$

Use the properties of logarithms to express the following as an algebraic expression involving $\log x$, $\log y$, and/or $\log z$ (or $\ln *$).

8. $\log\sqrt{16x^8 y^4 z^2}$

$\log\sqrt{16} + \log\sqrt{x^8} + \log\sqrt{y^4} + \log\sqrt{z^2}$

$\log 4 + \log x^4 + \log y^2 + \log|z|$

9. $\ln\left(\dfrac{81\sqrt{x}}{\sqrt{yz^3}}\right)$

$\ln 81 + \ln\sqrt{x} - \ln\sqrt{y} - \ln\sqrt{z^3}$

$\ln 81 + \dfrac{1}{2}\ln x - \dfrac{1}{2}\ln y - \dfrac{3}{2}\ln z$

10. $\ln\left(\dfrac{\sqrt[3]{x}}{10\sqrt{yz}}\right)$

$\ln\sqrt[3]{x} - \ln 10 - \ln\sqrt{y} - \ln\sqrt{z}$

$\dfrac{1}{3}\ln x - \ln 10 - \dfrac{1}{2}\ln y - \dfrac{1}{2}\ln z$

11. $\ln\left(\dfrac{\sqrt[4]{x^3}\,y^7}{z^8}\right)$

$\ln\sqrt[4]{x^3} + \ln y^7 - \ln z^8$

$\dfrac{3}{4}\ln x + 7\ln y - 8\ln z$

10

Solve for x.

12. $3^{x-1} = 81$

$3^{x-1} = 3^4$

$x - 1 = 4$

$x = 5$

13. $-14 + 3e^x = 11$

$3e^x = 25$

$e^x = 25/3$

$x = \ln(25/3)$

14. $2e^{2x} - 5e^x - 3 = 0$

$(2e^x + 1)(e^x - 3) = 0$

$2e^x + 1 = 0 \qquad e^x - 3 = 0$

$2e^x = -1 \qquad\quad e^x = 3$

$e^x = -\frac{1}{2} \qquad\qquad x = \ln 3$

$x = \ln(-\frac{1}{2})$

$\hookleftarrow$ DNE

15. $25(1 - e^x) = 12$

$1 - e^x = 12/25$

$e^x = 1 - \frac{12}{25}$

$e^x = \frac{13}{25}$

$x = \ln(13/25)$

16. $4^{3x-3} \cdot 4^{2-2x} = 16^{-x}$

$4^{x-1} = 4^{-2x}$

$x - 1 = -2x$

$3x = 1$

$x = 1/3$

17. $-5e^{-x} + 9 = 6$

$-5e^{-x} = -3$

$e^{-x} = 3/5$

$-x = \ln(3/5)$

$x = -\ln(3/5)$

18. $5A = 2Ae^{x+k}$

$e^{x+k} = 5/2$

$x + k = \ln(5/2)$

$x = \ln(5/2) - k$

19. $2e^{bx} = e \cdot 3^{bx}$

$\left(\frac{e}{3}\right)^{bx} = \frac{e}{2}$

$bx \ln\left(\frac{e}{3}\right) = \ln\left(\frac{e}{2}\right)$

$bx(1 - \ln 3) = 1 - \ln 2$

$bx = \frac{1 - \ln 2}{1 - \ln 3}$

$x = \frac{1 - \ln 2}{b(1 - \ln 3)}$

20. $7(3^x) - 10x(3^x) + 3x^2(3^x) = 0$

$3^x(3x^2 - 10x + 7) = 0$

$3^x(3x - 7)(x - 1) = 0$

$3^x = 0 \qquad 3x - 7 = 0 \qquad x - 1 = 0$

$\hookleftarrow$ DNE $\quad x = 7/3 \qquad x = 1$

21. $\ln(x) - \ln(3) = 4$

$\ln\frac{x}{3} = 4$

$\frac{x}{3} = e^4$

$x = 3e^4$

22. $\ln(x+2)^2 = 6$

$2\ln(x+2) = 6$

$\ln(x+2) = 3$

$x + 2 = e^3$

$x = e^3 - 2$

23. $\log_5(\log_5(x+10)) = 0$

$\log_5(x+10) = 5^0$

$x + 10 = 5^1$

$x = -5$

24. $4^{\log_4(x+2)} = 2x$

$x + 2 = 2x$

$x = 2$

25. $2\log_4 x - \log_4(x-1) = 1$

$\log_4\frac{x^2}{x-1} = 1$

$\frac{x^2}{x-1} = 4$

$x^2 = 4x - 4$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

26. $\ln x - \ln(6) = 2\ln(4)$

$\ln\frac{x}{6} = \ln 16$

$\frac{x}{6} = 16$

$x = 96$

Composition of Functions

Given the following functions $f(x)$ and $g(x)$, find the following.

Graph of f
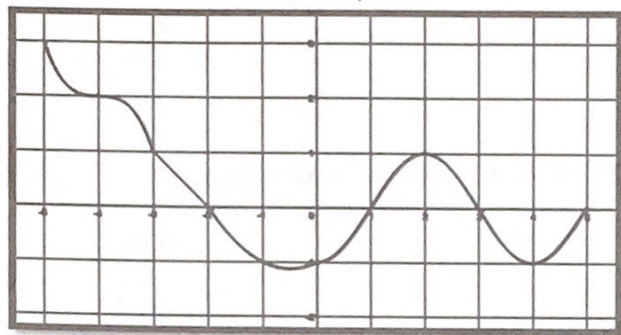
Graph of g
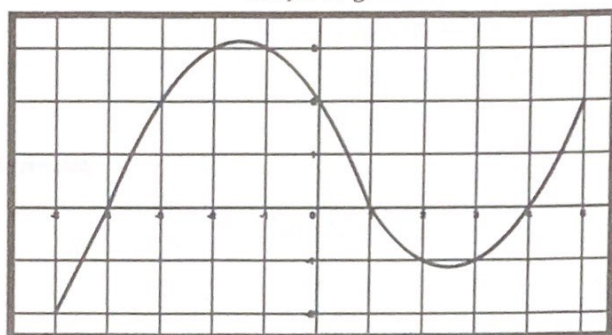
1. $f(g(-2)) = f(3)$
   $= 0$

2. $f(g(2)) = f(-1)$
   $= -1$

3. $g(f(-1)) = g(-1)$
   $= 3$

4. $f(f(5)) = f(0)$
   $= -1$

5. $g(g(-2)) = g(3)$
   $= -1$

Given the tables for functions f and g, find the following.

x	$f(x)$
-1	2
0	4
1	3
2	0
3	1
4	-1

x	$g(x)$
-1	3
0	4
1	2
2	6
3	2
4	-1

6. $f(g(3)) = f(2)$
   $= 0$

7. $g(f(2)) = g(0)$
   $= 4$

8. $f(f(4)) = f(-1)$
   $= 2$

9. $g(g(4)) = g(-1)$
   $= 3$

Given the tables for functions f and g, find the following.

x	$f(x)$
-2	0
0	1
2	-1
4	2

x	$g(x)$
-1	-2
0	2
1	3
2	1

10. $f(f(-2)) = f(0)$
    $= 1$

11. $f(g(0)) = f(2)$
    $= -1$

12. $g(f(1))$ undefined

Let $f(x) = 2x - 3$, $g(x) = e^x$, and $h(x) = \ln x$. Find a formula for each function.

13. $f(f(x))$ 14. $f(g(x))$ 15. $g(h(x))$ 16. $h(g(x))$

$2(2x-3)-3$ $2e^x - 3$ $e^{\ln x}$ $\ln e^x$

$4x - 9$ x x

Let $f(x) = x^3$, $g(x) = 5x + 1$, and $h(x) = 2^x$, find the following.

17.. $f(g(x))$ 18. $h(f(x))$ 19. $h(g(x))$

$(5x+1)^3$ 2^{x^3} 2^{5x+1}

20. $g(h(x))$ 21. $g(f(x))$ 22. $f\left(g(h(x))\right)$

$5 \cdot 2^x + 1$ $5x^3 + 1$ $(5 \cdot 2^x + 1)^3$

In the composition of functions $f(g(x))$, g is known as the inner function and f is known as the outer function. For each problem below, specify the inner and outer function.

23.. $y = (7x - 3)^2$ 24. $y = \ln(x^2 + 4)$ 25. $y = 2^{3x-5}$

$g(x) = 7x - 3$ $g(x) = x^2 + 4$ $g(x) = 3x - 5$

$f(x) = x^2$ $f(x) = \ln x$ $f(x) = 2^x$

26. $y = e^{\ln x}$ 27. $y = \sqrt{x}$ 28. $y = (\ln x)^2 + 1$

$g(x) = \ln x$ $g(x) = x$ $g(x) = \ln x$

$f(x) = e^x$ $f(x) = \sqrt{x}$ $f(x) = x^2 + 1$

29. $y = (x^2 - 4)^3$ 30. $y = \dfrac{2}{(x-7)^2}$ 31. $y = \sqrt[5]{x + 3}$

$g(x) = x^2 - 4$ $g(x) = x - 7$ $g(x) = x + 3$

$f(x) = x^3$ $f(x) = \dfrac{2}{x^2}$ $f(x) = \sqrt{x}$

13

Inequalities

1. $x^2 - 6x - 30 > -3$

$x^2 - 6x - 27 > 0$

$(x-9)(x+3)$

$\overset{+}{\underset{-3}{\quad}} \overset{-}{\quad} \overset{+}{\underset{9}{\quad}}$

$(-\infty, -3) \cup (9, \infty)$

2. $x^2 + 5x + 6 < 20$

$x^2 + 5x - 14 < 0$

$(x+7)(x-2) < 0$

$\overset{+}{\underset{-7}{\quad}} \overset{-}{\quad} \overset{+}{\underset{2}{\quad}}$

$(-7, 2)$

3. $(x-4)^2 > 4$

$x^2 - 8x + 12 > 0$

$(x-6)(x-2) > 0$

$\overset{+}{\underset{2}{\quad}} \overset{-}{\quad} \overset{+}{\underset{6}{\quad}}$

$(-\infty, 2) \cup (6, \infty)$

4. $-4x^2 - 13x - 6 \leq 0$

$4x^2 + 13x + 6 \geq 0$

$\dfrac{-13 \pm \sqrt{169 - 96}}{8}$

$\dfrac{-13 \pm \sqrt{73}}{8}$

$\overset{+}{\underset{\frac{-13-\sqrt{73}}{8}}{\quad}} \overset{-}{\quad} \overset{+}{\underset{\frac{-13+\sqrt{73}}{8}}{\quad}}$

$(-\infty, \frac{-13-\sqrt{73}}{8}] \cup [\frac{-13+\sqrt{73}}{8}, \infty)$

5. $2x^2 - 10x \leq 2x - 16$

$2x^2 - 12x + 16 \leq 0$

$x^2 - 6x + 8 \leq 0$

$(x-4)(x-2) \leq 0$

$\overset{+}{\underset{2}{\quad}} \overset{-}{\quad} \overset{+}{\underset{4}{\quad}}$

$[2, 4]$

6. $7x^2 - 12x - 45 \geq 0$

$\dfrac{12 \pm \sqrt{144 + 1260}}{14}$

$\dfrac{12 \pm \sqrt{1404}}{14}$

$\dfrac{12 \pm 6\sqrt{39}}{14}$

$\dfrac{6 \pm 3\sqrt{39}}{7}$

$\overset{+}{\underset{\frac{6-3\sqrt{39}}{7}}{\quad}} \overset{-}{\quad} \overset{+}{\underset{\frac{6+3\sqrt{39}}{7}}{\quad}}$

$(-\infty, \frac{6-3\sqrt{39}}{7}] \cup [\frac{6+3\sqrt{39}}{7}, \infty)$

7. $x^2 + 5x + 8 < 0$

$\dfrac{-5 \pm \sqrt{25 - 4(8)}}{2}$

$\dfrac{-5 \pm \sqrt{-7}}{2} \leftarrow$ Imaginary

$\overset{+ \; + \; + \; +}{\underset{-\infty \qquad \qquad \infty}{\quad}}$

No Solution

8. $x^2 + 5x + 8 \geq 0$

$\leftarrow$ SAME set up

$\overset{+ \; + \; + \; + \; + \; +}{\underset{-\infty \qquad \qquad \infty}{\quad}}$

$\mathbb{R}$

9. $x^2 - 10x + 25 > 0$

$(x-5)^2 > 0$

$\overset{+}{\underset{5}{\quad}} \overset{+}{\quad}$

$(-\infty, 5) \cup (5, \infty)$

10. $x^2 - 10x + 25 \leq 0$

$\leftarrow$ SAME set up

$\overset{+}{\underset{5}{\quad}} \overset{+}{\quad}$

$\{5\}$

11. $2x^2 + 16 \leq x^2 + 8x$

$x^2 - 8x + 16 \leq 0$

$(x-4)^2 \leq 0$

$\overset{+}{\underset{4}{\quad}} \overset{+}{\quad}$

$\{4\}$

12. $-x^2 \geq 4x + 4$

$-x^2 - 4x - 4 \geq 0$

$x^2 + 4x + 4 \leq 0$

$(x+2)^2 \leq 0$

$\overset{+}{\underset{-2}{\quad}} \overset{+}{\quad}$

$\{-2\}$

14

13. $2x^2 \leq -x - 4$

$2x^2 + x + 4 \leq 0$

$\dfrac{-1 \pm \sqrt{1 - 4(2)(4)}}{4}$ ← Imaginary

$+ + + + +$

No solution
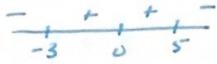
14. $2x^2 + 8x \geq 4x - 8$

$2x^2 + 4x + 8 \geq 0$

$x^2 + 2x + 4 \geq 0$

$\dfrac{-2 \pm \sqrt{4 - 4(1)(4)}}{2}$ ← Imaginary

$+ + + + +$

$\mathbb{R}$

15. $x^2(5 - x)(x + 3) < 0$

$\begin{array}{ccccc} - & + & + & - \\ \hline & -3 & 0 & 5 \end{array}$

$(-\infty, -3) \cup (5, \infty)$

16. $x^4 < 4x^2$

$x^4 - 4x^2 < 0$

$x^2(x^2 - 4) < 0$

$x^2(x + 2)(x - 2) < 0$

$\begin{array}{ccccc} + & - & - & + \\ \hline & -2 & 0 & 2 \end{array}$

$(-2, 0) \cup (0, 2)$

17. $\sqrt{3x + 4} \geq 2$

$\sqrt{3x + 4} \geq 2$ And $3x + 4 \geq 0$

$3x + 4 \geq 4$ $3x \geq -4$

$3x \geq 0$ $x \geq -4/3$

$x \geq 0$

$[0, \infty)$
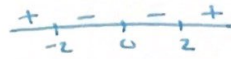
18. $\sqrt{3x + 4} \leq 2$

$\sqrt{3x + 4} \leq 2$ and $3x + 4 \geq 0$

$3x + 4 \leq 4$ $3x \geq -4$

$3x \leq 0$ $x \geq -4/3$

$x \leq 0$

$[-4/3, 0]$

19. $-\sqrt{x} \geq 2$

↳ Neg #

$-\# \ngeq 2$

No Solution

20. $\sqrt{2x + 1} - 3 < 0$

$\sqrt{2x + 1} < 3$ And $2x + 1 \geq 0$

$2x + 1 < 9$ $2x \geq -1$

$2x < 8$ $x \geq -1/2$

$x < 4$

$[-1/2, 4)$

21. $\dfrac{x^2 - 11x + 28}{x + 3} \geq 0$

$\dfrac{(x - 7)(x - 4)}{x + 3} \geq 0$

$\begin{array}{ccccc} - & + & - & + \\ \hline & -3 & 4 & 7 \end{array}$

$(-3, +] \cup [7, \infty)$

22. $\dfrac{x^2 - 10x + 24}{x + 5} \geq 0$

$\dfrac{(x - 6)(x - 4)}{x + 5} \geq 0$

$\begin{array}{ccccc} - & + & - & + \\ \hline & -5 & 4 & 6 \end{array}$

$(-5, 4] \cup [6, \infty)$

15

23. $32^{5x+2} \geq 16^{5x}$

$2^{5(5x+2)} \geq 2^{4(5x)}$

$25x+10 \geq 20x$

$5x \geq -10$

$x \geq -2$

$[-2, \infty)$

24. $2^{4x-5} > \left(\frac{1}{2}\right)^{x-5}$

$2^{4x-5} > 2^{-1(x-5)}$

$4x-5 > -x+5$

$5x > 10$

$x > 2$

$(2, \infty)$

25. $\log_4 x \leq \frac{3}{2}$

$\log_4 x \leq 3/2$ and $x > 0$

$x \leq 4^{3/2}$

$x \leq 8$

$(0, 8]$

26. $\log_5 x - 7 \geq -6$

$\log_5 x \geq 1$

$x \geq 5^1$

$x \geq 5$

$[5, \infty)$

27. $8 \ln x \geq 1$

$\ln x \geq 1/8$

$x \geq e^{1/8}$

$[e^{1/8}, \infty)$

28. $-2 \log_7 x + 9 > 11$

$-2 \log_7 x > 2$ and $x > 0$

$\log_7 x < -1$

$x < 7^{-1}$

$x < 1/7$ $\qquad (0, 1/7)$

29. $\log_5(x - 4) + 6 \leq 8$

$\log_5 (x-4) \leq 2$ and $x-4 > 0$

$x - 4 \leq 25$ $\qquad x > 4$

$x \leq 29$

$(4, 29]$

30. $|x| \leq 2$

$x \leq 2 \qquad x \geq -2$

$[-2, 2]$

31. $|x + 3| > 4$

$x+3 > 4 \qquad x+3 < -4$

$x > 1 \qquad\qquad x < -7$

$(-\infty, -7) \cup (1, \infty)$

32. $|x + 3| < 6$

$x+3 < 6 \qquad x+3 > -6$

$x < 3 \qquad\qquad x > -9$

$(-9, 3)$

33. $3|2x - 4| \geq -9$

$|2x-4| \geq -3$

$\mathbb{R}$

34. $2|x - 9| + 6 > 6$

$2|x-9| > 0$

$|x-9| > 0$

$(-\infty, 9) \cup (9, \infty)$

16

Absolute Value & Piecewise Functions

Solve the absolute value equation/inequality.

1. $\|3x + 12\| + 7 = 7$ $\|3x+12\| = 0$ $3x = -12$ $x = -4$	2. $\|3x - 7\| + 7 = 2$ $\|3x-7\| = -5$ No Solution
3. $\|3x - 7\| + 7 = 9$ $\|3x-7\| = 2$ $3x-7 = 2 \qquad 3x-7 = -2$ $3x = 9 \qquad\quad 3x = 5$ $x = 3 \qquad\quad x = 5/3$	4. $\|x + 5\| = \|2x - 1\|$ $x+5 = 2x-1 \qquad x+5 = -2x+1$ $x = 6 \qquad\qquad 3x = -4$ $\qquad\qquad\qquad x = -4/3$
5. $\|x - 4\| \geq 0$ $\mathbb{R}$	6. $\|2x - 1\| + 4 < 4$ $\|2x-1\| < 0$ No Solution
7. $-3 + \|x + 1\| \leq -3$ $\|x+1\| \leq 0$ $\{-1\}$	8. $\|3x + 4\| + 5 \leq 3$ $\|3x+4\| \leq -2$ No Solution

9. $2|x - 1| - 4 \geq 2$	10. $|x - 6| + 6 \geq -4$
$2\lvert x-1\rvert \geq 6$ $\lvert x-1\rvert \geq 3$ $x-1 \geq 3 \qquad x-1 \leq -3$ $x \geq 4 \qquad\quad x \leq -2$ $(-\infty, -2] \cup [4, \infty)$	$\lvert x-6\rvert \geq -10$ $\mathbb{R}$
11. $|2 - x| < 8$	12. $3|4x - 1| \leq 9$
$2-x < 8 \qquad 2-x > -8$ $x > -6 \qquad\quad x < 10$ $(-6, 10)$	$\lvert 4x-1\rvert \leq 3$ $4x-1 \leq 3 \qquad 4x-1 \geq -3$ $4x \leq 4 \qquad\quad 4x \geq -2$ $x \leq 1 \qquad\quad x \geq -\tfrac{1}{2}$ $\left[-\tfrac{1}{2}, 1\right]$

Rewrite as a piecewise function.

13. $f(x) = |x - 2|$	14. $f(x) = \left\lvert\dfrac{x-3}{x+1}\right\rvert$	15. $f(x) = |x^2 - 3x - 4|$
$\xleftarrow{\ -\ \ \underset{2}{+}\ +\ }$ $f(x) = \begin{cases} -(x-2), & x < 2 \\ x - 2, & x \geq 2 \end{cases}$	$\xleftarrow{\ +\ \underset{-1}{\ }\ -\ \underset{3}{\ }\ +\ }$ $f(x) = \begin{cases} \dfrac{x-3}{x+1}, & x < -1 \\ -\left(\dfrac{x-3}{x+1}\right), & -1 \leq x \leq 3 \\ \dfrac{x-3}{x+1}, & x > 3 \end{cases}$	$(x-4)(x+1)$ $\xleftarrow{\ +\ \underset{-1}{\ }\ -\ \underset{4}{\ }\ +\ }$ $f(x) = \begin{cases} x^2-3x-4, & x < -1 \\ -(x^2-3x-4), & -1 \leq x \leq 4 \\ x^2-3x-4, & x > 4 \end{cases}$
16. $f(x) = |x - 4|$	17. $f(x) = |x^2 - 5x - 6|$	18. $f(x) = \left\lvert\dfrac{2x+1}{4-x}\right\rvert$
$\xleftarrow{\ -\ \ \underset{4}{+}\ +\ }$ $f(x) = \begin{cases} -(x-4), & x < 4 \\ x - 4, & x \geq 4 \end{cases}$	$(x-6)(x+1)$ $\xleftarrow{\ +\ \underset{-1}{\ }\ -\ \underset{6}{\ }\ +\ }$ $f(x) = \begin{cases} x^2-5x-6, & x < -1 \\ -(x^2-5x-6), & -1 \leq x \leq 6 \\ x^2-5x-6, & x > 6 \end{cases}$	$\xleftarrow{\ -\ \underset{-1/2}{\ }\ +\ \underset{4}{\ }\ -\ }$ $f(x) = \begin{cases} -\left(\dfrac{2x+1}{4-x}\right), & x < -\tfrac{1}{2} \\ \dfrac{2x+1}{4-x}, & -\tfrac{1}{2} \leq x \leq 4 \\ -\left(\dfrac{2x+1}{4-x}\right), & x > 4 \end{cases}$