

Intro to Conic Sections

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Conic Sections

A conic section is formed by the intersection of a plane with a right circular cone. The "kind" of curve produced is determined by the angle at which the plane intersects the surface.

Diagonal Slice	Horizontal Slice	Deep Vertical Slice	Vertical Slice
Ellipse	Circle	Hyperbola	Parabola

$(x - h)^2 + (y - k)^2 = r^2$	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} \geq 1$

$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$	$(y - k) = 4p(x - h)^2$

Circles

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$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{where } (h, k) \text{ is the center & } r \text{ is the radius}$$

→ opposite sign of what you see
in the equation

Graph

$$1. x^2 + y^2 = 9$$

$$(x-0)^2 + (y-0)^2 = 9$$

center $(0, 0)$

$$\text{radius: } r^2 = 9$$

$$2. (x+3)^2 + (y-2)^2 = 20$$

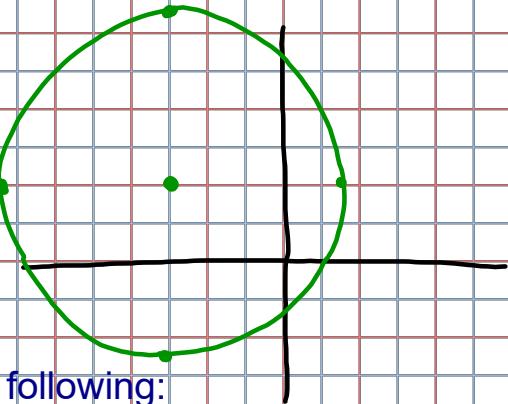
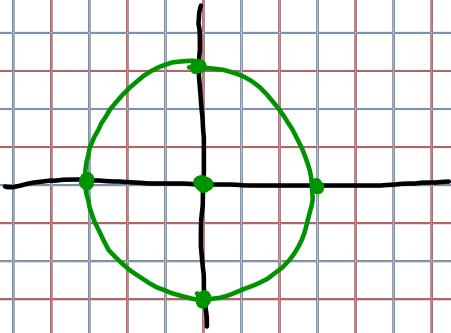
$$h \quad k \quad r^2$$

center $(-3, 2)$

$$\text{radius: } \sqrt{r^2} = \sqrt{20}$$

$$\sqrt{4} = 4 \quad \frac{20}{4} = 5 \quad r = \sqrt{20} \approx 4.5$$

$$\sqrt{25} = 5 \quad r = 2\sqrt{5}$$



Write the equation of the circle given the following:

$$1. \text{center: } (-6, 2) \text{ & radius: } 3\sqrt{2}$$

• plug in $h, k, \text{ & } r$

• simplify

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-(-6))^2 + (y-2)^2 = (3\sqrt{2})^2$$

$$(x+6)^2 + (y-2)^2 = 18$$

$$2. \text{center: } (4, -3) \text{ & a point on the circle: } (-1, 3)$$

• find radius using formula

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(-1-4)^2 + (3-(-3))^2 = r^2$$

$$(-5)^2 + (6)^2 = r^2$$

$$25 + 36 = r^2$$

$$61 = r^2$$

$$\text{or } \sqrt{61} = r$$

• write eq. using (h, k) & r^2

$$(x-4)^2 + (y+3)^2 = 61$$

To rewrite an equation to standard form, you must complete the square.

- move the constant (c) to the other side & group x's & y's
- complete the square $(\frac{b}{2})^2$ & add to both sides
- factor to a binomial squared $(x \pm \frac{b}{2})^2$

Write in standard form. Then state center & radius.

1. $x^2 + y^2 - 8x - 6y + 21 = 0$

$$(x^2 - 8x + 16) + (y^2 - 6y + 9) = -21 + 16 + 9$$

$$(\frac{b}{2})^2 \quad (\frac{-8}{2})^2 = (-4)^2 \quad (-6/2)^2 = (-3)^2$$

$$(x - 4)^2 + (y - 3)^2 = 4$$

ctr: (4, 3) r = 2

2. $2x^2 - 4x + 2y^2 + 12y - 12 = 0$

$$(2x^2 - 4x -) + (2y^2 + 12y -) = 12 - -$$

$$2(x^2 - 2x + 1) + 2(y^2 + 6y + 9) = 12 + 2 + 18$$

$$(-2/2)^2 = (-1)^2 \quad (6/2)^2 = (3)^2$$

$$\frac{2(x-1)^2}{2} + \frac{2(y+3)^2}{2} = \frac{32}{2}$$

$$(x-1)^2 + (y+3)^2 = 16$$

ctr: (1, -3) r = 4

* must factor out leading coefficients before $(\frac{b}{2})^2$

* divide by LC