Calculus : Limits and Continuity Name ____________

Limits and Continuity Review Date ____________ Period ______

LIMITS REVIEW

$(3-x)(3-x)$

$9-6x+x^2$

Use the properties and factoring techniques to find each limit.

1. $\lim_{x\to 0}\dfrac{9-4x}{2x^3-4x^2+3}$

$\dfrac{9-4(0)}{2(0)^3-4(0)^2+3}=\dfrac{9}{3}$

3

2. $\lim_{x\to 2}\dfrac{2x^2+x-10}{x^2+x-6}=\dfrac{(2x+5)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$

$\dfrac{2\cdot 2+5}{2+3}=\dfrac{9}{5}$

3. $\lim_{x\to -3}\dfrac{x^3+27}{x+3}$ $\dfrac{\cancel{(x+3)}(x^2-3x+9)}{\cancel{(x+3)}}$

$(-3)^2-3(-3)+9$

$9+9+9=27$

4. $\lim_{x\to 5}\dfrac{x}{x^2-25}=\dfrac{5}{0}$

$\varnothing$ DNE

5. $\lim_{x\to 0}\dfrac{x^3-8}{x^2-4}$

$\dfrac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}$

$\dfrac{4}{2}=2$

6. $\lim_{x\to 3}\dfrac{(3-x)^2}{x-3}$ $\dfrac{x^2-6x+9}{x-3}$

$\dfrac{(x-3)\cancel{(x-3)}}{\cancel{(x-3)}}=x-3=0$

7. $\lim_{\theta\to 0}\dfrac{1-\cos\theta}{2\sin^2\theta}$

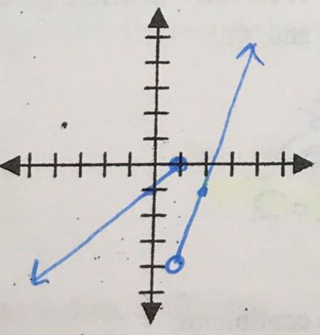
8. $\lim_{x\to -1}\dfrac{x^4-1}{x+1}$

$\dfrac{(x^2+1)\cancel{(x+1)}(x-1)}{\cancel{(x+1)}}=((-1)^2+1)(-1-1)$

$2(-2)=-4$

For problems 9-12, use the function $f(x)=\begin{cases}x-1, & x\le 1\\ 3x-7, & x>1\end{cases}$.

9. Graph the function.



10. $\lim_{x\to 1^-} f(x)$ 0

11. $\lim_{x\to 1^+} f(x)$ -4

12. $\lim_{x\to 1} f(x)$ DNE

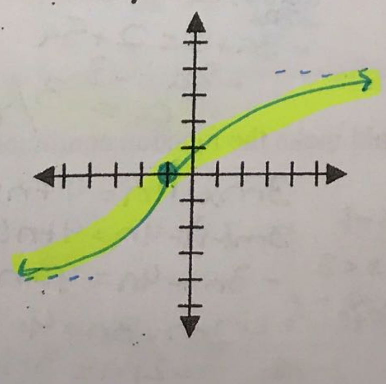
13. Draw a function that meets the following conditions. Is this function continuous? Explain.

$\lim_{x\to\infty} f(x)=4$

$\lim_{x\to -1} f(x)=0$

$\lim_{x\to -\infty} f(x)=-4$

$f(-1)=0$



yes

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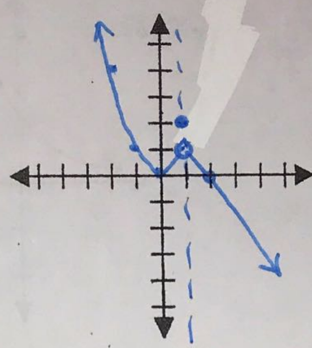
14. Draw a function that meets the following conditions. Find the indicated limit if it exists. Is this function continuous? Explain.

$$f(x)=\begin{cases} x^2, & x<1 \\ 2-x, & x>1 \\ 2, & x=1 \end{cases}$$



15. $\lim\limits_{x\to1^+} f(x)$ 1

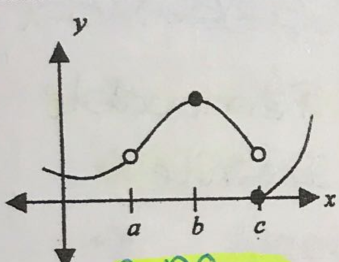
16. $\lim\limits_{x\to1^-} f(x)$ 1

17. $\lim\limits_{x\to1} f(x)$ 1

18. $f(1)$ 2

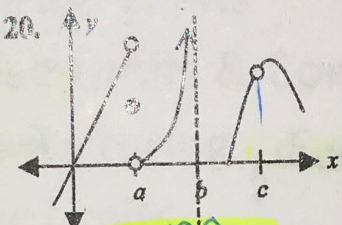
Indicate whether the function whose graph is given is continuous at each of the points *a*, *b*, and *c*.

19.



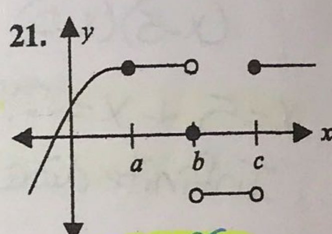
a. no
b. yes
c. no

20.



a. no
b. no
c. no

21.



a. yes
b. no
c. no

CONTINUITY REVIEW

Find a value for *k* which will cause $f(x)$ to be continuous for all real *x*.

22. $f(x)=\begin{cases} kx^2, & \text{if } x<-3 \\ 5-4x, & \text{if } x\ge -3 \end{cases}$

$9k=5+12$

$9k=17$

$k=17/9$

23. $f(x)=\begin{cases} x^3, & \text{if } x<\frac{1}{2} \\ kx^2, & \text{if } x\ge\frac{1}{2} \end{cases}$

$(1/2)^3=k(\frac{1}{2})^2$

$\frac{1}{8}=\frac{1}{4}k$

$k=1/2$

24. Define $f(3)$ so that $f(x)=\dfrac{x^2-9}{x-3}$ is continuous at $x=3$.

$\dfrac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}=x+3$

$3+3$

$x=3$ $(3,6)$

25. Define $f(1)$ so that $f(x)=\dfrac{x^3-1}{x^2-1}$ is continuous at $x=1$.

$\dfrac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)}=\dfrac{(1)^2+1+1}{1+1}=3/2$

hole at $x=1$

$(1,3/2)$

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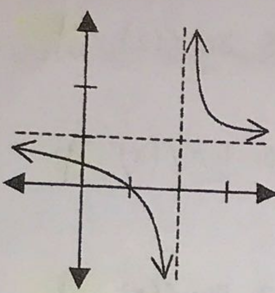
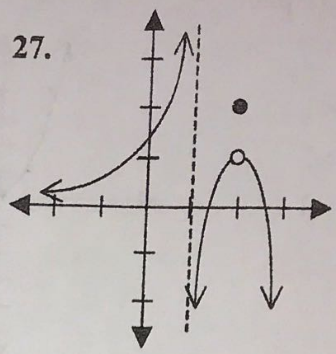
Use the graph to determine the intervals for which the function is continuous.

26.



$(-\infty, 2) \cup (2, \infty)$

27.



$(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

At what values are the following functions <u>discontinuous</u>? Explain the type of discontinuity.

28. $f(x)=\dfrac{x+3}{x^2-3x-10}$

$(x-5)(x+2)$

$x=5$ + $x=-2$

Infinite discontinuity

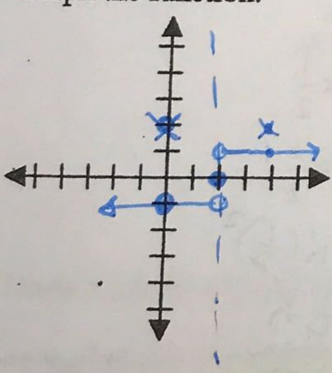
29. $f(x)=\dfrac{x+2}{4-x^2}$ $= \dfrac{\cancel{x+2}}{(2-x)\cancel{(2+x)}}$ $\dfrac{1}{2-x}$

$x=-2$ Removable

$x=2$ Infinite

For problems 30-42, use the function $f(x)=\begin{cases}\dfrac{|x-2|}{x-2}, & x\neq 2\\ 0, & x=2\end{cases}$.

30. Graph the function.



31. domain: $(-\infty, 2) \cup (2, \infty)$

range: $\{-1, 0, 1\}$

32. $f(0)$ -1

33. $f(2)$ 0

34. $f(4)$ 1

35. $\lim\limits_{x\to 0^+} f(x)$ -1

36. $\lim\limits_{x\to 0^-} f(x)$ -1

37. $\lim\limits_{x\to 0} f(x)$ ~~DNE~~ -1

38. Is $f(x)$ continuous at $x=0$? Explain.

yes

39. $\lim\limits_{x\to 2^-} f(x)$ -1

40. $\lim\limits_{x\to 2^+} f(x)$ 1

41. $\lim\limits_{x\to 2} f(x)$ DNE

42. Is $f(x)$ continuous at $x=2$? Explain. no

$\lim\limits_{x\to 2}$ = DNE + $f(2)=0$

Jump