Related Rates Notes Day 1

1. Joe inflates a spherical balloon. Air is entering the balloon at a rate of $15\frac{cm^3}{sec}$. How fast is the radius changing when the radius is $10\ cm$.

K: $\frac{dV}{dt} = 15 cm^3/sec$

F: $\frac{dr}{dt}$

W: $10cm = r$

$V = \frac{4}{3}\pi r^3$ volume of a sphere

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$15 = 4\pi(10)^2 \frac{dr}{dt}$

$\frac{15}{400\pi} = \frac{\cancel{400\pi}}{\cancel{400\pi}} \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{3}{80\pi}$ cm/sec

2. A pebble is thrown into a pond forming ripples whose radius increases at the rate of 4 in/sec. How fast is the area of the ripple changing when the radius is one foot?

K: $\frac{dr}{dt} = 4 in/sec$

F: dA/dt

W: $1ft = 12in = r$

$A = \pi r^2$ area of a circle

$dA/dt = 2\pi r\, dr/dt$

$dA/dt = 2\pi(12)(4)$

$\frac{dA}{dt} = 96\pi\ in^2/sec$

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3. The radius of a circle is increasing at the rate of 2 in/sec. At what rate is the area increasing when the circumference of the circle is 12π in.?

K: $dr/dt = 2$ in/sec

F: dA/dt

W: 12π in $= C$

find r using circumference

$A = \pi r^2$

$dA/dt = 2\pi r \frac{dr}{dt}$

$dA/dt = 2\pi(6)(2)$

$\frac{dA}{dt} = 24\pi$ in²/sec

$C = 2\pi r$ Circumference of a circle

$\frac{12\pi}{2\pi} = \frac{2\pi r}{2\pi}$

6 in $= r$

4. A circular cotton doily with radius 22 cm is inadvertently thrown in the dryer and starts shrinking so that the radius is decreasing at a rate of 2 cm/min. At what rate is the area enclosed by the circle decreasing 5 minutes after the doily is thrown the dryer?

K: $dr/dt = -2$ cm/min

F: dA/dt

W: $t = 5$ min $r = 22$ cm at $t = 0$

t	r
0	22
1	20
2	18
3	16
4	14
5	12cm

$r = 12$cm after 5 min.

find radius at $t = 5$

$A = \pi r^2$

$dA/dt = 2\pi r \frac{dr}{dt}$

$dA/dt = 2\pi(12\text{cm})(-2\text{cm/min})$

$\frac{dA}{dt} = -48\pi$ cm²/min

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5. A piece of ice cut in the shape of a cube melts uniformly so that its volume decreases at 3 cm^3/sec. How fast is the surface area decreasing when the edge of the cube is 5 cm.

K: $\frac{dV}{dt} = -3\ cm^3/sec$

F: dS/dt

W: $e = 5\ cm$

rewrite for e

$V = e^3$ volume of cube

$e = \sqrt[3]{V}$

$V = (5)^3$

$V = 125$

$S = 6e^2$ surface area of cube

$S = 6\sqrt[3]{V^2} \rightarrow 6V^{2/3}$

$\frac{dS}{dt} = 4V^{-1/3}\,dV/dt$

$\frac{dS}{dt} = \frac{4}{\sqrt[3]{V}}\frac{dV}{dt}$

$\frac{dS}{dt} = \frac{4}{\sqrt[3]{125}}\frac{(-3)}{1}$

$\frac{dS}{dt} = \frac{-12}{5}\ cm^2/sec$

6. Air is escaping from a spherical balloon at the rate of 2 cm^3 per minute. How fast is the surface area shrinking when the radius is 1 cm?

K: $dV/dt = -2\ cm^3/min$

F: dS/dt

W: $r = 1\ cm$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{-2}{4\pi} = \frac{4\pi(1)^2}{4\pi}\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-1}{2\pi}$

$S = 4\pi r^2$ surface area of a sphere

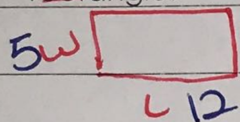
$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dS}{dt} = 8\pi(1)\left(\frac{-1}{2\pi}\right)$

$\frac{dS}{dt} = -4\ cm^2/min$

Related Rates Notes Day 1

7. The length l of a rectangle is decreasing at the rate of 2 cm/sec while the width w is increasing at the rate of 2 cm/sec. When $l = 12$ cm and $w = 5\ cm$, find the rates of change in (a) the area (b) the perimeter ~~and (c) the length of the diagonal of the rectangle~~.



$A = L \cdot W$

$A = 60\text{cm}^2$

$P = 2L + 2W$

$P = 34\text{cm}$

K: $\frac{dL}{dt} = -2\text{cm/sec}$ $\frac{dw}{dt} = 2\text{cm/sec}$

F: dA/dt and dP/dt

W: $L = 12\text{cm}$ $W = 5\text{cm}$

a) Change in area

$A = L \cdot W$

$dA/dt = L \cdot 1\frac{dw}{dt} + W \cdot 1\frac{dL}{dt}$

$dA/dt = 12(2) + 5(-2)$

$\boxed{dA/dt = 14\text{cm}^2/\text{sec}}$

b) Change in perimeter

$P = 2L + 2W$

$\frac{dP}{dt} = 2\frac{dL}{dt} + 2\frac{dw}{dt}$

$dP/dt = 2(-2) + 2(2)$

$dP/dt = 0$ Perimeter is constant