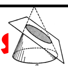
Ellipses 

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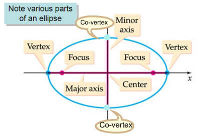
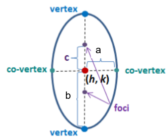
Graph of an Ellipse



or



$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Center (h, k) *remember it is the opposite sign that you see

Vertices: a tells you how many places to move left & right from center

b tells you how many places to move up & down from center

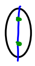
Major Axis: the longer axis (line between vertices)

Minor Axis: the shorter axis (line between co-vertices)

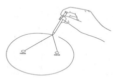
Foci: 2 focus points on the major axis that make up an ellipse

- c tells you how many places to move up & down from center
- To get the foci, you need to find c using $c^2 = a^2 - b^2$ or $c^2 = b^2 - a^2$ (larger denominator 1st)
- Then add c to the coordinate of the center that has the larger denominator.

$(h \pm c, k)$ if the major axis is horizontal 

$(h, k \pm c)$ if the major axis is vertical 

Definition demonstrated by using two tacks and a length of string to draw an ellipse



Jan 24-9:08 AM

Graph & label important features (center, vertices, & foci).

1. $\frac{(x-5)^2}{25} + \frac{y^2}{4} = 1$

$a^2 \rightarrow 25$ $\quad 4 \leftarrow b^2$

h k

Center: (5, 0)

$a^2 = 25$
$a = 5$ left/right

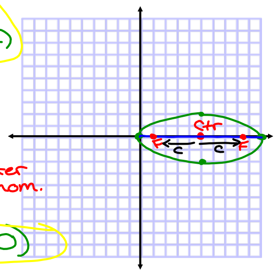
$b^2 = 4$
$b = 2$ up/down

Vertices: (0,0) (10,0)
co-vertices: (5,2) (5,-2)

$c^2 = a^2 - b^2$
$c^2 = 25 - 4$
$\sqrt{c^2} = \pm\sqrt{21}$
$c = \pm\sqrt{21}$

add to h of the center bc it has the larger denom. (horiz. major axis)

foci: $(5 \pm \sqrt{21}, 0)$



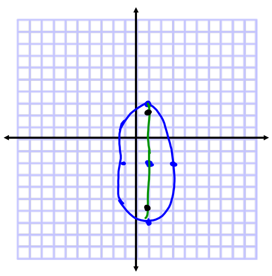
2. $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{25} = 1$

$a^2 = 4$
$a = 2$ L/R

$b^2 = 25$
$b = 5$ ↑/↓

center: (1, -2)
co-vertices: (3, -2) (-1, -2)
vertices: (1, 3) (1, -7)
foci: $(1, -2 \pm \sqrt{21})$

$c^2 = b^2 - a^2$
$c^2 = 25 - 4$
$c^2 = 21$
$c = \pm\sqrt{21}$



Oct 22-10:38 AM

3. $9x^2 + 4y^2 - 18x + 16y - 11 = 0$

$(9x^2 - 18x\_\_) + (4y^2 + 16y\_\_) = 11\_\_\ \_\_$

$9(x^2 - 2x \underline{+1}) + 4(y^2 + 4y \underline{+4}) = 11 \underline{+9} + \underline{16}$

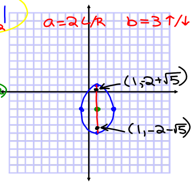
$\left(\frac{b}{2}\right)^2 \quad \left(\frac{-2}{2}\right)^2 = (-1)^2 \quad \left(\frac{4}{2}\right)^2 = (2)^2$

reduce fractions $\frac{9(x-1)^2}{36} + \frac{4(y+2)^2}{36} = \frac{36}{36}$ should $= 1$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$

$a^2 \rightarrow 4 \quad 9 \leftarrow b^2$

$a = 2$ L/R $\quad b = 3$ ↑/↓



center $(1, -2)$

vertices: $(1, 1)$ $(1, -5)$

co-vertices: $(2, -2)$ $(-1, -2)$

foci: $(1, -2 \pm \sqrt{5})$

$c^2 = 9 - 4$

$c^2 = 5$

$c = \pm\sqrt{5}$

4. $16x^2 + 65y^2 + 130y - 975 = 0$

$(16x^2) + (65y^2 + 130y\_\_) = 975\_\_$

$16x^2 + 65(y^2 + 2y \underline{+1}) = 975 \underline{+65}$

$\left(\frac{2}{2}\right)^2 = (1)^2$

$\frac{16x^2}{1040} + \frac{65(y+1)^2}{1040} = \frac{1040}{1040}$

$\frac{x^2}{65} + \frac{(y+1)^2}{16} = 1$

ctr: $(0, -1)$ $\quad a = \sqrt{65} \quad b = 4$

vertices: $(0+\sqrt{65}, -1)$ $(0-\sqrt{65}, -1)$

co-vertices: $(0, 3)$ $(0, -5)$

foci: $(0 \pm 7, -1)$

$(7, -1)$ + $(-7, -1)$

$c^2 = 65 - 16$

$c^2 = 49$

$c = \pm 7$

5. Write the equation of the ellipse with vertices at (2, -2) (-2, -2) (0, 3) (0, -7)

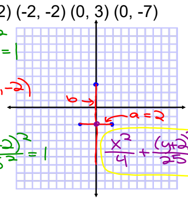
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$(h, k) \rightarrow (0, -2)$

$a = 2$

$b = 5$

$\frac{(x-0)^2}{2^2} + \frac{(y--2)^2}{5^2} = 1$



$\frac{x^2}{4} + \frac{(y+2)^2}{25} = 1$

Oct 22-11:11 AM