(HA) (SA)

Finding Horizontal & Slant Asymptotes of Rational Functions

Degree of numerator $<$ Degree of denominator

less than

Ex. $f(x) = \frac{x}{x^2-1}$ ← deg 1 ← deg 2

HA is always $y=0$

Degree of Numerator $=$ Degree of Denominator

Ex. $f(x) = \frac{x^2+3x+2}{2x^2+4x}$ degree 2

$y=$ ratio of leading coefficients

Degree of Numerator $>$ Degree of Denominator

greater than

Ex. $f(x) = \frac{3x^2+4x+1}{x-2}$ deg 2 deg 1

No HA! but could have a Slant Asym.

When the degree in the numerator is less than the degree in the denominator, the horizontal asymptote is ALWAYS $y=0$.

* Write as y = 0 not just 0.

When the degree in the numerator is equal to the degree in the denominator, the horizontal asymptote is a ratio of leading coefficients.

* Looks like y = #

If degree in the numerator is greater than the degree in the denominator, there is no horizontal asymptote, but there may be a slant asymptote.

* Use synthetic ÷ to find the slant asymptote when the numerator's degree is exactly 1 more than the denominator's degree.

SA: $y=mx+b$ form

1. $f(x) = \frac{x^{2}+2x}{x^3+x^2-x+1}$

num. deg 2 < denom. deg 3

Horizontal Asymptote : $y=0$

2. $f(x) = \frac{3x^{1}+5}{4x^{1}-1}$ degrees are same (equal)

HA : $y=\frac{3}{4}$

3. $f(x) = \frac{1x^{2}-5x+3}{3x^{2}-2x+1}$ same deg

HA : $y=\frac{1}{3}$

4. $f(x) = \frac{x^2-x+1}{x^1-5}$ $x-5=0$, $x=5$

No HA but we can divide to find SA

$$\begin{array}{r|rrr} 5 & 1x^2 & -1 & 1 \\ & \downarrow & 5 & 20 \\ \hline & 1 & 4 & : \cancel{21} \end{array}$$

*Ignore remainder

SA: $y=1x+4$ or $y=x+4$

5. $f(x) = \frac{x^2-3x+10}{x+1}$ $x+1=0$, $x=-1$

$$\begin{array}{r|rrr} -1 & 1x^2 & -3 & 10 \\ & \downarrow & -1 & 4 \\ \hline & 1x & -4 & : \cancel{14} \end{array}$$

No HA

SA : $y=x-4$