Meaning of Integration Unit Review

Section 1: Indefinite Integrals

1. $\int \sec^2 x\,dx$ $\tan x + C$

2. $\int dx$ $x + C$

3. $\int 4\sin x\,dx$ $-4\cos x + C$

4. $\int \frac{5}{x^3}dx$ $5x^{-3}$ $\frac{5x^{-2}}{-2} + C = -\frac{5}{2x^2} + C$

5. $\int \sqrt[2]{x^5}\,dx$ $\frac{x^{5/2+1}}{7/2} + C$ $\frac{2x^{7/2}}{7} + C$

6. $\int \csc x \cot x\,dx$ $-\csc x + C$

7. $\int \frac{-5}{(2x)^3}dx = \frac{-5x^{-3+1}}{8\cdot -2}$ $\frac{5}{16x^2} + C$

8. $\int (5x+1)^2\,dx$ $(5x+1)(5x+1)$ $\int 25x^2 + 10x + 1$ $\frac{25x^3}{3} + \frac{10x^2}{2} + x + C$ $\frac{25x^3}{3} + 5x^2 + x + C$

9. $\int (x^{-4} + 3x^3)dx$ $\frac{x^{-3}}{-3} + \frac{3x^4}{4} + C$ $\frac{-1}{3x^3} + \frac{3x^4}{4} + C$

10. $\int 3\sec x \tan x\,dx$ $3\sec x + C$

11. $\int (\frac{1}{\sqrt[3]{x^2}} - \sec^2 x)dx$ $x^{-2/3} - \sec^2 x$ $\frac{x^{1/3}}{1/3} - \tan x + C$ $3x^{1/3} - \tan x + C$

12. $\int (\pi e^x + 7\cos x)dx$ $\pi e^x + 7\sin x + C$

13. $\int (2x-5)^2\,dx$ $\int 4x^2 - 20x + 25$ $\frac{4x^3}{3} - \frac{20x^2}{2} + 25x + C$ $\frac{4}{3}x^3 - 10x^2 + 25x + C$

14. $\int \csc x \cot x\,dx$ $-\csc x + C$

15. $\int 3x^{-1}dx$ $3\ln|x| + C$

16. $\int (3x-4)^2\,dx$ $\int 9x^2 - 24x + 16\,dx$ $\frac{9x^3}{3} - \frac{24x^2}{2} + 16x + C$ $3x^3 - 12x^2 + 16x + C$

17. $\int ex^{-1}dx$ $e\ln|x| + C$

18. $\int \frac{1\,dx}{\cos^2 x} = \int \frac{1}{\cos^2 x}dx = \int \sec^2 x$ $\tan x + C$

Section 2: Definite Integrals – Fundamental Theorem of Calculus

19. $\int_1^8 (x^{-1/3} + \frac{1}{x})dx$ $\frac{3x^{2/3}}{2} + \ln|x| \Big|_1^8$ $(\frac{3\sqrt[3]{8}^2}{2} + \ln 8) - (\frac{3\sqrt[3]{1}^2}{2} + \ln 1)$ $6 + \ln 8 - 1.5 + 0 = 4.5 + \ln 8$

20. $\int_0^{\pi} (\sin t + 1)dt$ $-\cos t + t \Big|_0^{\pi}$ $(\cos\pi + \pi) - (\cos 0 + 0)$ $+1 + \pi + 1 - 0 = \ldots\pi$ $2 + \pi$

21. $\frac{d}{dx}\int_2^x \sqrt{1+4t^2}\,dt$ $\sqrt{1+4x^2}$

22. $\frac{d}{dx}\int_3^{x^2} \frac{1}{\sqrt{2-t^5}}dt$ $\frac{1}{\sqrt{2-(x^2)^5}} \cdot 2x = \frac{2x}{\sqrt{2-x^{10}}}$

23. $\frac{d}{dx}\int_{\tan x}^{x^2} \frac{1}{4-t^3}dt$ $\frac{1}{4-(x^2)^3} \cdot 2x - \frac{1}{4-\tan^3 x} \cdot \sec^2 x$ $\frac{2x}{4-x^6} - \frac{\sec^2 x}{4-\tan x^3}$

24. $\frac{d}{dx}\int_2^x \sec(t)dt$ $\sec x$

25. $\frac{d}{dx}\int_{-5}^x \sec(t)dt$ $\sec x$

26. $\frac{d}{dx}\int_x^2 \sec(t)dt$ $-\sec x$

27. $\frac{d}{dx}\int_2^{2x^5} \sec(t)dt$ $10x^4 \sec(2x^5)$

15

28. $f(x)=\int_{-1}^{x}\ln(t)dt \quad f'(x)=$

$\ln x$

29. $f(x)=\int_{x}^{4}\ln(t)dt \quad f'(x)=$

$-\ln x$

30. $f(x)=\int_{0}^{\sin x}\ln(t)dt \quad f'(x)=$

$\ln(\sin x)\cdot\cos x$

$\cos x\ln(\sin x)$

31. $f(x)=\int_{3+x^2}^{x}\ln(t)dt \quad f'(x)=$

$\ln x\cdot 1-\ln(3+x^2)\cdot 2x$

32. $\frac{d}{dx}\int_{-1}^{x}t^2dt$

x^2

33. $\int_{-1}^{x}t^2dt$

$\left.\frac{t^3}{3}\right|_{-1}^{x}$

$\frac{x^3}{3}+\frac{1}{3}$

34. $-\frac{d}{dx}\int_{x^3}^{5}\left(\frac{t^2-3}{2t}\right)dt$

$-\frac{(x^3)^2-3}{2x^3}\cdot 3x^2=\frac{-3x^2(x^4-3)}{2x^3}$

35. $\int_{1}^{25}x^{1/2}dx$

$\left.\frac{2x^{3/2}}{3}\right|_{1}^{25}=$

$\frac{2\sqrt{25}^3}{3}-\frac{2\sqrt{1}^3}{3}=\frac{250}{3}-\frac{2}{3}=\frac{148}{3}$

36. $\int_{0}^{3\pi/2}(\sin t+1)dt$

$\left.-\cos t+t\right|_{0}^{3\pi/2}$

$\left(\cos\frac{3\pi}{2}+\frac{3\pi}{2}\right)-(\cos 0+0)$

$0+\frac{3\pi}{2}-1-0=\frac{3\pi}{2}-1$

37. $\frac{d}{dx}\int_{2}^{x}\sec^3 t\,dt$

$\sec^3 x$

38. $\frac{d}{dx}\int_{e^x}^{x^3}\frac{1}{1+t^2}dt$

$\frac{1}{1+(x^3)^2}\cdot 3x^2-\frac{1}{1+(e^x)^2}\cdot e^x=\frac{3x^2}{1+x^6}-\frac{e^x}{1+e^{2x}}$

39. $\frac{d}{dx}\int_{3}^{x^2}\sqrt{5t-x^2}\,dt$

$\sqrt{5x^2-(x^2)^2}\cdot 2x$

$2x\sqrt{5x^2-x^4}$

40. If $\int_{2}^{7}f(x)dx=8$ and $\int_{2}^{7}g(x)dx=-3$ and $\int_{5}^{7}f(x)dx=-1$, find the following:

a. $\int_{2}^{7}[3f(x)+2g(x)]dx$

$3(8)+2(-3)=18$

b. $\int_{7}^{2}3g(x)dx$

$-3(-3)=+9$

c. $\int_{2}^{5}f(x)dx$

$\int_{2}^{7}f(x)+-1\int_{7}^{5}f(x)$

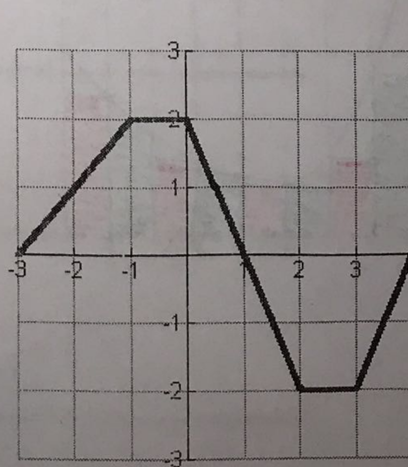
$8--1=9$

d. $\int_{3}^{3}g(x)dx$

0

Section 3: Area

Given the graph below, if $g(x)=\int_{-2}^{x}f(t)dt$, find the following



41. g(2)

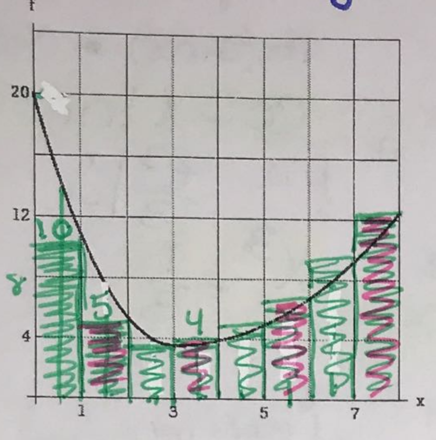
42. g(0)

43. g(4)

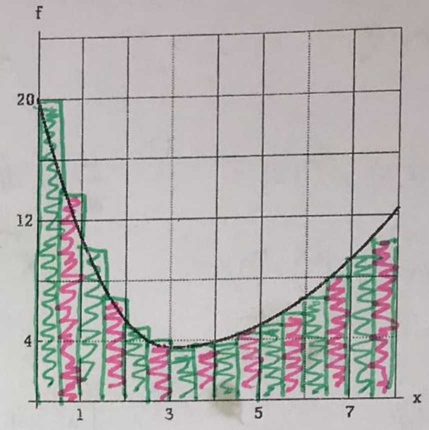
44. g(-3)

16

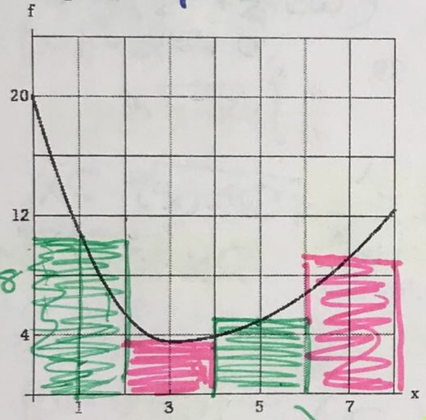
49. Right Riemann Sum 8



50. Left Riemann Sum 16

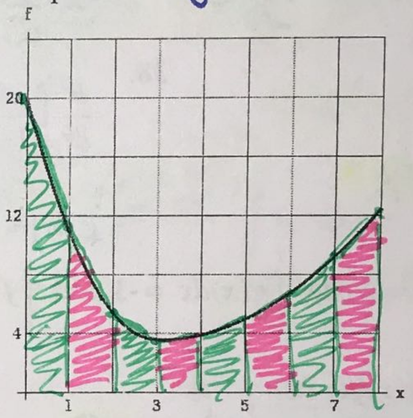


51. Midpoint 4

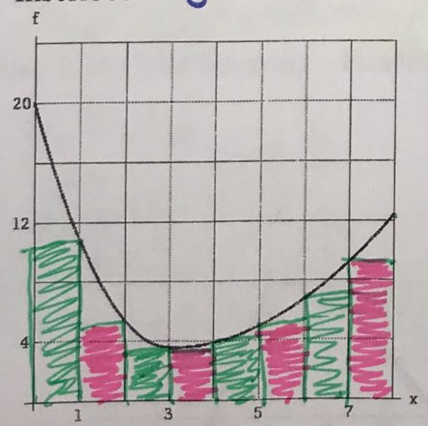


$2(10+3+5+9) = 54$

52. Trapezoid 8



53. Inscribed 8



54. Circumscribed 4