INTERPRETING GRAPHS USING DERIVATIVES

Honors Calculus

Keeper 23

INTERCEPTS

- X-intercept(s): plug in a 0 for y and solve for x
- Y-intercept: plug in a 0 for x and solve for y

CRITICAL POINTS

We say that x = c is a critical point of the function $f(x)$ if $f(c)$ exists and if either of the following are true:

Set $f' = 0$
& solve
for x

$$f'(c) = 0 \text{ or } f'(c) = \text{DNE}$$

***If a point is not in the domain of the function then it is not a critical point.

INTERVALS OF INCREASE AND DECREASE

*Make an f' line using the critical points & undefined values of f. Then plug in numbers to $f'(x)$ to find out + or –

+ + 0 − −

−2

Intervals of Increase: Intervals where $f'(x)$ is positive

$(-\infty, -2)$

Intervals of Decrease: Intervals where $f'(x)$ is negative

$(-2, \infty)$

RELATIVE/LOCAL EXTREMA: MAX AND MIN

A local max occurs when $f'(x)$ changes from + to –

peak

A local min occurs when $f'(x)$ changes from – to +

valley

*To find the Y-value, plug into f(x) original

CONCAVITY

$- \; - \; 0 \; + \; + \; 0 \; - \; -$

$2 \quad 5$

Concave Up – Occurs on the interval where $f''(x)$ is positive

$(2, 5)$

Concave Down – Occurs on the interval where $f''(x)$ is negative

$(-\infty, 2) \cup (5, \infty)$

POINTS OF INFLECTION (POI)

A POSSIBLE point of inflection occurs where $f''(x) = 0$ or DNE but is in the domain of f

f'' + + 0 – –

∪ -2 ∩

The actual point of inflection occurs when $f''(x)$ changes signs. where concavity changes

Plug into f(x) (original) to get y-coordinate

1. $f(x) = -x^2 - 4x$ $\quad y = -0^2 - 4(0)$

x-int $(0,0)$ $(-4,0)$

y-int $(0,0)$

p. 24 notes

VA none

HA none

Critical Points $x = -2$

Increases $(-\infty, -2)$

Decrease $(-2, \infty)$

Concave Up none

Concave Down $(-\infty, \infty)$

Relative Maxima $(-2, 4)$

Relative Minimum none

Point of Inflection none

$0 = -x^2 - 4x$

$0 = -x(x+4)$

$-x = 0 \qquad x+4 = 0$

$x = 0 \qquad x = -4$

$f'(x) = -2x - 4$

$0 = -2x - 4$

$4 = -2x$

$x = -2$ ← critical pt

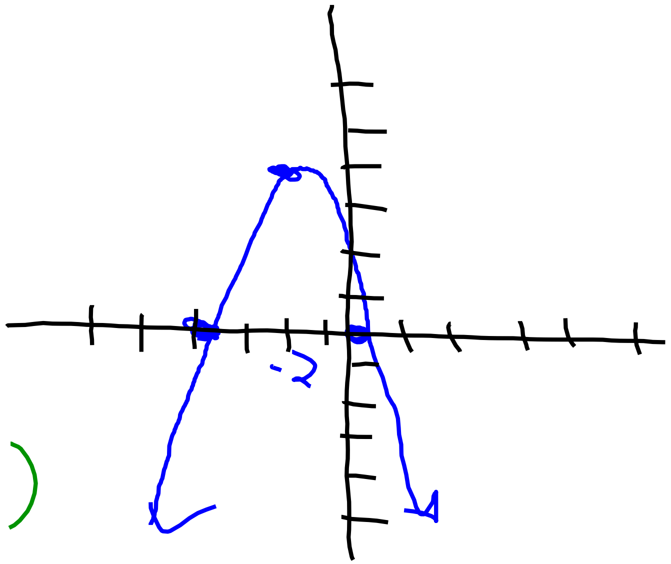
f': $-\infty$ $\quad + \; + \; + \; 0 \; - \; - \; -$ $\quad +\infty$

$-4, \quad -2 \quad 0$

max

$f(-2) = -(-2)^2 - 4(-2)$

$f(-2) = 4$



$f''(x) = -2$

f'' $\quad - \; - \; - \; -$

2. $f(x) = -\dfrac{1x^0}{x-3}$

$0 = \dfrac{-1}{x-3}$

$-1 \neq 0$

x-int none

y-int $(0, 1/3)$

$y = \dfrac{-1}{0-3}$

P.26

VA $x = 3$

$x - 3 = 0$

$x = 3$

HA $y = 0$

Critical Points $x = 3$

Increases $(-\infty, 3) \cup (3, \infty)$

Decrease none

Concave Up $(-\infty, 3)$

Concave Down $(3, \infty)$

Relative Maxima none

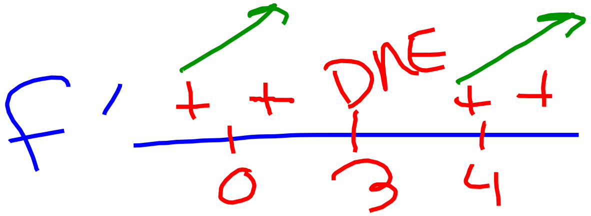
Relative Minimum none

Point of Inflection none

$f(x) = -1(x-3)^{-1}$

$f'(x) = 1(x-3)^{-2}$

$f'(x) = \dfrac{1}{(x-3)^2}$



$0 = \dfrac{1}{(x-3)^2}$

$0 \neq 1$

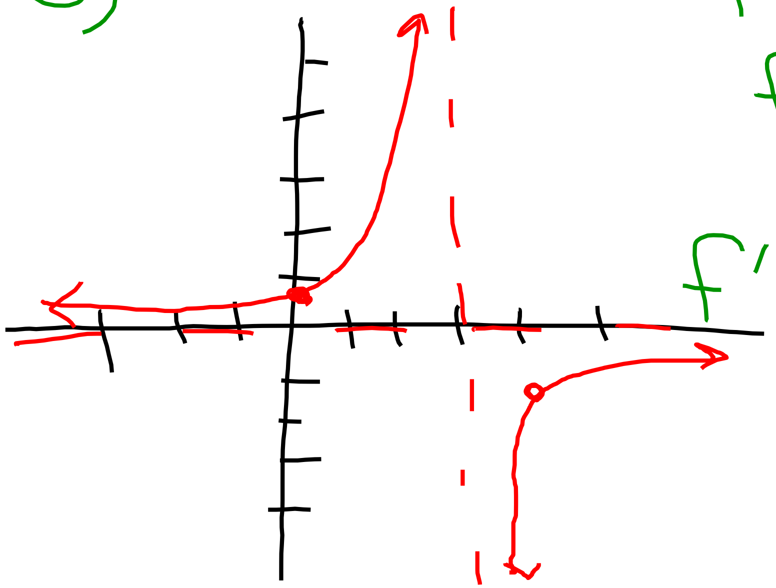
$x - 3 = 0$

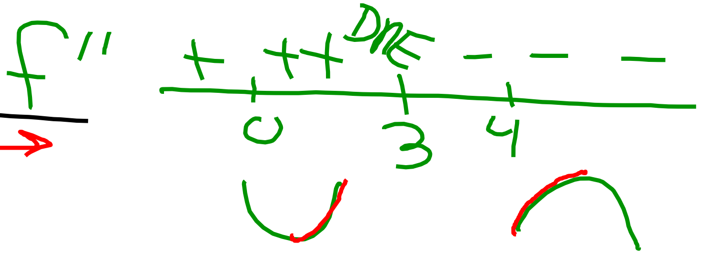
$x = 3$

DNE

$f''(x) = -2(x-3)^{-3}$

$f''(x) = \dfrac{-2}{(x-3)^3}$





3. $f(x) = -\frac{x^3}{6} - \frac{x^2}{2}$ $= -\frac{1}{6}x^3 - \frac{1}{2}x^2$

$0 = x^2(-\frac{1}{6}x - \frac{1}{2})$

$\sqrt{x^2} = \sqrt{0}$ $\quad -\frac{1}{6}x - \frac{1}{2} = 0$

$x = 0$ $\quad -6 \cdot -\frac{1}{6}x = \frac{1}{2} \cdot -6$

$x = -3$

$+\frac{1}{6} - \frac{3}{6}$

$f'(x) = -\frac{1}{2}x^2 - x$

$0 = x(-\frac{1}{2}x - 1)$ $\quad -\frac{1}{2}x - 1 = 0$

$x = 0$ $\quad -2 \cdot -\frac{1}{2}x = 1 \cdot -2$

$x = -2$

f': $- - 0 + + 0 - -$ at -2, 0: min, max

$-\frac{8}{6} - \frac{4}{2}\,\frac{12}{6} = -\frac{4}{6} = -\frac{2}{3}$

$f'' = -x - 1$

$-x - 1 = 0$

$-x = 1$

$x = -1$

f'': $+ + 0 - -$ at -1

x-int $(0,0)$ $(-3,0)$

y-int $(0,0)$

VA none

HA none

Critical Points $x = 0$ $x = -2$

Increases $(-2, 0)$

Decrease $(-\infty, -2) \cup (0, \infty)$

Concave Up $(-\infty, -1)$

Concave Down $(-1, \infty)$

Relative Maxima $(0, 0)$

Relative Minimum $(-2, -2/3)$

Point of Inflection $(-1, -1/3)$