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Polynomial Division

If given ONE factor
Ex: 1. Find the other factors

Given One Factor or Root...

Find the Other Factors or Roots

Remember that roots, zeros, x-
intercepts & solutions are synonyms!

If given ONE factor:

Ex. 1: Find the other factors.

$f(x) = 2x^3 - 14x^2 - 56x - 40$; $x - 10$ is a factor

$$\begin{array}{r|rrrr} 10 & 2 & -14 & -56 & -40 \\ & \downarrow & 20 & 60 & 40 \\ \hline & 2 & 6 & 4 & :0 \end{array}$$

$2x^2 + 6x + 4$
 $2(x^2 + 3x + 2)$
 $2(x+2)(x+1)$

- ① Divide with synthetic \div (You should get a remainder of 0)
- ② Factor quotient completely to get other factors

If given ONE factor:

Ex. 2: Find all roots.

$f(x) = 15x^3 + 14x^2 - 3x - 2$; $x + 1$ is a factor

$x + 1 = 0$
 $x = -1$ outside of synth. \div

$$\begin{array}{r|rrrr} -1 & 15 & 14 & -3 & -2 \\ & \downarrow & -15 & 1 & 2 \\ \hline & 15 & -1 & -2 & :0 \end{array}$$

Use synth. \div to get to degree 2 (quadratic)
 Then solve by factoring, square root method, quadratic formula or completing the square.

$15x^2 - x - 2 = 0$
 $(15x^2 - 6x)(5x - 2) = 0$
 $3x(5x - 2) + 1(5x - 2) = 0$
 $(3x + 1)(5x - 2) = 0$
 $3x + 1 = 0 \quad 5x - 2 = 0$

$x = -1 \quad x = -\frac{1}{3} \quad x = \frac{2}{5}$

If given ONE root:

Ex. 3: Find the other roots.

$f(x) = 6x^3 + x^2 - 10x + 3$; 1 is a root (or zero)
 $x = 1$

$$\begin{array}{r|rrrr} 1 & 6 & 1 & -10 & 3 \\ & \downarrow & 6 & 7 & -3 \\ \hline & 6 & 7 & -3 & :0 \end{array}$$

$6x^2 + 7x - 3 = 0$
 $6x^2 + 9x - 2x - 3 = 0$
 $3x(2x + 3) - 1(2x + 3) = 0$
 $(3x - 1)(2x + 3) = 0$

Factor or use quadratic formula to solve

$3x - 1 = 0 \quad 2x + 3 = 0$
 $x = \frac{1}{3} \quad x = -\frac{3}{2}$

$x = 1$ given

If given TWO zeros:

Ex. 4: Find the other zeros.

$f(x) = 2x^4 - 9x^3 + 4x^2 + 21x - 18$; -2 & 3 are

zeros Since this is degree 4, you have to divide twice to get to degree 2.

$$\begin{array}{r|rrrrr} x^4 & 2 & -9 & 4 & 21 & -18 \\ x^4 & \downarrow & 4 & -10 & -12 & 18 \\ \hline x^3 & 2 & -5 & -6 & 9 & :0 \checkmark \\ x^3 & \downarrow & 6 & 3 & -9 & \\ \hline x^2 & 2 & 1 & -3 & :0 \checkmark \\ & 2 & 1 & -3 & & \\ \hline & 2x^2 + x - 3 = 0 \end{array}$$

Factor or use quad. formula to solve.

$\frac{-1 \pm \sqrt{(1)^2 - 4(2)(-3)}}{2(2)} = \frac{-1 \pm \sqrt{25}}{4} = \frac{-1 \pm 5}{4}$
 $\frac{-1+5}{4} = 1$
 $\frac{-1-5}{4} = -\frac{3}{2}$

$x = 1, -\frac{3}{2}, 2, 3$