Honors Calculus Mid-Term Review

Name ____________________

1. Find the domain of $f(x) = \frac{1}{\sqrt{5-3x}}$

$5-3x > 0$
$-3x > -5$
$x < 5/3$

DNE
$5/3$

$(-\infty, 5/3)$

2. What is the average rate of change of $f(x) = \frac{1}{x+2}$ over [-3, 2]?

$f(-3) = \frac{1}{-1}$ $\quad f(2) = \frac{1}{4}$
$(-3, -1)$ $\quad (2, 1/4)$
$x_1 \; y_1$ $\quad x_2 \; y_2$

$m = \frac{\frac{1}{4} - -1}{2 - -3} = \frac{\frac{5}{4}}{5} = \frac{5}{4} \cdot \frac{1}{5} = \frac{1}{4}$

3. Evaluate $f(x) = \begin{cases} 5x-8, \ x < 7 \\ \sqrt{x-7}, \ 7 \le x < 12 \\ \frac{4}{x}, \ x \ge 12 \end{cases}$ for f(3), f(7), & f(12).

$f(3) = 5(3) - 8 = 7$
$f(7) = \sqrt{7-7} = 0$
$f(12) = \frac{4}{12} = \frac{1}{3}$

4. Find the exact value of $\cos\frac{3\pi}{4}$, $\tan\frac{7\pi}{6}$, & $\sin\frac{5\pi}{3}$.

$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

$\tan\frac{7\pi}{6} = 1$

$\sin\frac{5\pi}{3} = -\frac{\sqrt{2}}{2}$

5. Find the horizontal & vertical asymptote of $f(x) = \frac{3x^2+2x-16}{x^2-4}$

$f(x) = \frac{(3x+8)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$ hole at $x=2$ (removable discont.)

VA: $x+2=0$
$x=-2$

HA: $y=3$ (same degree)

6. If $f(x) = 2x^2+1$ and $g(x) = x+2$,

a. find $(f \circ g)(x) =$ $2(x+2)^2+1$
$2(x^2+4x+4)+1$
$2x^2+8x+9$

b. find $(g \circ f)(x) =$ $(2x^2+1)+2$
$2x^2+3$

7. $f(x) = \begin{cases} \frac{x^2-25}{x+5}, & for\ x \ne -5 \\ 3, & for\ x = -5 \end{cases}$ $\quad \frac{\cancel{(x+5)}(x-5)}{\cancel{x+5}}$

Which of the following are true about f?

I. $f(x)$ is continuous at $x=-5$ no, hole at $x=-5$
II. $\lim_{x\to-5} f(x)$ exists yes
III. $f(-5)$ exists yes

$\lim_{x\to-5} \frac{x^2-25}{x+5}$ $\quad f(-5) = 3$

$\lim_{x\to-5} x-5 = -5-5 = -10$

II & III are true

8. $f(x) = \begin{cases} x^2, & for\ x > 2, \\ 5ax, & for\ x \le 2 \end{cases}$

For what value of a is the function continuous?

$x^2 = 5ax$
$(2)^2 = 5a(2)$
$4 = 10a$
$\frac{4}{10} = a$
$a = 2/5$

9. Determine if the function is continuous. If not, name the x-value & type of discontinuity.

$f(x) = \begin{cases} x^2-25, & x > -3 \\ 3x+8, & x \le -3 \end{cases}$

$\lim_{x\to-3^+} (-3)^2-25 = -16$

$\lim_{x\to-3^-} (3)(-3)+8 = -1$

$\lim_{x\to-3^+} f(x) \ne \lim_{x\to-3^-} f(x)$

$\therefore f(x)$ is not continuous

Jump Discontinuity at $x=-3$

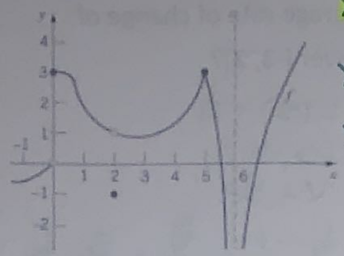
10. Given a function defined by $f(x) = \frac{x+1}{x^2-4x+12}$, for what values of x is the function discontinuous?

$f(x) = \frac{x+1}{(x+2)(x-6)}$

VA: $x = -2, 6$

Infinite discontinuity at $x=-2$ & $x=6$

11. List places & types of discontinuities for the graph below.

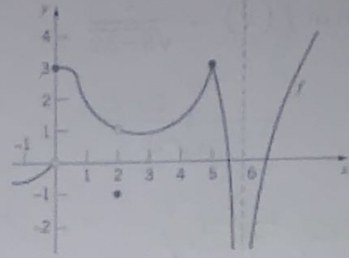


x = 0 Jump

x = 2 Removable

x = 6 Infinite

12. List places where the graph below is continuous but not differentiable.



x = 5 is a cusp

13. $\lim_{x\to-1} -3x^2 + 2x = -3(-1)^2 + 2(-1)$

$= -3(1) + 2(-1)$

$= -3 - 2$

$= -5$

14. $\lim_{x\to-2} \frac{5x^2}{x-3} = \frac{5(-2)^2}{2-3} = \frac{5(4)}{-1} = -20$

15. $\lim_{x\to-5} \frac{-1}{(x+5)^2} = -\infty$

$\lim_{x\to-5^-} \frac{-1}{(-5.001+5)^2} = -\#$ $\quad \lim_{x\to-5^+} \frac{-1}{(-4.999+5)^2}$

16. $\lim_{x\to9} \frac{x-9}{\sqrt{x}-3} = 6$

$\lim_{x\to9} \frac{(\sqrt{x}+3)\cancel{(\sqrt{x}-3)}}{\cancel{(\sqrt{x}-3)}} = \sqrt{9}+3 = 6$

17. $\lim_{x\to0^-} \frac{1}{x} = -\infty$

$\frac{1}{-.001} = -\#$

18. $\lim_{x\to0} \frac{\sqrt{x+25}-5}{x} = \frac{1}{10}$

$\lim_{x\to0} \frac{(\sqrt{x+25}-5)(\sqrt{x+25}+5)}{x(\sqrt{x+25}+5)}$

$\lim_{x\to0} \frac{x+25-25}{x(\sqrt{x+25}+5)}$

$\lim_{x\to0} \frac{1}{\sqrt{x+25}+5}$

$= \frac{1}{\sqrt{0+25}+5} = \frac{1}{10}$

19. $\lim_{x\to-2^-} \frac{|x+2|}{x+2} = -1$

$\frac{-\cancel{(x+2)}}{\cancel{(x+2)}}$

20. $\lim_{x\to\infty} \frac{x^2-x}{-4x} = -\infty$

$\frac{(\infty)^2-\infty}{-4(\infty)} = \frac{+\infty}{-\infty}$

21. $\lim_{x\to3^+} \frac{|x-3|}{x-3} = 1$

$\frac{+\cancel{(x-3)}}{\cancel{x-3}}$

22. $\lim_{x\to-\infty} \frac{x^2-x}{-4x} = \infty$

$\frac{(-\infty)^2-(-\infty)}{-4(-\infty)} = \frac{+\infty}{+\infty}$

23. Use the definition of the derivative to find the derivative of $y = \frac{2}{x}$.

$y' = \lim_{h\to0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$

$y' = \lim_{h\to0} \frac{\frac{2x-2x-2h}{x(x+h)}}{h}$

$y' = \lim_{h\to0} \frac{-2\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$

$y' = \frac{-2}{x(x+0)}$

$y' = -\frac{2}{x^2}$

24. Use the definition of the derivative to find the derivative of $y = \sqrt{x-7}$

$y' = \lim_{h\to0} \frac{\sqrt{x+h-7}-\sqrt{x-7}}{h}$

$y' = \lim_{h\to0} \frac{(\sqrt{x+h-7}-\sqrt{x-7})(\sqrt{x+h-7}+\sqrt{x-7})}{h(\sqrt{x+h-7}+\sqrt{x-7})}$

$y' = \lim_{h\to0} \frac{x+h-7-x+7}{h(\sqrt{x+h-7}+\sqrt{x-7})}$

$y' = \frac{1}{\sqrt{x+0-7}+\sqrt{x-7}}$

$y' = \frac{1}{2\sqrt{x-7}}$

$f(x) = -4x^{-1/4}$

25. If $f(x) = -\frac{4}{\sqrt[4]{x}}$, then $f'(16) =$

$f'(x) = x^{-5/4} = \frac{1}{x^{5/4}}$

$f'(16) = \frac{1}{\sqrt[4]{16}^5} = \frac{1}{32}$

26. Find the derivative, $\frac{dy}{dx}$, of $y = \frac{3x}{x^2+1}$

$\frac{dy}{dx} = \frac{(x^2+1)3 - 3x(2x)}{(x^2+1)^2}$

$\frac{dy}{dx} = \frac{3x^2+3-6x^2}{(x^2+1)^2}$

$\frac{dy}{dx} = \frac{-3x^2+3}{(x^2+1)^2}$

27. If $y = -\frac{4}{\sqrt[3]{x+5}}$, then $\frac{dy}{dx} =$

$y = -4(x+5)^{-1/3}$

$\frac{dy}{dx} = \frac{4}{3}(x+5)^{-4/3}$

$\frac{dy}{dx} = \frac{4}{3\sqrt[3]{(x+5)^4}}$

28. Find the derivative of $y = \sqrt[3]{x^2+x}$ $= (x^2+x)^{1/3}$

$y' = \frac{1}{3}(x^2+x)^{-2/3}(2x+1)$

$y' = \frac{2x+1}{3(x^2+x)^{2/3}}$

29. Find the derivative of $y = (x^2+2x+5)^6$

$y' = 6(x^2+2x+5)^5(2x+2)$

$y' = (12x+12)(x^2+2x+5)^5$

30. Find $f'(x)$ given $f(x) = \sin^3(4x)$ $[\sin(4x)]^3$

$f'(x) = 3[\sin(4x)]^2 \cos(4x) \cdot 4$

$f'(x) = 12\sin^2(4x)\cos(4x)$

31. Given $y = \cos^3 x$, then $\frac{dy}{dx} =$

$y = (\cos x)^3$

$\frac{dy}{dx} = 3(\cos x)^2 \cdot -\sin x$

$\frac{dy}{dx} = -3\cos^2 x \sin x$

32. If $y = \cot(2x^3)$, then $\frac{dy}{dx} =$

$\frac{dy}{dx} = -\csc^2(2x^3) \cdot 6x^2$

$\frac{dy}{dx} = -6x^2 \csc^2(2x^3)$

33. Find $\frac{dy}{dx}$ if $y = x^2 \cdot e^x$

$\frac{dy}{dx} = x^2e^x + 2xe^x$

or

$xe^x(x+2)$

34. Find $\frac{d}{dx}$ given $y = \ln e^4$

$y = 4\ln e$

$y = 4$

$y' = 0$

35. If $y = e^{x^4-3x^2}$, then $y' =$

$y' = e^{x^4-3x^2}(4x^3-6x)$

36. $\frac{d}{dx} e^{\ln(\cos x)} =$

$\frac{d}{dx}\cos x = -\sin x$

37. $\frac{d}{dx}\ln(e^{4x^2}+3)$

$\frac{1}{e^{4x^2}+3}\cdot e^{4x^2}\cdot \ln e \cdot 8x = \frac{8xe^{4x^2}}{e^{4x^2}+3}$

38. Find $\frac{dy}{dx}$, given $y = \frac{x^3}{3^x}$

$y' = \frac{3^x\cdot 3x^2 - x^3\cdot 3^x \ln 3}{(3^x)^2}$

$y' = \frac{3x^2 - x^3 \ln 3}{3^x}$

39. If $y = \ln(2x^2-5)$, then $\frac{dy}{dx} =$

$y' = \frac{1}{2x^2-5}\cdot 4x$

$y' = \frac{4x}{2x^2-5}$

40. Find $\frac{d^2y}{dx^2}$ if $y = \ln 5x^2$

$\frac{dy}{dx} = \frac{1}{5x^2}\cdot 10x$

$\frac{dy}{dx} = \frac{2}{x}$ or $2x^{-1}$

$\frac{d^2y}{dx^2} = \frac{-2}{x^2}$

41. Find the slope of the tangent line to the graph $f(x) = 2x(2x^2-1)$ at the point where $x = 1$

$f(x) = 4x^3 - 2x$

$f'(x) = 12x^2 - 2$

$f'(1) = 10$

$m_{tan} = 10$

42. Find an equation of the tangent line to the curve $f(x) = -x^2 + 12$ passing when $x = 4$.

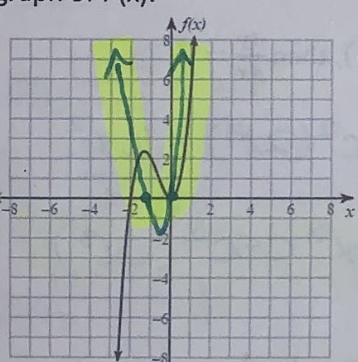
$f'(x) = -2x$

$f'(4) = -8$

$f(4) = -(4)^2 + 12$
$= -16 + 12$
$= -4$

$y + 4 = -8(x-4)$

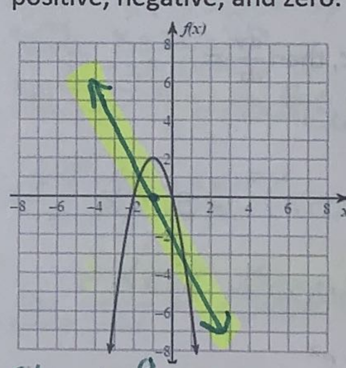
43. Given the graph of f(x) below, sketch a possible graph of f′(x).



f′(x) + 0 − 0 +

f(x) ↗ ↘ ↗

44. Given the graph of f(x) below, list where f′(x) will be positive, negative, and zero.

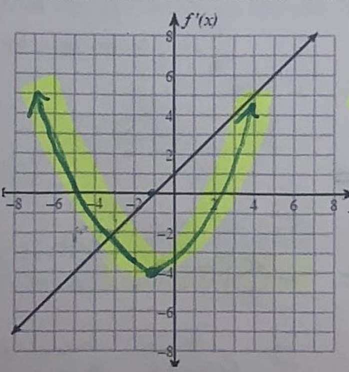


f′(x) is positive $(-\infty, -1)$, f′(x) is negative $(-1, \infty)$ & f′(x) = 0 at −1

f′ + 0 −

f ↗ ↘

45. Given the graph of f', list where the graph of $f(x)$ has intervals of increase & decrease.



f(x) increases from $(-1, \infty)$ & decreases from $(-\infty, -1)$.

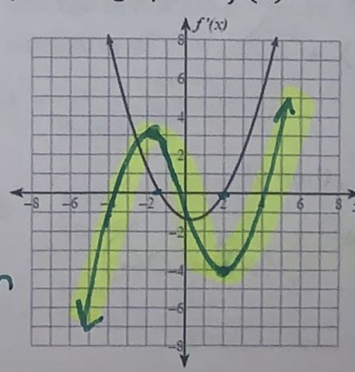
f(x) has a minimum at x = −1

f′ − − 0 + + +

f ↘ −1 ↗

min

46. Given the graph of f', the derivative of f. Sketch a possible graph for $f(x)$.



f′ + + 0 − 0 + +

f ↗ max ↘ ↗

min

Use the table for # 47 – 50

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	2	4	-2
2	4	$\frac{1}{2}$	2	$-\frac{3}{2}$
3	3	$-\frac{3}{2}$	1	$\frac{1}{2}$
4	1	-2	3	2

47. $h(x) = f(x) - g(x)$, find $h'(3)$

$h'(x) = f'(x) - g'(x)$

$h'(3) = f'(3) - g'(3)$

$= -\frac{3}{2} - \frac{1}{2}$ $\quad h'(3) = -2$

48. $h(x) = f(x) \cdot g(x)$, find $h'(4)$

$h'(4) = f(4)g'(4) + g(4)f'(4)$

$h'(4) = 1 \cdot 2 + 3 \cdot -2$

$= 2 - 6$

$= -4$

49. $h(x) = \frac{f(x)}{g(x)}$, find $h'(2)$

$h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$

$= \frac{(2)(1/2) - (4)(-3/2)}{(2)^2} = \frac{1+6}{4} = \frac{7}{4}$

50. $h(x) = f(g(x))$, find $h'(1)$

$h'(1) = f'(g(1)) \cdot g'(1)$

$= f'(4) \cdot g'(1)$

$= (-2)(-2)$

$h'(1) = 4$

51. Evaluate the limit using L'Hopital's Rule

$$\lim_{x \to 0} \frac{5x}{\ln(x+1)} \quad \frac{0}{0}$$

$\lim_{x \to 0} \frac{5}{\frac{1}{x+1}} = \frac{5}{\frac{1}{0+1}} = \frac{5}{1}$

5

52. Evaluate the limit using L'Hopital's Rule

$$\lim_{x \to 0} \frac{e^{4x} - 1}{\sin(5x)}$$

$\lim_{x \to 0} \frac{4e^{4x}}{5\cos(5x)} = \frac{4e^{4 \cdot 0}}{5\cos(5 \cdot 0)} = \frac{4e^0}{5\cos(0)}$

$= \frac{4 \cdot 1}{5 \cdot 1} = \frac{4}{5}$

53. Evaluate the limit using L'Hopital's Rule

$$\lim_{x \to \infty} \frac{e^{3x}}{x}$$

$\lim_{x \to \infty} \frac{3e^{3x}}{1} = 3e^{3\infty}$

$= \infty$

54. Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y for $-2x + 2 = 3x^2 + 4y^2$.

$-2 = 6x + 8y\frac{dy}{dx}$

$-2 - 6x = 8y\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-2-6x}{8y}$ or $\frac{-1-3x}{4y} = \frac{dy}{dx}$

55. Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y for $3y + 4xy^2 = 3x$.

$3\frac{dy}{dx} + 4x \cdot 2y\frac{dy}{dx} + y^2 \cdot 4 = 3$

$3\frac{dy}{dx} + 8xy\frac{dy}{dx} = 3 - 4y^2$

$(3 + 8xy)\frac{dy}{dx} = 3 - 4y^2$

$\frac{dy}{dx} = \frac{3-4y^2}{3+8xy}$

56. Use implicit differentiation to find $\frac{dy}{dx}$ for $x^3 = 4y^2 + 5x^3$ at the point (-1, -1).

$3x^2 = 8y\frac{dy}{dx} + 15x^2$

$-12x^2 = 8y\frac{dy}{dx}$

$\frac{-12x^2}{8y} = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-3x^2}{2y}$