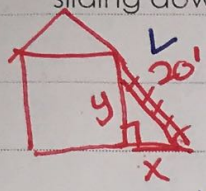
Related Rates Day 2 **Triangles, Boats, Cars, and Ladders**

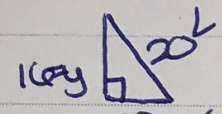
1. A 20 foot ladder is leaning against a house. The foot of the ladder begins to slide away from the house at a rate of 2 feet/second. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 12 feet from the house?



K: $\frac{dx}{dt} = 2$ ft/sec

F: $\frac{dy}{dt}$

W: $x = 12'$



① Find y

$x^2 + y^2 = L^2$

$12^2 + y^2 = 20^2$

$144 + y^2 = 400$

$y^2 = 256$

$y = 16'$

② Find $\frac{dy}{dt}$

$x^2 + y^2 = L^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2L\frac{dL}{dt}$ ladder is constant

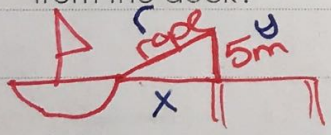
$2(12)(2) + 2(16)\frac{dy}{dt} = 2(20)(0)$

$48 + 32\frac{dy}{dt} = 0$

$\frac{32}{32}\frac{dy}{dt} = \frac{-48}{32}$

$\frac{dy}{dt} = -\frac{3}{2}$ ft/sec

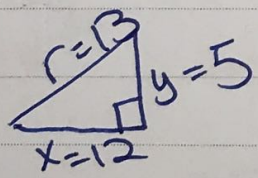
2. A boat is pulled into a dock by a rope attached to it and passing through a pulley on the dock positioned 5 meters higher than the boat. If the rope is being pulled in at a rate of 2 m/sec, how fast is the boat approaching the dock when it is 12 meters away from the dock?



K: $\frac{dr}{dt} = -2$ m/s

F: $\frac{dx}{dt}$

W: $x = 12$m $y = 5$



① Find r

$x^2 + y^2 = r^2$

$5^2 + 12^2 = r^2$

$r = 13$

② Find $\frac{dx}{dt}$

$x^2 + y^2 = r^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$

$2(12)\frac{dx}{dt} + 2(5)(0) = 2(13)(-2)$

$24\frac{dx}{dt} = -52$

$\frac{dx}{dt} = -\frac{52}{24}$

$\frac{dx}{dt} = \frac{-13}{6}$ m/s

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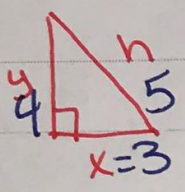
3. In a right triangle, leg x is increasing at the rate of 2 m/s while leg y is decreasing so that the area of the triangle is always equal to $6\ m^2$. How fast is the hypotenuse changing when $x = 3$ m?



K: $\frac{dx}{dt} = 2$ m/s $\quad \frac{dy}{dt} = ?$

F: $\frac{dh}{dt}$

W: $x = 3$m $\quad A = 6$m^2

$\frac{dh}{dt} = \frac{-14}{15}$ m/s

① Find y using Area

$A = \frac{1}{2}xy$

$6 = \frac{1}{2}(3)y$

$\frac{2}{3}6 = \frac{2}{3}\frac{3}{2}y$

$y = 4$

② Find h

$x^2 + y^2 = h^2$

$3^2 + 4^2 = h^2$

$h = 5$

③ Find $\frac{dy}{dt}$

$A = \frac{1}{2}xy$ ← Product: $\frac{1}{2}x$ (1st) · y (2nd)

derivative of $\frac{dA}{dt} = \frac{1}{2}x \cdot 1\frac{dy}{dt} + y \cdot \frac{1}{2}\frac{dx}{dt}$

$0 = \frac{1}{2}(3)\cdot\frac{dy}{dt} + 4\left(\frac{1}{2}\right)(2)$

$\frac{2}{3}\cdot\frac{-4}{1} = \frac{3}{2}\frac{2}{3}dy/dt$

$dy/dt = -8/3$ m/s

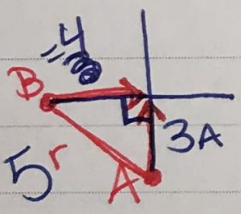
④ Find dh/dt: $x^2 + y^2 = h^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2h\frac{dh}{dt}$

$2(3)(2) + 2(4)(-8/3) = 2(5)\,dh/dt$

$dh/dt = -14/15$ m/s

4. Cars A and B are approaching each other at an intersection. Car A is approaching north at 70 km/h & Car B is approaching east at 60 km/h. What rate are the cars approaching when car A is 3 km from the intersection & Car B is 4 km from the intersection?



K: $\frac{dA}{dt} = -70$ Km/hr $\quad \frac{dB}{dt} = -60$ Km/hr

F: dr/dt

W: $A = 3$ Km $\quad B = 4$ Km

$A^2 + B^2 = r^2$

$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2r\frac{dr}{dt}$

$2(3)(-70) + 2(4)(-60) = 2(5)\frac{dr}{dt}$

$\frac{-900}{10} = \frac{10}{10}\frac{dr}{dt}$

$\frac{dr}{dt} = -90$ Km/hr