Algebra II
Ch. 4: Matrices Review Sheet

Name Key

Area?

CALCULATOR SECTION

Find the determinant. For #1&2

1. $\begin{bmatrix} 2 & 1 & 1 \\ 7 & 4 & -3 \\ -1 & 5 & 1 \end{bmatrix}$ 73

2. $\begin{bmatrix} 10 & 4 & 6 \\ 2 & -3 & 1 \\ -3 & 2 & 0 \end{bmatrix}$ −62

3. Multiply if possible. $\begin{bmatrix} -1 & 4 & -2 \\ 1 & -1 & -4 \end{bmatrix} \cdot \begin{bmatrix} -6 & 2 & -5 \\ 1 & -4 & 2 \\ 3 & 0 & -1 \end{bmatrix}$

2×3 3×3

$\begin{bmatrix} 4 & -18 & 15 \\ -19 & 6 & -3 \end{bmatrix}$

Find the inverse, if it exists.

4. $\begin{bmatrix} 21 & 12 \\ 7 & 4 \end{bmatrix}$ DNE

5. $\begin{bmatrix} -7 & -1 & 2 \\ 3 & 6 & 4 \\ 0 & 11 & -2 \end{bmatrix}$

$\begin{bmatrix} \frac{-14}{113} & \frac{5}{113} & \frac{-4}{113} \\ \frac{3}{226} & \frac{7}{226} & \frac{17}{226} \\ \frac{33}{452} & \frac{77}{452} & \frac{-39}{452} \end{bmatrix}$

6. Write as a matrix equation & solve.

$3x + 4y + 2z = 12$
$-2x - 3y - 4z = -12$
$5x + 5y + 6z = 8$

$\begin{bmatrix} 3 & 4 & 2 \\ -2 & -3 & -4 \\ 5 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ 8 \end{bmatrix}$

$\begin{bmatrix} -7 \\ 8 \\ \frac{1}{2} \end{bmatrix}$ $(-7, 8, \frac{1}{2})$

7. Write as a matrix equation & solve.

$2x + z = 6$
$3x - 2y + 4z = 13$
$-y - 3z = -15$

$\begin{bmatrix} 2 & 0 & 1 \\ 3 & -2 & 4 \\ 0 & -1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \\ -15 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$ $(1, 3, 4)$

NON-CALCULATOR SECTION

Name the dimensions of the matrices.

8. $\begin{bmatrix} 3 & 2 & 1 \\ -5 & 6 & -3 \end{bmatrix}$ 2×3

9. $\begin{bmatrix} 8 \\ 7 \\ -2 \\ 3 \end{bmatrix}$ 4×1

10. $\begin{bmatrix} 2 & 43 \end{bmatrix}$ 1×2

Perform indicated operations. #11-15

11. $-3\begin{bmatrix} 8 & -3 & 2 \\ 4 & -1 & 7 \end{bmatrix}$

$\begin{bmatrix} -24 & 9 & -6 \\ -12 & 3 & -21 \end{bmatrix}$

12. $2\begin{bmatrix} 8 & -1 \\ 3 & 4 \end{bmatrix} - 3\begin{bmatrix} -1 & 2 \\ 6 & 0 \end{bmatrix}$

$\begin{bmatrix} 16 & -2 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 3 & -6 \\ -18 & 0 \end{bmatrix}$

$\begin{bmatrix} 19 & -8 \\ -12 & 8 \end{bmatrix}$

13. $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \bullet \begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix}$

2×2 2×3

$\begin{bmatrix} 6+24 & -4+0 & 14-20 \\ 9-6 & -6+0 & 21+5 \end{bmatrix}$

$\begin{bmatrix} 30 & -4 & -6 \\ 3 & -6 & 26 \end{bmatrix}$

14. $\begin{bmatrix} -4 & 0 & -8 \\ 7 & -2 & 10 \end{bmatrix} \bullet \begin{bmatrix} -1 & 2 \\ 6 & 0 \end{bmatrix}$

2×3 2×2

CANNOT MULTIPLY!

15. $2\begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} + 4\begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ 9 \\ -6 \end{bmatrix}$

$\begin{bmatrix} 6 \\ -2 \\ 8 \end{bmatrix} + \begin{bmatrix} 0 \\ -20 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ -9 \\ 6 \end{bmatrix}$

$\begin{bmatrix} 7 \\ -31 \\ 18 \end{bmatrix}$

- Know identity matrices on test (#1)
- Determinant: write matrix (#2)
- Dimensions (#3)
- (#4)

Solve for x.
Hint: There are bars, not brackets, around the matrices.

16. $\begin{vmatrix} 3 & -4 \\ 2x & 6 \end{vmatrix} = 34$

$18 + 8x = 34$
$-18 \quad -18$
$8x = 16$
$8 \quad 8$
$x = 2$

17. $\begin{vmatrix} 2 & -1 \\ 3 & 4x \end{vmatrix} = -16$

$8x + 3 = -16$
-3
$8x = -19$
8
$x = -\frac{19}{8}$

18. Write the system as a matrix equation. DO NOT SOLVE!

$2w - x + 5y - z = 1$
$x + 3y - 6z = 2$
$-3w - 9z = 12$
$2z = 6$

$\begin{bmatrix} 2 & -1 & 5 & -1 \\ 0 & 1 & 3 & -6 \\ -3 & 0 & 0 & -9 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 12 \\ 6 \end{bmatrix}$

Fill in the blank.

19a. Two matrices may be added if dimensions are exactly the same.

19b. Two matrices may be multiplied if the inner dimensions match

19c. A matrix must be SQUARE in order for it to have a determinant or an inverse.

Solve the system using matrices. Write the solutions as ordered pairs.

20. $2x + 3y = 7$
$4x - 4y = 4$

$\begin{bmatrix} 2 & 3 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$

$\frac{1}{-8-12} \begin{bmatrix} -4 & -3 \\ -4 & 2 \end{bmatrix}$

$\frac{1}{-20} \begin{bmatrix} -4 & -3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ 4 \end{bmatrix} = \frac{1}{-20} \begin{bmatrix} -28 + -12 \\ -28 + 8 \end{bmatrix}$

$\frac{1}{-20} \begin{bmatrix} -40 \\ -20 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $(2, 1)$

21. $-5x - y = 2$
$10x + 3y = 1$

$\begin{bmatrix} -5 & -1 \\ 10 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\frac{1}{-15+10} \begin{bmatrix} 3 & 1 \\ -10 & -5 \end{bmatrix}$

$\frac{1}{-5} \begin{bmatrix} 3 & 1 \\ -10 & -5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} 6+1 \\ -20-5 \end{bmatrix}$

$= -\frac{1}{5} \begin{bmatrix} 7 \\ -25 \end{bmatrix} = \begin{bmatrix} -\frac{7}{5} \\ +5 \end{bmatrix}$ $\left(-\frac{7}{5}, +5\right)$

Pre-Calculus
REVIEW 2: Matrices

Name Key

NO CALCULATOR!!

D 1. Which of the following matrices has an inverse?

A. $\begin{bmatrix} 4 & 8 \\ 2 & 4 \end{bmatrix}$ B. $[1]$ C. $[3 \quad 5 \quad 1]$ D. $\begin{bmatrix} 3 & 4 \\ 4 & 6 \end{bmatrix}$

2. Find A^{-1} if $A = \begin{bmatrix} 10 & -5 \\ 7 & 2 \end{bmatrix}$. *(Circle your final answer.)*

$\frac{1}{20--35}\begin{bmatrix} 2 & 5 \\ -7 & 10 \end{bmatrix} = \frac{1}{55}\begin{bmatrix} 2 & 5 \\ -7 & 10 \end{bmatrix} = \begin{bmatrix} \frac{2}{55} & \frac{5}{55} \\ \frac{-7}{55} & \frac{10}{55} \end{bmatrix} = \begin{bmatrix} \frac{2}{55} & \frac{1}{11} \\ \frac{-7}{55} & \frac{2}{11} \end{bmatrix}$

3. What would make a square matrix have an inverse that does not exist? Give an example. Det = 0

$\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ → Answers will vary

4. What are the dimensions of matrix M if $M \cdot A_{1\times4} = B_{2\times4}$? 2 x 1

5. Let $A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 2 \\ 1 & -1 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ $D = \begin{bmatrix} 1 & -3 \\ -1 & 2 \\ -2 & 3 \end{bmatrix}$

Evaluate. If not possible, state why. *(Circle your final answer.)*

a) $2C - B$

$\begin{bmatrix} 4 & 6 \\ -2 & 8 \end{bmatrix} + \begin{bmatrix} +4 & -2 \\ -1 & +1 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ -3 & 9 \end{bmatrix}$

b) $AB + C$ Not possible

c) C^{-1}

$\frac{1}{8--3}\begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$

$\frac{1}{11}\begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{-3}{11} \\ \frac{1}{11} & \frac{2}{11} \end{bmatrix}$

d) Det D → Not possible

e) $|C|$

$8 + 3 = 11$

6. **Identity Matrix**: Know properties of identity matrix (section 1.11). (e.g. what do you multiply to get the identity matrix as your answer, what do you get if multiply by identity matrix, etc…)

2×2 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 3×3 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ etc.

___7. You are planning a birthday party for your younger brother at a skating rink. The cost of admission is $3.50 per adult and $2.25 per child. The cost of skate rental is $5.25 for an adult and $3.75 for a child. The cost of refreshments provided at the party is $3 regardless of age.

a) Display this information in a matrix C. Label the rows (age) and columns (purchase).

	Admission	Skate Rental	Refreshments
Adult	$3.50	5.25	3
Child	2.25	3.75	3

b) Explain (in words) C_{12} and C_{23}.

C_{12} = Skate rental for adult is $5.25

C_{23} = Ref. for child is $3.

7. Find x and y: $2x\begin{bmatrix} -2 & -1 \\ -10 & 5 \end{bmatrix} = \begin{bmatrix} -16 & -8 \\ y & 40 \end{bmatrix}$ *(Circle your final answer.)*

$\begin{bmatrix} -4x & -2x \\ -20x & 10x \end{bmatrix} = \begin{bmatrix} -16 & -8 \\ y & 40 \end{bmatrix}$

$\frac{-4x}{-4} = \frac{-16}{-4}$

$x = 4$

$-20(4) = y$

$-80 = y$

$(4, -80)$

CALCULATOR

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8. What is the area of the triangle in square units with the given vertices?

(Circle your final answer.)

$A = \pm\frac{1}{2}\begin{vmatrix} -3 & 4 & 1 \\ 6 & 3 & 1 \\ 2 & -1 & 1 \end{vmatrix}$

$\pm\frac{1}{2}(-40) =$ 20 units²