

## U Substitution Definite and Arc Trig Extra Practice

1.  $\int_1^2 \frac{1}{r^2} dr$

$$\int_1^2 r^{-2} dr$$

$$-\frac{1}{r} \Big|_1^2 = -\frac{1}{2} - (-1)$$

$$= \frac{1}{2}$$

2.  $\int \frac{3}{3x+5} dx$

$$\int \frac{3}{u} \frac{du}{3}$$

$$\int \frac{1}{u} du$$

$$\ln|u| + C$$

$$\ln|3x+5| + C$$

$$u = 3x+5$$

$$du = 3 dx$$

$$\frac{du}{3} = dx$$

3.  $\int (4x+1)^8 dx$

$$\frac{(4x+1)^9}{9 \cdot 4} + C$$

$$\frac{(4x+1)^9}{36} + C$$

4.  $\int_0^6 6y dy$

$$\frac{6y^2}{2} \Big|_0^6 = 3y^2 \Big|_0^6$$

$$3(6)^2 - 3(0)^2 = 108$$

5.  $\int 4x(2x^2+1)^5 dx$

$$\int u^5 du$$

$$\frac{u^6}{6} + C$$

$$\frac{(2x^2+1)^6}{6} + C$$

$$u = 2x^2+1$$

$$du = 4x dx$$

6.  $\int_0^3 t^3 dt$

$$\frac{t^4}{4} \Big|_0^3$$

$$\frac{3^4}{4} - \frac{0^4}{4} = \frac{81}{4}$$

7.  $\int_0^1 \frac{3s^2+2}{2s^3+4s+3} ds$

$$\int_0^1 \frac{1}{u} du$$

$$\frac{1}{2} \ln(2s^3+4s+3) \Big|_0^1$$

$$\frac{1}{2} \ln 9 - \frac{1}{2} \ln 3$$

$$\ln \left( \frac{9}{3} \right)^{\frac{1}{2}} = \ln \sqrt{3}$$

$$\text{or } \frac{1}{2} \ln 3$$

$$u = 2s^3+4s+3$$

$$du = 6s^2+4 ds$$

$$\frac{du}{2} = 3s^2+2 ds$$

8.  $\int \frac{3x^2}{x^3+8} dx$

$$\int \frac{1}{u} du$$

$$\ln|x^3+8| + C$$

$$u = x^3+8$$

$$du = 3x^2 dx$$

9.  $\int_{-2}^7 12s^2 + 1 ds$

$$4s^3 + s \Big|_{-2}^7$$

$$4(7)^3 + 7 - 4(-2)^3 - (-2)$$

$$1372 + 7 + 32 + 2$$

$$1413$$

11.  $\int \frac{x}{(2-x)(2+x)} dx$

$$\int \frac{x}{4-x^2} dx$$

$$-\frac{1}{2} \int \frac{1}{u} du$$

$$-\frac{1}{2} \ln|4-x^2| + c$$

$$u = 4-x^2$$

$$du = -2x dx$$

$$\frac{du}{-2} = x dx$$

10.  $\int_0^1 (2x-1)^6 dx$

$$\frac{(2x-1)^7}{14} \Big|_0^1 = \frac{1}{14} + \frac{1}{14} = \frac{1}{7}$$

12.  $\int 4te^{t^2} dt$

$$2 \int 2te^{t^2} dt$$

$$2 \int e^u du$$

$$2e^{t^2} + c$$

$$u = t^2$$

$$du = 2t dt$$

13.  $\int \frac{6x}{3x^2+5} dx$

$$\int \frac{1}{u} + c$$

$$\ln(3x^2+5) + c$$

$$u = 3x^2+5$$

$$du = 6x dx$$

14.  $\int_0^2 3y\sqrt{4-y^2} dy$

$$\frac{3}{-2} \int_0^2 u^{1/2} du$$

$$-\frac{3}{2} \cdot \frac{2}{3} u^{3/2} \Big|_0^2$$

$$-u^{3/2} \Big|_0^2 = 0 + 4^{3/2} = 8$$

$$u = 4-y^2$$

$$du = -2y dy$$

$$\frac{du}{-2} = y dy$$

15.  $\int \frac{x}{2x^2+3} dx$

$$\frac{1}{4} \int \frac{1}{u} du$$

$$\frac{1}{4} \ln(2x^2+3) + c$$

$$u = 2x^2+3$$

$$du = 4x dx$$

$$\frac{du}{4} = x dx$$

16.  $\int_0^1 6x^2 e^{x^3} dx$

$$\frac{6}{3} \int_0^1 e^u du$$

$$2e^{x^3} \Big|_0^1$$

$$2e^1 - 2e^0$$

$$2e - 2$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$\frac{du}{3} = x^2 dx$$



$$17. \int \frac{dx}{x^2+9}$$

$$\int \frac{1}{9\left(\frac{x^2}{9}+1\right)} dx$$

$$\frac{1}{9} \int \frac{1}{\left(\frac{x}{3}\right)^2+1} dx$$

$$\frac{1}{3} \int \frac{1}{u^2+1} du$$

$$\frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$$

$$u = \frac{x}{3}$$

$$du = \frac{1}{3} dx$$

$$3du = dx$$

$$18. \int \frac{9r^2}{\sqrt{1-r^3}} dr$$

$$u = 1-r^3$$

$$du = -3r^2 dr$$

$$\int \frac{-3}{u^{1/2}} du$$

$$-3 \int u^{-1/2} du$$

$$-3 \cdot 2u^{1/2} + C$$

$$-6\sqrt{1-r^3} + C$$

$$19. \int \cos(3z+4) dz$$

$$\frac{1}{3} \sin(3z+4) + C$$

$$20. \int \frac{dx}{\sin^2 3x}$$

$$\int \csc^2 3x dx$$

$$-\frac{1}{3} \cot(3x) + C$$

$$21. \int \frac{6 \cos t}{(2+\sin t)^2} dt$$

$$u = 2+\sin t$$

$$du = \cos t dt$$

$$6 \int \frac{1}{u^2} du$$

$$6 \int u^{-2} du$$

$$6 \cdot \frac{u^{-1}}{-1} + C$$

$$-\frac{6}{u} + C$$

$$-\frac{6}{2+\sin t} + C$$

$$22. \int \frac{x}{x^2+1} dx$$

$$u = x^2+1$$

$$du = 2x dx$$

$$\frac{du}{2} = x dx$$

$$\frac{1}{2} \int \frac{1}{u} du$$

$$\frac{1}{2} \ln(x^2+1) + C$$

$$23. \int \frac{40}{x^2+25} dx$$

$$40 \int \frac{1}{25 \left( \frac{x^2}{25} + 1 \right)} dx$$

$$\frac{8}{5} \int \frac{1}{\left( \frac{x}{5} \right)^2 + 1} dx$$

$$8 \int \frac{1}{u^2 + 1} dx$$

$$8 \arctan\left(\frac{x}{5}\right) + C$$

$$u = \frac{x}{5}$$

$$du = \frac{1}{5} dx$$

$$5 du = dx$$

$$24. \int \frac{dx}{\sqrt{1-4x^2}}$$

$$\int \frac{1}{\sqrt{1-(2x)^2}}$$

$$\frac{1}{2} \int \frac{1}{\sqrt{1-u^2}}$$

$$\frac{1}{2} \sin^{-1}(2x) + C$$

$$u = 2x$$

$$du = 2 dx$$

$$\frac{du}{2} = dx$$

$$25. \int \frac{dy}{2y\sqrt{4y^2-1}}$$

$$\frac{1}{2} \int \frac{1}{y\sqrt{(2y)^2-1}} dy$$

$$\frac{1}{2} \cdot \frac{1}{2} \int \frac{1}{\frac{u}{2}\sqrt{u^2-1}} du$$

$$2 \cdot \frac{1}{4} \int \frac{1}{u\sqrt{u^2-1}} du$$

$$\frac{1}{2} \operatorname{arcsec}(2y) + C$$

$$u = 2y$$

$$du = 2 dy$$

$$\frac{du}{2} = dy$$

$$y = \frac{u}{2}$$

$$26. \int \frac{dx}{\sqrt{9-x^2}}$$

$$\int \frac{1}{\sqrt{9\left(1-\frac{x^2}{9}\right)}} dx$$

$$\frac{1}{3} \int \frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}} dx$$

$$\frac{1}{3} \int \frac{1}{\sqrt{1-u^2}} du$$

$$\frac{1}{3} \arcsin\left(\frac{x}{3}\right) + C$$

$$u = \frac{x}{3}$$

$$du = \frac{1}{3} dx$$