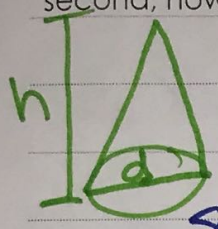
Related Rates Day 3 **Shadows and Cones**

1. Assume that sand allowed to pour onto a level surface will form a pile in the shape of a cone, with height equal to the diameter of the base. If sand is poured at 2 cubic meters per second, how fast is the height of the pile increasing when the base is 8 meters in diameter?



$h=d$
$h=2r$

K: $\frac{dV}{dt}=2m^3/sec$
F: $\frac{dh}{dt}$
W: $d=8m$ $h=8m$ $r=4m$

Solve for r

① Get r in terms of h

$\frac{r}{h}=\frac{4}{8}$

$\frac{4h}{8}=\frac{8r}{8}$

$r=\frac{1}{2}h$

② Volume of Cone

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(\frac{1}{2}h)^2 h$

$V=\frac{1}{3}\pi\frac{1}{4}h^2\cdot h$

$V=\frac{1}{12}\pi h^3$

③ Find $\frac{dV}{dt}$

$\frac{dV}{dt}=\frac{1}{4}\pi h^2\frac{dh}{dt}$

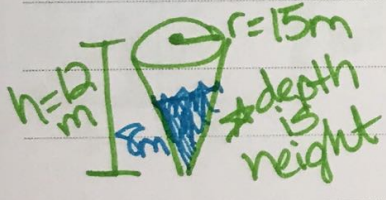
$2=\frac{1}{4}\pi(8)^2\frac{dh}{dt}$

$2=16\pi\frac{dh}{dt}$

$\frac{2}{16\pi}=\frac{16\pi}{16\pi}\frac{dh}{dt}$

$\frac{dh}{dt}=\frac{1}{8\pi}$ m/sec

2. A water tank has the shape of an inverted right-circular cone, with radius at the top 15 meters and depth 12 meters. Water is flowing into the tank at the rate of 2 cubic meters per minute. How fast is the depth of water in the tank increasing when the depth is 8 meters?



K: $\frac{dV}{dt}=2m^3/min$
F: $\frac{dh}{dt}$
W: $h=8m$

① Proportion of cone dim.

$\frac{r}{h}=\frac{15}{12}$

$\frac{15h}{12}=\frac{12r}{12}$

$r=\frac{5}{4}h$

② $V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(\frac{5}{4}h)^2 h$

$V=\frac{1}{3}\pi\frac{25}{16}h^2\cdot h$

$V=\frac{25}{48}\pi h^3$

③ $V=\frac{25}{48}\pi h^3$

$\frac{dV}{dt}=\frac{25}{16}\pi h^2\frac{dh}{dt}$

$2=\frac{25}{16}\pi(8)^2\frac{dh}{dt}$

$2=\frac{25}{16}\cdot\frac{64}{1}\pi\frac{dh}{dt}$

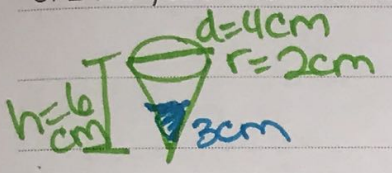
$2=100\pi\frac{dh}{dt}$

$\frac{2}{100\pi}=\frac{100\pi}{100\pi}\frac{dh}{dt}$

$\frac{dh}{dt}=\frac{1}{50\pi}$ m/min

Related Rates Day 3 **Shadows and Cones**

3. A conical cup is 4 cm across and 6 cm deep. Water leaks out of the bottom at the rate of 2 cm^3/sec. How fast is the water level dropping when the height of the water is 3 cm?



K: $dV/dt = -2cm^3/sec$

F: dh/dt

W: $h = 3cm$

① $\frac{h}{r} = \frac{6}{2}$

$\frac{6r}{6} = \frac{2h}{6}$

$r = \frac{1}{3}h$

② $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(\frac{1}{3}h)^2 h$

$V = \frac{1}{3}\pi \frac{1}{9}h^2 h$

$V = \frac{1}{27}\pi h^3$

③ $V = \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

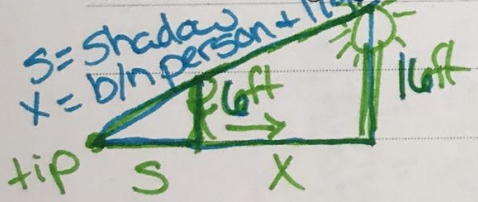
$-2 = \frac{1}{9}\pi(3)^2 \frac{dh}{dt}$

$-2 = \frac{1}{9}\pi 9 \frac{dh}{dt}$

$\frac{-2}{\pi} = \frac{\pi}{\pi}$

$\frac{dh}{dt} = \frac{-2}{\pi}$ cm/sec

4. A man 6 feet tall walks at the rate of 5 ft/sec toward a street light that is 16 ft above the ground. ② At what rate is the tip of the shadow moving? ① At what rate is the length of his shadow changing when he is 10 feet from the base of the light?



K: $dx/dt = -5ft/sec$

F: ds/dt (shadow length) and $\frac{dtip}{dt}$ (tip of shadow)

W: $x = 10ft$

① Find $\frac{ds}{dt}$

small Δ big Δ

$\frac{6}{s} \times \frac{16}{s+x}$

$16s = 6s + 6x$

$10s = 6x$

Find deriv.

$10\frac{ds}{dt} = 6\frac{dx}{dt}$

$\frac{10\frac{ds}{dt}}{10} = \frac{6(-5)}{10}$

$\frac{ds}{dt} = -3ft/s$

② Find $dtip/dt$

$Tip = x + s$

Find deriv.

$\frac{dtip}{dt} = \frac{dx}{dt} + \frac{ds}{dt}$

$\frac{dtip}{dt} = -5 + -3$

$\frac{dtip}{dt} = -8$ ft/sec

or

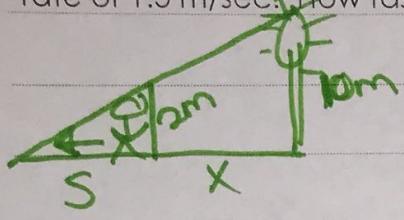
$\frac{10s}{10} = \frac{6x}{10}$

$s = \frac{3}{5}x$

$\frac{ds}{dt} = \frac{3}{5}\frac{dx}{dt}$ $\frac{ds}{dt} = \frac{3}{5}(-5)$

Related Rates Day 3 **Shadows and Cones**

5. A pickpocket walking away from a 10 meter tall lamppost is 2 meters tall. He walks at a rate of 1.5 m/sec. How fast is his shadow growing when he is 5 meter from the lamppost?



K: $\frac{dx}{dt} = 1.5 m/sec$

F: ds/dt

W: $x = 5 m$

Small Δ Big Δ

$$\frac{2}{S} = \frac{10}{X+S}$$

$$10S = 2X + 2S$$

$-2S \quad -2S$

$$8S = 2X \quad \text{or} \quad 8\frac{ds}{dt} = 2\frac{dx}{dt}$$

$\div 8 \quad \div 8$

$$S = \frac{1}{4}X \qquad \frac{ds}{dt} = \frac{1}{4}\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{1}{4}\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{1}{4}(1.5) \qquad \frac{1}{4}\cdot\frac{3}{2}$$

$$\frac{ds}{dt} = \frac{3}{8} \text{ m/sec}$$

p. 14-15 #1-3 & #6-8