Mid-Term Review

1. Find the domain of $f(x) = \frac{1}{\sqrt{5-3x}}$

2. What is the average rate of change of $f(x) = \frac{1}{x+2}$ over [-3, 2]?

3. Evaluate $f(x) = \begin{cases} 5x-8, & x < 7 \\ \sqrt{x-7}, & 7 \le x < 12 \\ \frac{4}{x}, & x \ge 12 \end{cases}$ for f(3), f(7), & f(12).

4. Find the exact value of $\cos\frac{3\pi}{4}$, $\tan\frac{7\pi}{6}$, & $\sin\frac{5\pi}{3}$.

5. Find the horizontal & vertical asymptote of $f(x) = \frac{3x^2+2x-16}{x^2-4}$

6. If $f(x) = 2x^2 + 1$ and $g(x) = x + 2$, then find $(f \circ g)(x) =$

7. Let f be defined as follows:
$$f(x) = \begin{cases} \frac{x^2-25}{x+5}, & for\ x \neq -5 \\ 3, & for\ x = -5 \end{cases}$$
Which of the following are true about f?
   I. $f(x)$ is continuous at $x = -5$
   II. $\lim_{x\to-5} f(x)$ exists
   III. $f(-5)$ exists

8. $f(x) = \begin{cases} x^2, & for\ x > 2, \\ 5ax, & for\ x \le 2 \end{cases}$
For what value of a is the function continuous?

9. Given a function is defined by $f(x) = \frac{2x+2}{x^2+5x+4}$, for what value(s) of x does the function have one or more vertical asymptotes?

10. Given a function defined by $f(x) = \frac{2x+1}{x^2+5x+4}$, for what values of x is the function discontinuous?

11. $\lim_{x\to -5} \frac{-1}{(x+5)^2} =$

12. $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3} =$

13. $\lim_{x\to 0^-} \frac{1}{x} =$

14. $\lim_{x\to 0} \frac{\sqrt{x+25}-5}{x} =$

15. $\lim_{x\to -2^-} \frac{|x+2|}{x+2} =$

16. $\lim_{x\to \infty} \frac{x^2-x}{-4x} =$

17. If $f(x) = -\frac{4}{\sqrt[4]{x}}$, then $f'(16) =$

18. Find the derivative, $\frac{dy}{dx}$, of $y = \frac{3x}{x^2+1}$

19. If $y = -\frac{4}{\sqrt[3]{x+5}}$, then $\frac{dy}{dx} =$

20. Find the derivative of $y = \sqrt[3]{x^2+x}$

21. Find the derivative of $y = (x^2+2x+5)^6$

22. Find $f'(x)$ fiven $f(x) = \sin^3(4x)$

23. Given $y = \sin(2x^5)$, then $\frac{dy}{dx} =$

24. If $y = \cos(e^x)$, then $\frac{dy}{dx} =$

25. Find $\frac{dy}{dx}$ if $y = x^2 \cdot e^x$

26. Find $\frac{d}{dx}$ given $y = \ln\frac{5}{5-x}$

27. If $y = e^{x^4-3x^2}$, then $y' =$

28. $\frac{d}{dx}\ e^{\ln(\cos x)} =$

29. $\frac{d}{dx}\ln\left(e^{4x^2}+3\right)$

30. Find $\frac{dy}{dx}$ given $y = \frac{x^3}{3^x}$

31. If $y = \ln(2x^2-5)$, then $\frac{dy}{dx} =$

32. Find $\frac{d^2y}{dx^2}$ for $y = \ln 5x^2$

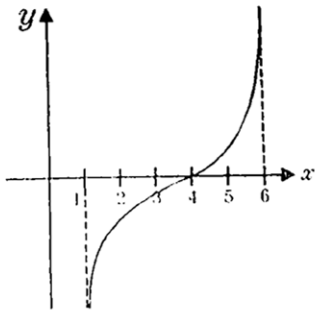
33. Find the slope of the tangent line to the graph $f(x) = 2x(2x^2 - 1)$ at the point where $x = 1$

34. Find an equation of the tangent line to the curve $f(x) = -x^2 + 12$ passing through the point $(4,0)$

35. Find the critical numbers of $f(x) = x^3 - 12x^2$

36. Let $f(x) = x^2(x - 3)$. Over what interval is the function decreasing?

37. The figure shows the graph of ff', the derivative of the function f. The domain of the function f is $-10 \leq x \leq 10$. For what value(s) does the function have a relative maximum?



38. Refer to the previous figure. For what value(s) does the function have a relative minimum?

39. A particle's motion is described by $x(t) = 4t^3 - 5t^2,\ t \geq 0$, where t is in seconds and distance in meters. Find the velocity in the third second.

40. The position of a particle moving in a straight line at any time t is $x(t) = 2t^2 + 6t + 5$. What is the acceleration of the particle at $t = 3$?

41. Find all points of inflection for $f(x) = x^4 - 4x^3 + 2$

42. Find the interval(s) on which the curve $y = x^3 - 3x^2 - 9x + 6$ is concave upward or concave downward.

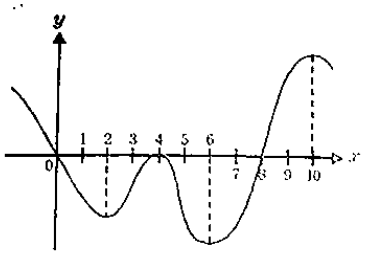
43. Given that $f(x) = \frac{4}{x}$, determine where the function is concave up and concave down.

44. Given that $f(x) = -x^2 + 12x - 34$ has a relative maximum at $x = 6$, determine where $f'(x)$ is positive and negative.

45. Find the point of inflection of $f(x) = x^3 - 3x^2 - x + 7$

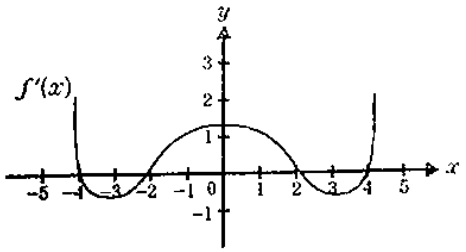
46. Given a function defined by $f(x) = 3x^5 - 5x^3 - 8$, for what value(s) of x is there a point of relative minimum?

47. The figure shows the graph of f', the derivative of the function f. The domain of the function f is $-10 \leq x \leq 10$. For what value(s) does the function have a relative minimum?

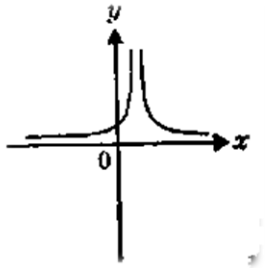


48. Refer to the previous figure. For what value(s) does the function have a relative maximum?

49. The graph $f(x)$ has horizontal tangents when $x =$



50. The graph of the derivative is shown. Draw the graph of f.



51. A ladder 10 feet long is leaning against a wall, with the foot of the ladder 8 feet away from the wall. I the foot of the ladder is being pulled away from the wall at 3 ft/sec how fast is the top of the ladder sliding down the wall?

52. Find all value(s) of x (if any) that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = \frac{1}{1+x}$ on the interval $[0,1]$.

53. A farmer has 20 feet of fence, and he wishes to make from it a rectangular pen for his pig Wilbur, using a barn as one of the sides. In square feet, What is the maximum area possible for his pet?

54. One person is walking south toward an intersection that is 60 ft away at a rate of 2 ft/s while a second person on a bicycle is riding east away from the same intersection at 10 ft/s. If the bicyclist is 80 ft from the intersection, how fast is the distance between he and the person walking increasing?

55. A rectangle is inscribed between the parabola $y = 7 - x^2$ and the x-axis, with its base on the x-axis. Find the value of x that maximizes the area of the rectangle.

56. A circular conical reservoir, vertex down, has a depth 20 ft and radius of the top 10 ft. Water is leaking out so that the surface is falling at the rate of ½ ft/hr. Find the rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 feet deep.