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Determinants of 2x2 & 3x3 Matrices

THE DETERMINANT OF A MATRIX

Determinant of a 2 × 2 matrix

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Determinant of a 3 × 3 matrix

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} =$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}\begin{matrix} a & b \\ d & e \\ g & h \end{matrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

- used in the inverse formula
- you can only find the determinants of square matrices
- represented as "det A" or |A| (you need to remember those bars mean determinant & not absolute value)

Find the determinant, if possible.

1. $\det\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

$\det = ad - bc$

$(1)(5) - (3)(2)$

$5 - 6$

-1

2. $\begin{vmatrix} 7 & 2 \\ 2 & 3 \end{vmatrix}$ ← means det

$(7)(3) - (2)(2)$

$21 - 4$

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3. $\begin{vmatrix} 2 & -1 & 0 \\ 3 & 4 & 1 \end{vmatrix}$

no det bc it's not a square

4. $\begin{vmatrix} 1 & 2 & 3 \\ 0 & -4 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$\begin{matrix} 1 & 2 & 3 & 1 & 2 \\ 0 & -4 & 1 & 0 & -4 \\ 0 & 3 & -1 & 0 & 3 \end{matrix}$

$0 + 3 + 0 = 3$

$4 + 0 + 0 = 4$

down − up

$4 - 3 = 1$

1. Rewrite 1st 2 columns behind the matrix
2. Multiply along the 3 down diagonals & add
3. Multiply along the up 3 diagonals & add
4. Subtract: down sum - up sum

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Inverses of 2x2 Matrices

THE INVERSE OF A 2 × 2 MATRIX

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

provided det ≠ 0.

*Switch a & d

*Change signs of b & c

* You can't divide matrices, so you need to multiply by the inverse

* Represented as A^{-1} or B^{-1} means inverse

* Must be a square matrix

* The inverse is undefined when the det=0

Find the inverse if it's defined.

1. $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

$\det = ad - bc$

$6 - 4$

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$A^{-1} = \frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$ switch a+d change signs of b+c

$A^{-1} = \begin{bmatrix} 2/2 & -1/2 \\ -4/2 & 3/2 \end{bmatrix}$ distribute + reduce fractions

$A^{-1} = \begin{bmatrix} 1 & -1/2 \\ -2 & 3/2 \end{bmatrix}$

2. $B = \begin{bmatrix} 4 & -3 \\ 6 & 0 \end{bmatrix}$

$\det = 0 - -18$

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$B^{-1} = \frac{1}{18}\begin{bmatrix} 0 & 3 \\ -6 & 4 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} 0/18 & 3/18 \\ -6/18 & 4/18 \end{bmatrix} = \begin{bmatrix} 0 & 1/6 \\ -1/3 & 2/9 \end{bmatrix}$

3. $C = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$

$\det = 12 - 12$

0

$C^{-1} = \frac{1}{0}$ can't ÷ by 0

no inverse bc the det = 0

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Solving Matrix Equations

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$\frac{1}{3}\cdot 3x = 6\cdot\frac{1}{3}$

Solve for Matrix X

$A \bullet X = B$

$X = A^{-1} \bullet B$

Since you can't divide matrices, you have to multiply by the inverse!

$$1.\ \underset{A}{\begin{bmatrix} 3 & -5 \\ 2 & 1 \end{bmatrix}} X = \underset{B}{\begin{bmatrix} 0 & -2 \\ 4 & -1 \end{bmatrix}}$$

det

$3 - -10$

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$X = A^{-1} \cdot B$

$$X = \frac{1}{13}\underset{2\times 2}{\begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}}\underset{2\times 2}{\begin{bmatrix} 0 & -2 \\ 4 & -1 \end{bmatrix}}$$

$$X = \frac{1}{13}\begin{bmatrix} 20 & -7 \\ 12 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 20/13 & -7/13 \\ 12/13 & 1/13 \end{bmatrix}$$

*The inverse must be in front

*Multiply matrices 1st!

Then distribute scalar #

*Hint: When you distribute the scalar #, the determinant becomes the denominator. Reduce fractions if possible.

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IDENTITY MATRIX

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- The identity matrix is like multiplying by 1. It leaves the original matrix unchanged.
- It must be a square matrix.
- Ones in main diagonal & zeros everywhere else.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fill in the missing matrix to make the statement true.

$$1.\ \underset{2\times 3}{\begin{bmatrix} 3 & -2 & 1 \\ 7 & 0 & -5 \end{bmatrix}} \cdot \underset{3\times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \underset{2\times 3}{\begin{bmatrix} 3 & -2 & 1 \\ 7 & 0 & -5 \end{bmatrix}}$$

$$2.\ \underset{4\times 2}{\begin{bmatrix} 0 & 2 \\ -3 & 8 \\ 10 & 3 \\ 5 & 6 \end{bmatrix}} \cdot \underset{2\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 0 & 2 \\ -3 & 8 \\ 10 & 3 \\ 5 & 6 \end{bmatrix}$$

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What are the dimensions of M if ...

1) $A_{2\times 7} \cdot M_{7 \times 3} = B_{2\times 3}$

$\sqrt{}$ $\underline{7} \times \underline{3}$

2) $M_{5 \times 8} \cdot A_{8\times 2} = B_{5\times 2}$

$\underline{5} \times \underline{8}$

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