

13

Solving Systems of Equations with Matrices

1. Write the system of equations as a matrix equation.
 [Coefficient Matrix] • [Variable Matrix] = [Constant Matrix]

Always a square matrix Always a column matrix

2. Then solve the matrix equation.

Solve by hand.

1. $-3x + 4y = 5$
 $2x - y = -10$

Coef matrix • Var matrix = Constant matrix

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$A \cdot X = B$
 $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} -1 & 4 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} -5 + 40 \\ -10 + 30 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} 35 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ -4 \end{bmatrix}$$

or $(-7, -4)$

2. $x + y = 2$
 $7x + 8y = 21$

$$\begin{bmatrix} 1 & 1 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 21 \end{bmatrix}$$

$A \cdot X = B$
 $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} 8 & -1 \\ -7 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} 16 - 21 \\ -14 + 21 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$$

or $(-5, 7)$

Solve using a graphing calculator.

3. $2x + y = 3$
 $x - 2y + z = 5$
 $-4x + 5y - 3z = 6$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 1 \\ -4 & 5 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}$$

$A \cdot X = B$
 $X = A^{-1}B$

calc command!

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24 \\ 45 \\ -109 \end{bmatrix}$$

or $(24, 45, -109)$

Aug 3-9:01 AM

Area of a Triangle with Matrices

14

Area of a Triangle

The area of a triangle with vertices at (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) is given by:

$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

.5 det [3x3]

- * Remember the bars mean determinant
- * Area is ALWAYS POSITIVE!

Find the area of the triangle.

1. $(1, 7)$, $(5, 8)$, $(7, -4)$

$$A = .5 \begin{vmatrix} 1 & 7 & 1 \\ 5 & 8 & 1 \\ 7 & -4 & 1 \end{vmatrix}$$

enter in calc

★ Calc command
.5 det A

$A = -25$ but must be positive so
 $A = 25$

2. Vertices: $(-2, 9)$; $(10, -5)$; $(3, 0)$

$$.5 \det \begin{bmatrix} -2 & 9 & 1 \\ 10 & -5 & 1 \\ 3 & 0 & 1 \end{bmatrix}$$

Aug 3-10:42 AM