

## Law of Cosines

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Use for triangles when you are given:

- **SSS** - Always find largest angle 1st with law of cosines, then middle angle with law of sines & then the smallest angle
- **SAS** - Use law of cosines to find the missing side, then use law of sines to find the angles (find smaller angle 1st)



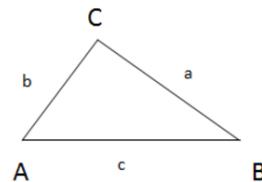
## Law of the Cosines

Law of Cosines:

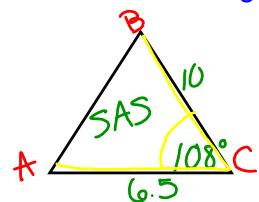
$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos(A)$$

$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos(B)$$

$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos(C)$$



1. Solve the triangle given  $C=108^\circ$ ,  $a=10$ ,  $b=6.5$



$$\begin{aligned} a &= 10 & A &= 44.7^\circ \\ b &= 6.5 & B &= 27.3^\circ \\ c &= 13.5 & C &= 108^\circ \end{aligned}$$

## ① Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = (10)^2 + (6.5)^2 - 2(10)(6.5) \cos 108^\circ$$

$$c = 13.5$$

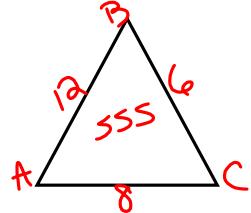
② Law of Sines to find  $\angle B$ 

$$\begin{aligned} \frac{c}{\sin C} &= \frac{b}{\sin B} \\ \frac{13.5}{\sin 108^\circ} &\propto \frac{6.5}{\sin B} \\ 6.5 \sin 108^\circ &= \frac{13.5 \sin B}{13.5} \\ B &= \sin^{-1}(\text{ANS}) \\ B &= 27.3^\circ \end{aligned}$$

③ Subtract 2 angles from  $180^\circ$ 

$$\begin{aligned} A &= 180^\circ - 27.3^\circ - 108^\circ \\ A &= 44.7^\circ \end{aligned}$$

2. Solve the triangle given  $a = 6$ ,  $b = 8$ ,  $c = 12$



$$\begin{aligned} a &= 6 \\ b &= 8 \\ c &= 12 \end{aligned}$$

$$\begin{aligned} A &= 26.4^\circ && 3rd \\ B &= 36.3^\circ && \text{and} \\ C &= 117^\circ && 1st \end{aligned}$$

① Find largest angle using LOC

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$12^2 = 6^2 + 8^2 - 2(6)(8) \cos C$$

$$\begin{aligned} 144 &= 36 + 64 - 96 \cos C \\ 144 &= 100 - 96 \cos C \end{aligned}$$

$$\frac{44}{-96} = \frac{-96 \cos C}{-96}$$

$$C = \cos^{-1}(\text{Ans})$$

$$C = 117.3^\circ$$

② LOS to find middle angle

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

③ Subtract 2 angles from  $180^\circ$  to find smallest  $\angle$ .

$$\begin{aligned} \frac{12}{\sin 117.3^\circ} &= \frac{8}{\sin B} \\ \sin^{-1}\left(\frac{8 \sin 117.3^\circ}{12}\right) &= B \\ B &= 36.3^\circ \end{aligned}$$

$$A = 180^\circ - 117.3^\circ - 36.3^\circ$$

$$A = 26.4^\circ$$

3. Solve the triangle given  $a = 9$ ,  $b = 3$ ,  $c = 11$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$11^2 = 9^2 + 3^2 - 2(9)(3) \cos C$$

$$121 = 90 - 54 \cos C$$

$$\frac{121}{54} = \frac{-90}{-54} \cos C$$

$$121 = 90 - 54x$$

$$C = 125^\circ$$