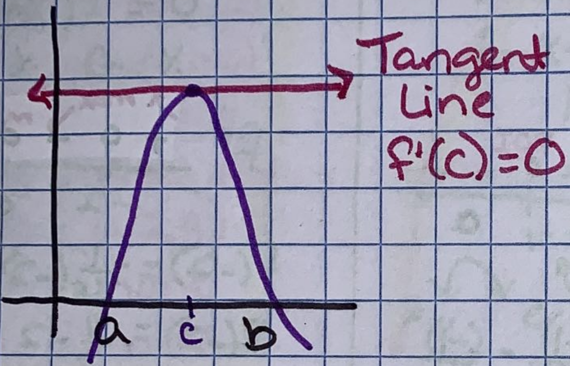
Mean Value Theorem (+ Rolle's Theorem)

Rolle's Theorem:

Suppose $f(x)$ is a function that satisfies all of the following:

1. $f(x)$ is continuous on the closed interval $[a,b]$
2. $f(x)$ is differentiable on the open interval (a,b)
3. $f(a) = f(b)$ (same y)

Then there is a number, c, such that $a < c < b$ + $f'(c) = 0$.
In other words, $f(x)$ has a critical pt. (max/min) in (a,b)



Basically, if a function is continuous + differentiable on an interval + the endpts have the same y-value, there is guaranteed to be at least one max/min within the interval (deriv = 0)

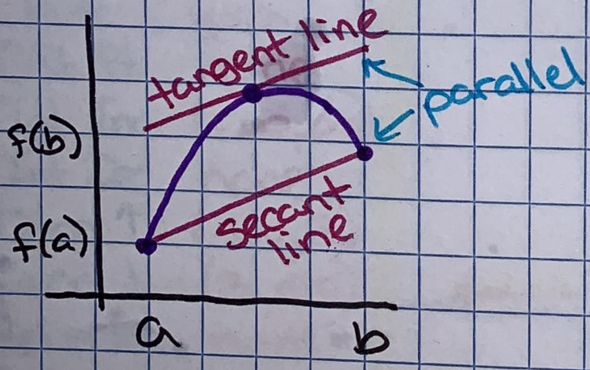
Mean Value Theorem:

Suppose $f(x)$ is a function that satisfies both of the following

1. $f(x)$ is continuous on the closed interval $[a,b]$
2. $f(x)$ is differentiable on the open interval (a,b)

Then there is a number, c, such that $a < c < b$ and

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



Basically, if a function is continuous + differentiable on an interval, then somewhere in $[a,b]$ there is a tangent line that is parallel to the secant line connecting the endpts

Note: Rolle's Theorem is a special case of Mean Value Theorem

1. Let f be a function given by $f(x)=x^3-3x^2$. What are all the values of c that satisfy the conclusion of the Mean Value Theorem on the closed interval $[0,3]$?

need to say this
$f(x)$ is continuous on $[0,3]$
$f(x)$ is differentiable on $(0,3)$
Therefore, there exist a 'c' in $(0,3)$ such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$\underset{a}{f(0)}=\underset{f(a)}{0}$ $\quad \underset{b}{f(3)}=\underset{f(b)}{0}$ $\quad f'(c)=3c^2-6c$

$3c^2-6c=\frac{0-0}{3-0}$

$3c^2-6c=0$

$3c(c-2)=0$

$c=0 \quad c=2$

$c=0$ isn't in $(0,3)$ so only

$\boxed{c=2}$

2. Let $f(x)=\cos 2x$. Find all values of c that satisfy the conclusion of the MVT for $0\le x\le 2\pi$

$f(x)$ is cont. on $[0,2\pi]$
$f(x)$ is diff. on $(0,2\pi)$ $\therefore$ (Therefore) $\exists$ (exists) $c\in$ (in the set) $(0,2\pi)$ $f'(c)=\frac{f(b)-f(a)}{b-a}$

$f(0)=\cos(2\cdot 0)$ $\quad f(2\pi)=\cos(2\cdot 2\pi)$ $\quad f'(x)=-\sin(2x)\cdot 2$

$f(0)=\cos 0=1$ $\quad f(2\pi)=\cos 4\pi=1$ $\quad f'(x)=-2\sin(2x)$

$(0,1)$ $\quad (2\pi,1)$

$-2\sin(2x)=\frac{1-0}{2\pi-0}$

$-2\sin(2x)=0$

$\sin(2x)=0$

$2x=\sin^{-1}0$

$2x=0 \quad + \quad 2x=\pi$

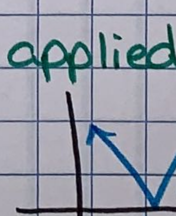
$x=0 \quad x=\frac{\pi}{2}$

not in open interval

$\boxed{c=\frac{\pi}{2}}$

3. Determine whether MVT can be applied for $f(x)=|x-4|$ on $[2,5]$

$f(x)$ is cont. on $[2,5]$
$f(x)$ is not diff. on $[2,5]$



cusp (not diff. at $x=4$)

MVT cannot be applied (or Rolle's)