

SOLVE RADICAL EQUATIONS

Radical on 1 Side

Radicals on Both Sides

Extraneous Solutions

To solve radical equations:

1. Isolate the radical on 1 side of the equation *Get radical by itself*
2. Square both sides of the equation to cancel the square root *or cube both sides to cancel cube root*
3. Finish solving the equation for the variable. *etc.*

EXAMPLE 1:

$$\begin{array}{r} \sqrt[3]{x} + 3 = 10 \\ -3 \quad -3 \\ \hline (\sqrt[3]{x})^3 = (7)^3 \\ x = 343 \end{array}$$

EXAMPLE 2:

$$\begin{array}{r} -3\sqrt{x} = -18 \\ -3 \quad -3 \\ \hline (\sqrt{x})^2 = (6)^2 \\ x = 36 \end{array}$$

check:
 $-3\sqrt{36} = -18$
 $-3 \cdot 6 = -18$
 $-18 = -18 \checkmark$

Example 3:

$$\begin{array}{r} 6\sqrt{\frac{x}{3}} = 12 \\ \div 6 \quad \div 6 \\ \hline (\sqrt{\frac{x}{3}})^2 = (2)^2 \\ 3 \cdot \frac{x}{3} = 32 \cdot 3 \\ x = 96 \end{array}$$

check:
 $6 \cdot \sqrt{\frac{96}{3}} = 12$
 $6 \cdot \sqrt{32} =$
 $6 \cdot 2 =$
 $12 = 12 \checkmark$

Example 4:

$$\begin{array}{r} 2\sqrt[4]{x} - 5 = -1 \\ +5 \quad +5 \\ \hline \frac{2\sqrt[4]{x}}{2} = \frac{4}{2} \\ (\sqrt[4]{x})^4 = (2)^4 \\ x = 16 \end{array}$$

check:
 $2\sqrt[4]{16} - 5 = -1$
 $2(2) - 5 =$
 $4 - 5 =$
 $-1 = -1 \checkmark$

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Example 5:

$$(\sqrt[4]{x+4})^4 = (\sqrt[4]{2x-1})^4$$

$$x+4 = 2x-1$$

$$\begin{array}{r} -2x \\ -2x \end{array}$$

$$\begin{array}{r} -x+4 = -1 \\ -4 \quad -4 \end{array}$$

$$\begin{array}{r} -x = -5 \\ -1 \quad -1 \end{array}$$

$$x = 5$$

radical = radical

① raise each side to the power that cancels the root

② then solve the equation

check:

$$\sqrt[4]{5+4} = \sqrt[4]{2(5)-1}$$

$$\sqrt[4]{9} = \sqrt[4]{9} \checkmark$$

example 6:

$$(\sqrt[4]{3x+8})^4 = (\sqrt[4]{x+4})^4$$

$$3x+8 = x+4$$

$$\begin{array}{r} -x \\ -x \end{array}$$

$$2x+8 = 4$$

$$\begin{array}{r} -8 \quad -8 \end{array}$$

$$\begin{array}{r} 2x = -4 \\ 2 \quad 2 \end{array}$$

$$x = -2$$

$$(\sqrt[4]{x})^4 = x$$

$$(x^{1/4})^4 = x$$

check:

$$\sqrt[4]{3(-2)+8} = \sqrt[4]{-2+4}$$

$$\sqrt[4]{2} = \sqrt[4]{2} \checkmark$$

Radicals on Both Sides

Extraneous Solutions

EXAMPLE 7: Solve the equation. Check for extraneous solutions.

$$(x)^2 = (\sqrt{42-x})^2$$

Square both sides to cancel sq. rt.
If quadratic, set it = 0

$$x^2 = 42 - x$$

$$+x - 42 - 42 + x$$

$$x^2 + x - 42 = 0$$

Solve by factoring or quadratic formula

$$(x-6)(x+7) = 0$$

$$\begin{array}{r} -42 \\ -6 \times 7 \\ 1 \end{array}$$

$$x-6=0 \quad x+7=0$$

$$x=6 \quad x \neq -7$$

check for extraneous solutions
 $\sqrt{\quad} = +\#$

$$6 = \sqrt{42-6} \quad -7 = \sqrt{42--7}$$

$$6 = \sqrt{36} \quad -7 = \sqrt{49}$$

$$6 = 6 \quad \checkmark$$

$$-7 \neq 7$$

-7 is extraneous

$$(\sqrt{2-x})^2 = (x+4)^2$$

$$2-x = (x+4)(x+4)$$

$$2-x = x^2 + 4x + 4x + 16$$

$$\begin{array}{r} 2-x = x^2 + 8x + 16 \\ -2+x \quad \quad \quad +x \quad -2 \\ \hline 0 = x^2 + 9x + 14 \end{array}$$

$$0 = x^2 + 9x + 14$$

$$0 = (x+7)(x+2)$$

$$x+7=0 \quad x+2=0$$

$$x \neq -7$$

$$x = -2$$

extraneous

$$(a+b)^2 = (a+b)(a+b)$$

$$\text{check: } x = -7$$

$$\sqrt{2--7} = -7+4$$

$$\sqrt{9} =$$

$$3 \neq -3$$

$$\text{check: } x = -2$$

$$\sqrt{2--2} = -2+4$$

$$\sqrt{4} = 2$$

$$2 = 2 \checkmark$$

Extraneous Solutions