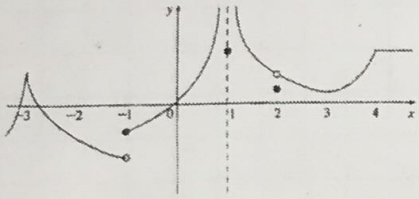
40 # Curve Sketching

A graph is differentiable anywhere EXCEPT where there is the following:

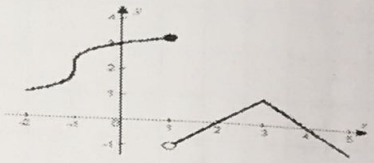
- cusp
- corner
- vertical tangent
- discontinuity (removable o, infinite ↕, jump ⊢ ⊣)

STATE THE X VALUES WHERE f IS NOT DIFFERENTIABLE AND THE REASON



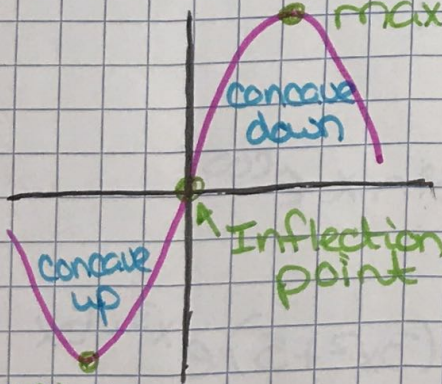
$x=-3$ cusp $x=-1$ jump
$x=1$ infinite $x=2$ hole

STATE THE X VALUES WHERE f IS NOT DIFFERENTIABLE AND THE REASON



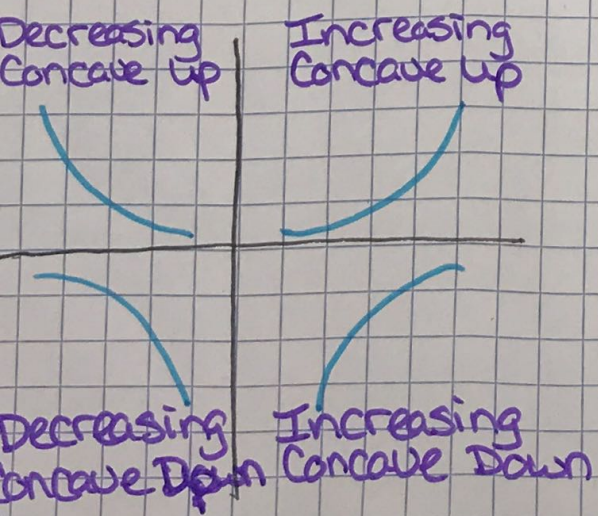
$x=-1$ vertical tangent
$x=1$ jump $x=3$ corner

Critical Points, Concavity, + Inflection Points



Critical points - the graph's turning points or the local max (peaks) + local min (valleys)

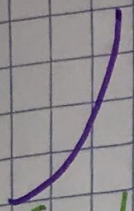
Inflection points - a point on a graph where it changes concavity 1 side is concave down + 1 side is concave up



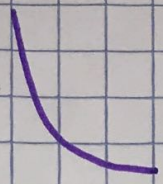
RELATIONSHIP BETWEEN f, f', f''

f	f'	f''
-Cusp -Corner -Discontinuity -Removable -Infinite -jump -Vertical Tangent	DNE	DNE
Local max, local min (local extrema), horizontal tangent	0 On the x-axis	
f increasing	Positive (Above the x-axis)	
f decreasing	Negative (Below the x-axis)	
f concave up	Increasing	Positive (Above the x-axis)
f concave down	Decreasing	Negative (Below the x-axis)
Points of Inflections	Local Extrema	Change Signs

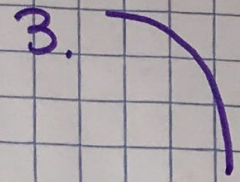
what can we say about g, g', g'' for each segment of the graph?

1. 

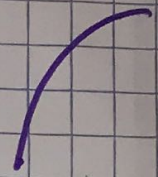
g: Incr / Conc up
g': positive / Incr
g'': Positive

2. 

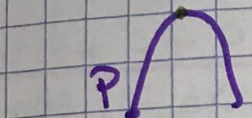
g: Decr / Conc up
g': negative / Incr
g'': Positive

3. 

Decr / Conc ↓
negative / decr
negative

4. 

Incr / Conc ↓
positive / Decr
negative

5. 

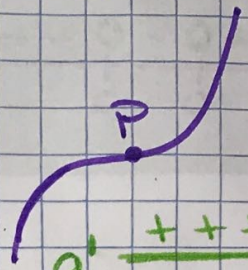
g' − − 0 + + 0 − −

− means negative (below x-axis)

g'' + + + 0 − − −

bc conc ↑ P bc conc ↓

+ means positive (above x-axis)

6. 

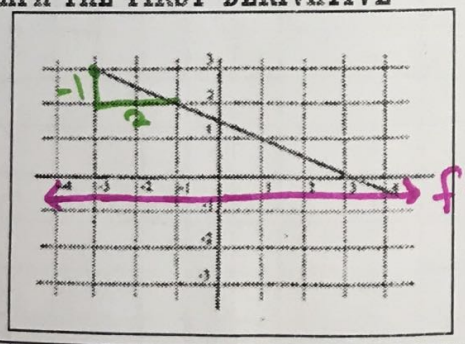
g' + + + + + + +

g'' − − − 0 + + + +

bc conc ↓ P bc conc ↑

★ concavity tells you whether the 2nd derivative of a function is positive or negative

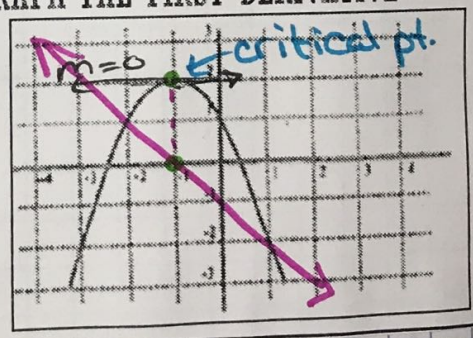
1. GRAPH THE FIRST DERIVATIVE



f' − − − − − − − all below x-axis

linear's derivative is constant $m = -\frac{1}{2}$

2. GRAPH THE FIRST DERIVATIVE

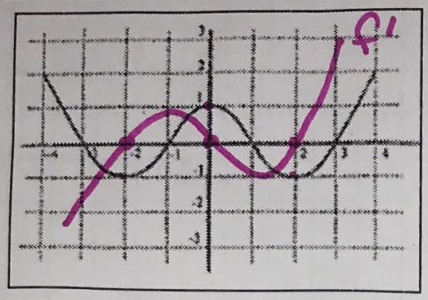


+ + + 0 − − − put critical pts on f' line

above x-axis | 0 | below x-axis

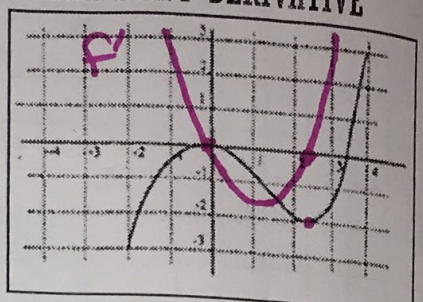
42 max + min become x-intercepts

3. GRAPH THE FIRST DERIVATIVE



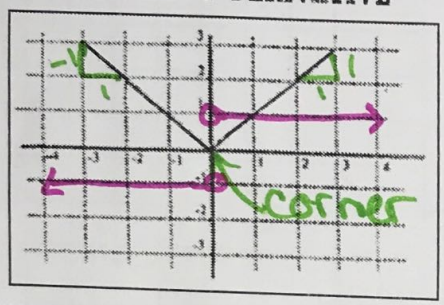
f' − − − 0 + 0 − 0 + + +
-2 0 2 critical pts

4. GRAPH THE FIRST DERIVATIVE



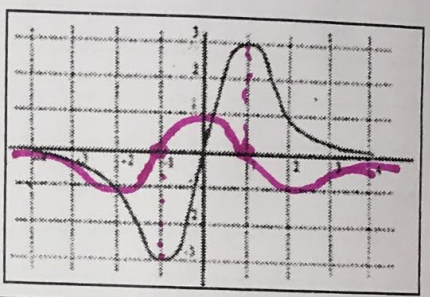
f' + + + 0 − 0 + + +
above 0 below 2, 1 above

5. GRAPH THE FIRST DERIVATIVE



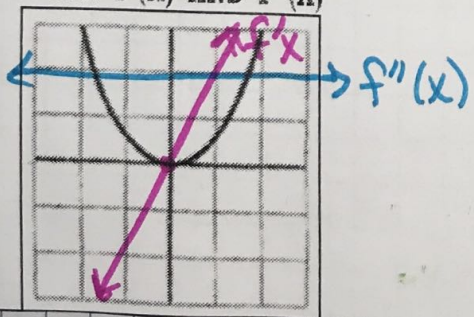
f' − − undef + + +
constant slope m=-1 | 0 | constant slope m=1

6. GRAPH THE FIRST DERIVATIVE



f' close to 0 − − 0 + + 0 − − close to 0
-1 1

7. GRAPH THE F'(X) AND F"(X)

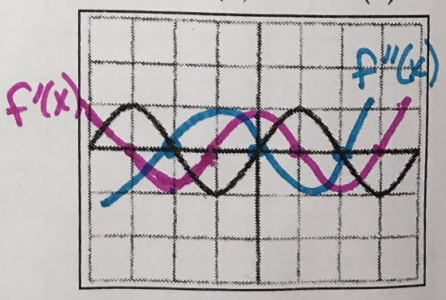


f' − − 0 + + +
0

f" + + + + + + + +
all positive w/ no inflection pt. (constant)

look at graph of f' for incr/decr

8. GRAPH THE F'(X) AND F"(X)



f' + 0 − 0 + 0 − 0 +
-3 -1 1 3

f" − − 0 + 0 − 0 + +
-2 0 2

★ Inflection pts of f(x) become zeros of f"(x)