Limits Review

Limit of a constant is a constant.

1. $\lim_{x\to e}\sqrt{7}$ $\sqrt{7}$

2. $\lim_{x\to\sqrt{5}}\pi$ π

Direct Substitution – ALWAYS try direct substitution first!

3. $\lim_{x\to5}\left(2x^2-x+3\right)$

$2(5)^2-5+3$

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4. $\lim_{y\to2^-}\frac{y^2-3y+2}{y+1}$

$\frac{(2)^2-3(2)+2}{2+1}=\frac{0}{3}=0$

5. $\lim_{x\to4}\frac{|5-3x|}{2x+1}$

$\frac{|5-12|}{8+1}=\frac{7}{9}$

6. $\lim_{x\to4}\cos\left(\frac{3\pi}{x}\right)$

$\cos\frac{3\pi}{4}=\frac{-\sqrt{2}}{2}$

If substitution results in $\frac{0}{0}$, Factor, reduce, and substitute again.

7. $\lim_{x\to1}\frac{x^4-1}{x-1}$ $\frac{(x^2+1)\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$\frac{(1^2+1)(1+1)}{}$

$2\cdot2=4$

8. $\lim_{x\to1}\frac{x-1}{x^3-x^2+x-1}=\frac{x-1}{(x^2+1)(x-1)}=\frac{1}{x^2+1}$

$\frac{1}{1^2+1}=\frac{1}{2}$

9. $\lim_{x\to9}\frac{\sqrt{x}-3}{x-9}=\frac{\cancel{\sqrt{x}-3}}{(\sqrt{x}+3)\cancel{(\sqrt{x}-3)}}$

$\frac{1}{\sqrt{x}+3}=\frac{1}{\sqrt{9}+3}=1/6$

10. $\lim_{x\to-2}\frac{x+2}{x^3+8}$ $\frac{\cancel{x+2}}{\cancel{(x+2)}(x^2-2x+4)}=\frac{1}{x^2-2x+4}$

$\frac{1}{(-2)^2-2(-2)+4}=\frac{1}{12}$

Multiply by the conjugate.

11. $\lim_{x\to2}\frac{\left(\sqrt{5x+6}-4\right)(\sqrt{5x+6}+4)}{(x-2)(\sqrt{5x+6}+4)}$

$\frac{5x+6-16}{(x-2)(\sqrt{5x+6}+4)}=\frac{5x-10}{\text{denom}}=\frac{5\cancel{(x-2)}}{\cancel{(x-2)}\sqrt{5x+6}+4}$

$\frac{5}{\sqrt{5\cdot2+6}+4}=\frac{5}{4+4}=5/8$

12. $\lim_{x\to4}\frac{3-\sqrt{x+5}}{x-4}$

$-1(x-4)$

$\frac{9-(x+5)}{(x-4)(3+\sqrt{x+5})}=\frac{-x+4}{\text{den.}}$

$\frac{-1}{3+\sqrt{x+5}}=\frac{-1}{3+\sqrt{9}}=\frac{-1}{6}$

13. $\lim_{x\to0}\frac{\left(\sqrt{x+3}-\sqrt{3}\right)(\sqrt{x+3}+\sqrt{3})}{x(\sqrt{x+3}+\sqrt{3})}=\frac{\cancel{x}+\cancel{3}-\cancel{3}}{\cancel{x}(\sqrt{x+3}+\sqrt{3})}$

$\frac{1}{\sqrt{x+3}+\sqrt{3}}=\frac{1}{\sqrt{3}+\sqrt{3}}=\frac{1}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$\frac{\sqrt{3}}{6}$

Complex fractions – clear the "little denominators"

14. $\lim_{h\to0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}$

$\frac{\frac{2}{2(2+h)}+\frac{-1(2+h)}{2(2+h)}}{h}=\frac{\frac{-h}{2(2+h)}}{h}$

$\frac{-1}{2(2+h)}=\frac{-1}{2(2+0)}=\frac{-1}{4}$

15. $\lim_{x\to10}\frac{\frac{x}{5}-2}{x-10}$

$\frac{\frac{x}{5}-\frac{10}{5}}{x-10}=\frac{\frac{x-10}{5}}{x-10}$

$\frac{\cancel{x-10}}{5}\cdot\frac{1}{\cancel{x-10}}=\frac{1}{5}$

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16. $\lim_{h\to-2}\frac{(h+5)^{-1}-3^{-1}}{h+2}=\frac{\frac{1}{h+5}-\frac{1}{3}}{h+2}$

$\frac{\frac{3}{3(h+5)}+\frac{-1(h+5)}{3(h+5)}}{h+2}=\frac{\frac{-h-2}{3(h+5)}}{h+2}=$

$\frac{-1\cancel{(h+2)}}{3(h+5)}\cdot\frac{1}{\cancel{h+2}}=\frac{-1}{3(h+5)}=\frac{-1}{3(-2+5)}=\frac{-1}{9}$

Rewrite the absolute value.

Reminder, if the inside is positive when you substitute in use the positive of the inside, if the inside is negative when you substitute in use the negative of the inside.

17. $\lim_{x\to 5^-} \frac{|2x-10|}{3x-15}$ $\frac{2|x-5|}{3(x-5)}$

18. $\lim_{x\to 7^-} \frac{3x-21}{|7-x|}$

19. $\lim_{x\to 2} \frac{|x-2|}{x-2}$

One sided limits when you get $\frac{\#}{0}$, do you get ∞ or $-\infty$? Reason it out!

20. $\lim_{x\to 3^-} \frac{5}{x-3}$

$\frac{5}{2.99-3} = \frac{5}{-.01} = -500$

$-\infty$

21. $\lim_{x\to 3^+} \frac{-4}{x-3}$

$\frac{-4}{3.01-3} = \frac{-4}{.01} = -400$

$-\infty$

22. $\lim_{x\to 6^+} \frac{x+6}{x^2-36}$ $\frac{\cancel{x+6}}{\cancel{(x+6)}(x-6)} = \frac{1}{x-6}$

$\frac{1}{6.01-6} = \frac{1}{.01} = 100$

∞

Limits to infinity. You can do a **behaves like** only in limits to infinity. You can also divide by the highest power in the denominator, simplify, and then take the limit.

$\lim_{x\to\pm\infty}$(polynomial) The highest power controls the behavior!

23. $\lim_{x\to\infty}(3x^2-4x+2)$ even +

$+\infty$

24. $\lim_{x\to-\infty}(5x^3-2x^2+1)$ odd +

$-\infty$

$\lim_{x\to\pm\infty} \frac{\text{degree smaller}}{\text{DEGREE LARGER}} = 0$

25. $\lim_{x\to\infty} \frac{3x-5}{x^2+1}$ 0

26. $\lim_{x\to-\infty} \frac{4x^2-3x}{6x^5-3x+1}$ 0

$\lim_{x\to\pm\infty} \frac{\text{degree} =}{\text{degree} =}$ = ratio of the leading coefficients

27. $\lim_{x\to\infty} \frac{5x-11}{4-3x} = \frac{5x-11}{-3x+4}$

$-5/3$

28. $\lim_{x\to-\infty} \frac{4x^2-5x+2}{3x^2+1}$

$4/3$

29. $\lim_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)}$ $\frac{2x^2+1}{4-x^2} = \frac{2x^2+1}{-x^2+4}$

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$\lim_{x\to\pm\infty} \frac{\text{DEGREE LARGER}}{\text{degree smaller}} = \infty \text{ or } -\infty$

30. $\lim_{x\to\infty} \frac{7-6x^5}{x+3}$

$\frac{-\infty}{\infty}$

$-\infty$

31. $\lim_{x\to-\infty} \frac{7-6x^5}{x+3}$

$\frac{\infty}{-\infty}$

$-\infty$

32. $\lim_{x\to\infty} \frac{5+x^3-3x^4}{2x-1}$ + + $-\infty$

$\frac{-\infty}{+\infty}$

$-\infty$

33. $\lim_{x\to-\infty} \frac{5+x^3-3x^4}{2x-1}$

$\frac{-\infty}{-\infty}$

$+\infty$

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$\lim_{x\to\pm\infty}$ involving square roots: Use the **behaves like** method and remember that $\sqrt{x^2} = |x|$!

34. $\lim_{x\to\infty} \frac{\sqrt{4x^2-2}}{x+1}$ 35. $\lim_{x\to-\infty} \frac{\sqrt{4x^2-2}}{x+1}$ 36. $\lim_{x\to\infty} \frac{2-x}{\sqrt{7+9x^2}}$ 37. $\lim_{x\to-\infty} \frac{2-x}{\sqrt{7+9x^2}}$

Write the equations of all vertical and horizontal asymptotes.

38. $y = \frac{2x^2-5x-3}{x^2-2x-3} = \frac{(2x+1)(x-3)}{(x+1)(x-3)}$

VA: $x=-1$
HA: $y=2$
hole at $x=3$

39. $y = \frac{3-x}{9-x^2} = \frac{3-x}{(3+x)(3-x)} = \frac{1}{x+3}$

VA: $x=-3$
HA: $y=0$
hole at $x=3$

Continuity: Limit from right = limit from left = value of $f(x)$ at the point.

Is $f(x)$ continuous? Why?

40. $f(x) = \begin{cases} -5-x, & x > -1 \\ 6x+2, & x \le -1 \end{cases}$

$\lim_{x\to-1^-} = -4$ $\lim_{x\to-1^+} = -4$

$f(-1) = -4$ continuous

41. $f(x) = \frac{|x+2|}{x+2}$

no

Jump discontinuity at $x=-2$

$x+2=0$
$x \neq -2$

x	y
2	1
1	1
0	1
-1	1
-2	DNE

x	y
-3	-1
-4	-1
-5	-1

Intermediate Value Theorem.

42. **Verify** the conditions of the Intermediate Value Theorem, and **find** c guaranteed by the theorem when $f(x) = x^2 - 6x + 7$ over the interval $[0,3]$ and $f(c) = -1$.

$f(0)=7$
$f(3)=-2$
$-2 < -1 < 7$

$-1 = x^2-6x+7$
$0 = x^2-6x+8$
$0=(x-4)(x-2)$
$x \neq 4$ $x=2$

Finding values that make a function continuous.

43. Find the value of a that would make the function continuous.

$f(x) = \begin{cases} 3xa+5 & \text{if } x \le -1 \\ -2x+5a & \text{if } x > -1 \end{cases}$

$3xa+5 = -2x+5a$
$3(-1)a+5 = -2(-1)+5a$
$-3a+5 = 2+5a$
$-8a = -3$
$a = 3/8$

44. Find the value of m and n that would make the function continuous.

$g(x) = \begin{cases} 3mx-4n & \text{if} & x \le -1 \\ 4+nx-mx^2 & \text{if} & -1 < x < 2 \\ x^2-mx+7n & \text{if} & x \ge 2 \end{cases}$

$3mx-4n = 4+nx-mx^2$
$3m(-1)-4n = 4+n(-1)-m(-1)^2$
$-3m-4n = 4-n-m$
$-2m-3n = 4$
$-2m = 3n+4$
$m = -\frac{3}{2}n-2 \rightarrow -\frac{3}{2}n-2 = -\frac{5}{2}n$
$-3n-4 = -5n$
$2n = 4$

$4+nx-mx^2 = x^2-mx+7n$
$4+2n-4m = 4-2m+7n$
$-5n-2m = 0$
$-2m = 5n$
$m = -5/2n$

$\boxed{n=2}$ $\boxed{m=-5}$