Steps for finding all ~~rational~~ roots/zeros:

1. Type polynomial into a graphing calculator in [Y=]
2. Look at table to see where the y-values = 0. The x-value is a root of the function
3. If you get all solutions from the table, then you are done.
4. If not, you must use a root or roots for synthetic division.
5. If the calculator does not give you any roots for synthetic division, you must type your possible rational roots into the table in the calculator. P/Q
6. Once you have roots to begin with, do synthetic division until you get a quadratic equation.
7. Now factor, use the square root method, or the quadratic formula to solve the quadratic equation.

Examples:

1. $f(x) = \frac{P}{Q}x^3 - 2x^2 - 11x + 12$

P: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

Q: ±1

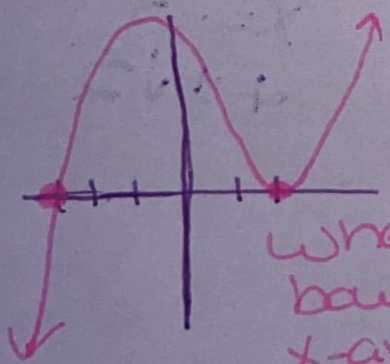
P/Q = $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

Calc:

$-3, 1, +4$

2. $f(x) = \frac{P}{Q}x^3 - x^2 - 8x + 12$

same P/Q list as #1



when the graph bounces off the x-axis, that is a root twice (multiplicity of 2)

Calc:

$x = -3, +2$ twice

or

$x = -3, 2, 2$

3. $f(x) = 6x^3 + x^2 - 10x + 3$ (Q, P)

P: $\pm 1, \pm 3$

Q: $\pm 1, \pm 2, \pm 3, \pm 6$

P/q: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm 3, \pm\frac{3}{2}, \pm\frac{3}{3}, \pm\frac{3}{6}$ already in list

Calc: $x = 1, \frac{1}{3}, -\frac{3}{2}$

4. $f(x) = x^3 + 3x^2 - 2x - 6$ P/q $= \pm 1, \pm 2, \pm 3, \pm 6$

Calc: $x = -3$

Synth ÷

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -2 & -6 \\ & \downarrow & -3 & 0 & 6 \\ \hline & 1 & 0 & -2 & :0 \end{array}$$ ✓

$x^2 - 2 = 0$

Solve the quadratic eq.

$x^2 - 2 = 0$

$\sqrt{x^2} = \sqrt{2}$

$x = \pm\sqrt{2}$

Roots

$x = -3$ & $\pm\sqrt{2}$

5. $f(x) = 2x^3 - 3x^2 - x + 1$ (Q, P) P: ± 1 Q: $\pm 2, \pm 1$ P/q: $\pm 1, \pm\frac{1}{2}$

Calc: $x = \frac{1}{2}$

Zeros: $\frac{1}{2}$ & $\frac{1 \pm \sqrt{5}}{2}$

Synth ÷

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -3 & -1 & 1 \\ & \downarrow & 1 & -1 & -1 \\ \hline & 2 & -2 & -2 & :0 \end{array}$$

$\frac{2x^2}{2} - \frac{2x}{2} - \frac{2}{2} = \frac{0}{2}$ or $x^2 - x - 1 = 0$

Quad. Formula $a=1$, $b=-1$, $c=-1$

$$\frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$\frac{1 \pm \sqrt{5}}{2}$$

6. $f(x) = x^4 - 3x^3 - 20x^2 - 24x - 8$

★Degree 4 so you need to do synth. twice to get to x^2

Calc: $x = -2, -1$

$$\begin{array}{r|rrrrr} x^4 \downarrow \quad -2 & 1 & -3 & -20 & -24 & -8 \\ & \downarrow & -2 & 10 & 20 & 8 \\ \hline x^3 \downarrow \quad -1 & 1 & -5 & -10 & -4 & :0 \\ & \downarrow & -1 & 6 & 4 & \\ \hline x^2 & 1 & -6 & -4 & :0 & \end{array}$$

$x^2 - 6x - 4 = 0$

$a = 1$ $b = -6$ $c = -4$

$$\frac{6 \pm \sqrt{(-6)^2 - 4(1)(-4)}}{2(1)}$$

$$\frac{6 \pm \sqrt{52}}{2} \leftarrow \sqrt{4 \cdot 13} = \frac{6 \pm 2\sqrt{13}}{2} = \frac{3 \pm \sqrt{13}}{1}$$

Roots: $-2, -1, 3 \pm \sqrt{13}$