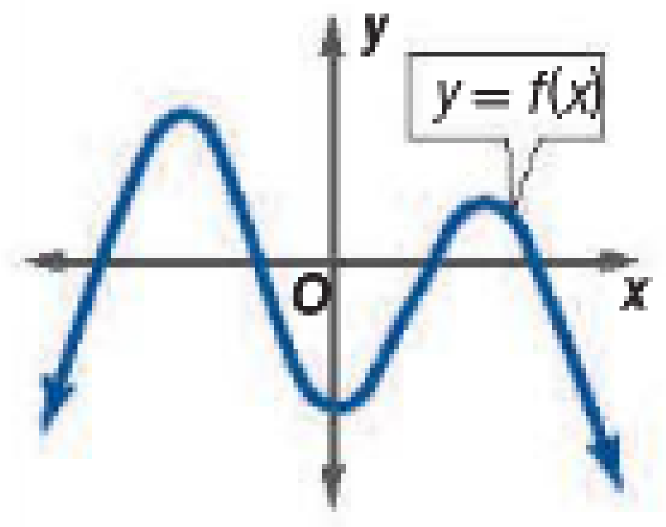
CONTINUITY

Keeper 9b

Honors Calculus

CONTINUOUS FUNCTIONS

The graph of a continuous function has no breaks, holes, or gaps. You can trace the graph of a continuous function without lifting your pencil.



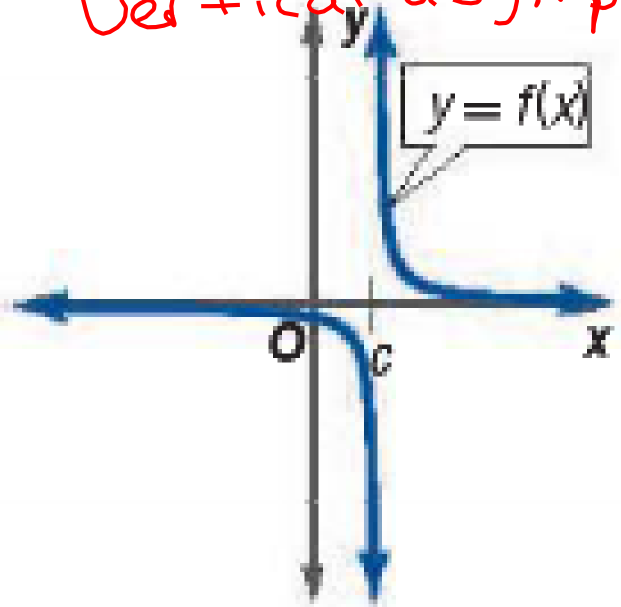
$f(x)$ is continuous for all x.

TYPES OF DISCONTINUITY

A function has an **infinite discontinuity** at $x = c$ if the function value increases or decreases indefinitely as x approaches c from the left and right.

Example

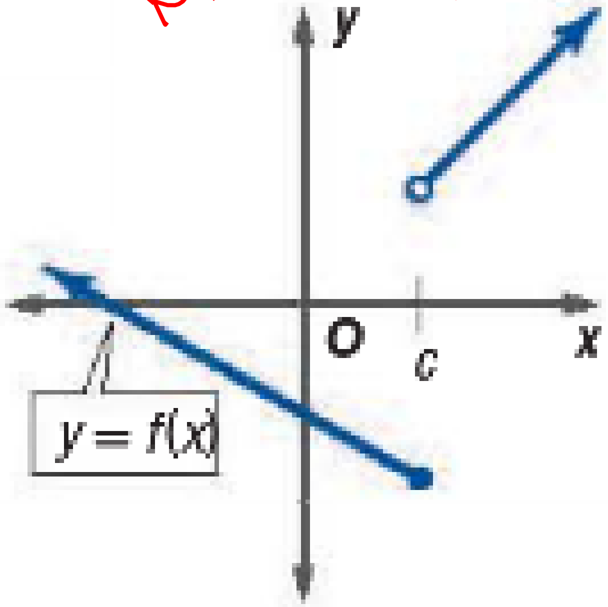
occurs at vertical asymptotes



A function has a **jump discontinuity** at $x = c$ if the limits of the function as x approaches c from the left and right exist but have two distinct values.

Example

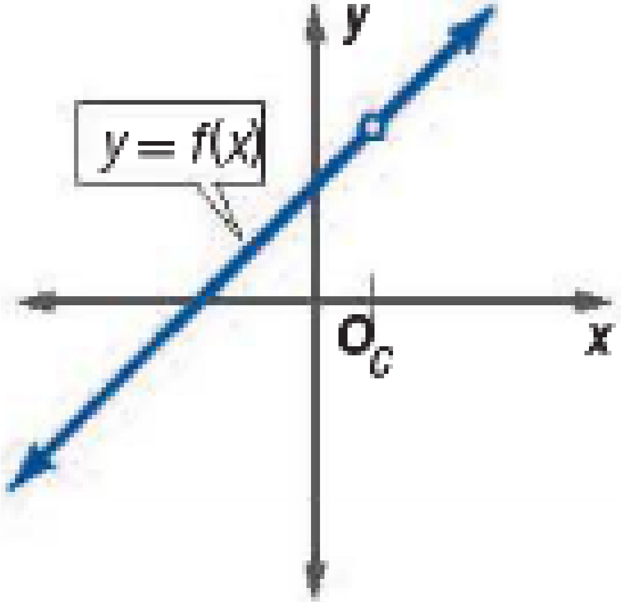
occurs in piecewise functions



A function has a **removable discontinuity** if the function is continuous everywhere except for a hole at $x = c$.

occurs when there is a hole in the graph

Example



CONTINUITY TEST

A function $f(x)$ is continuous at $x = c$ if it satisfies the following conditions.

$$\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = \lim_{x \to c} f(x) = f(c)$$

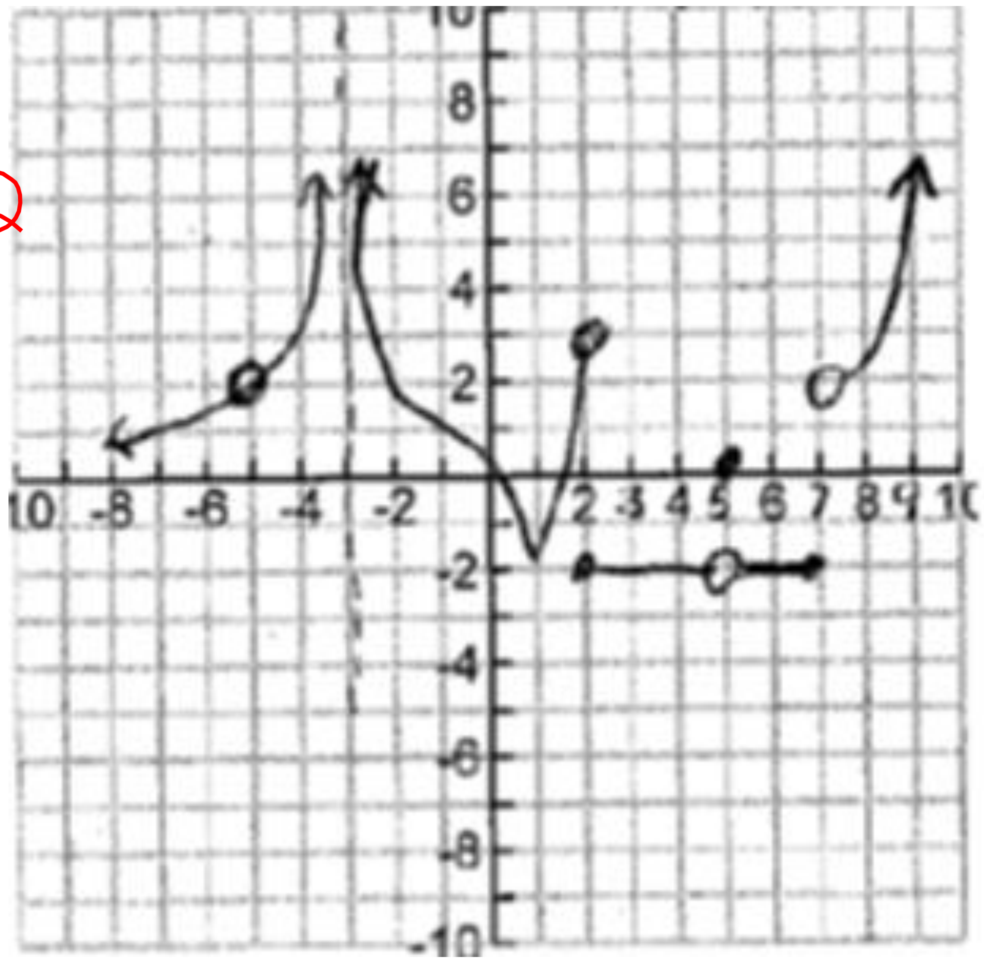
UNDERSTANDING CONTINUITY

a. Does $f(5)$ exist? yes; at 0

b. Does $\lim_{x \to 5} f(x)$ exist? yes; at -2

c. Is $f(x)$ continuous at $x = 5$? Justify. no; $f(5) \neq \lim_{x \to 5} f(x)$

d. What new value should be assigned to $f(5)$ to remove the discontinuity? -2

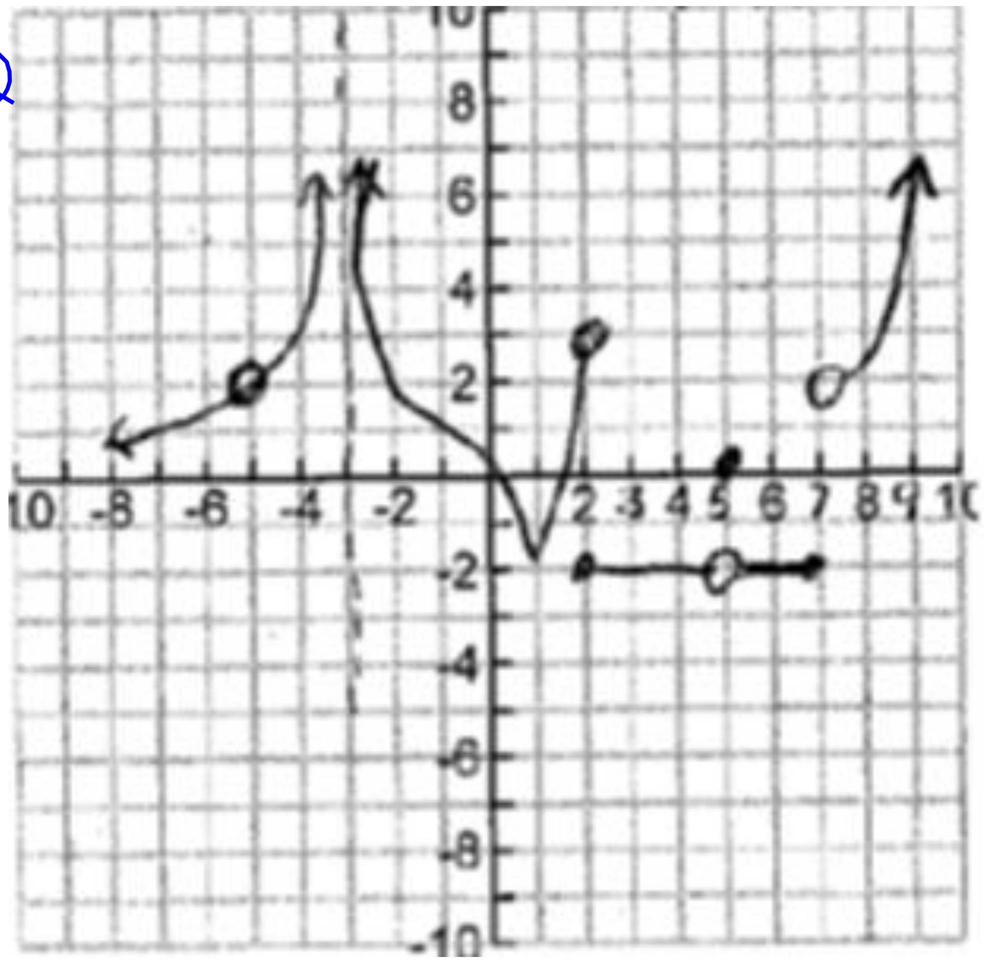


UNDERSTANDING CONTINUITY

e. Does $f(2)$ exist? yes at -2

f. Does $\lim\limits_{x \to 2} f(x)$ exist?

no



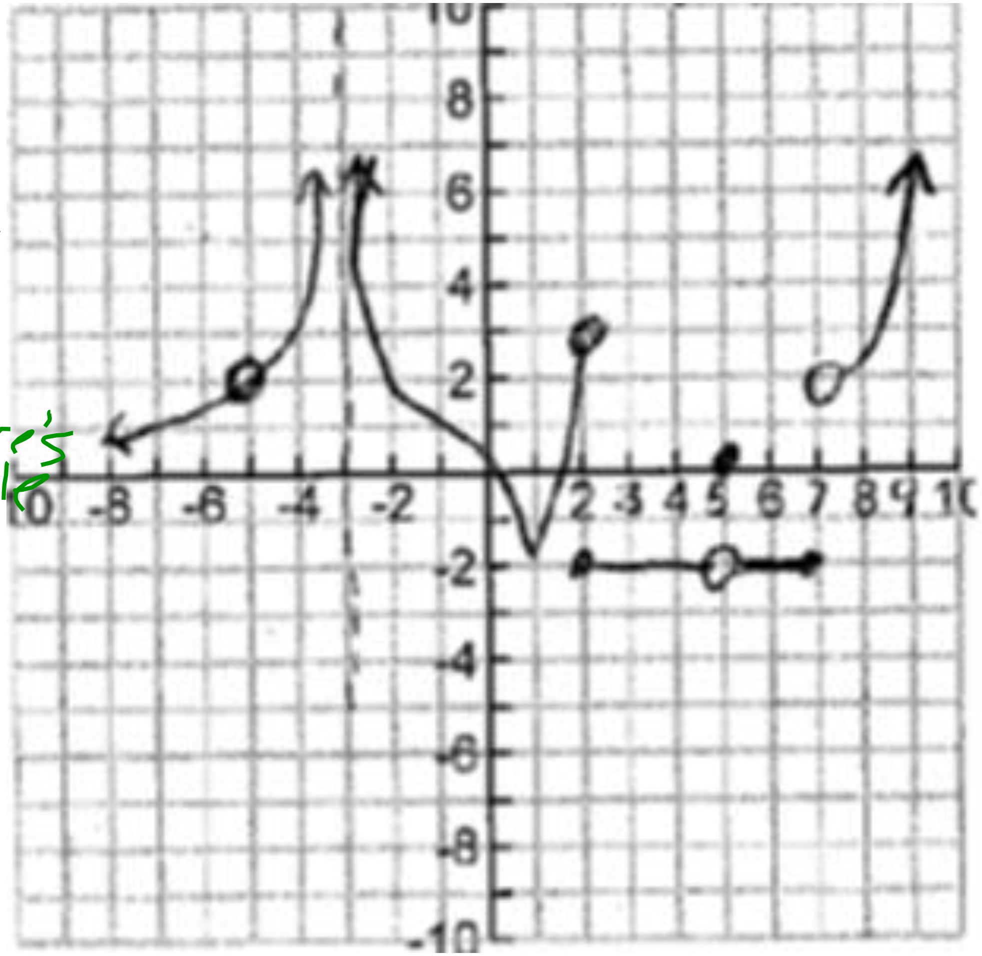
UNDERSTANDING CONTINUITY

g. Does $f(-5)$ exist? no

h. Does $\lim_{x \to -5} f(x)$ exist? yes at 2

i. Is $f(x)$ continuous at $x = -5$? Justify. no ; $f(-5) \neq \lim_{x \to -5} f(x)$ There's a hole

j. What new value should be assigned to $f(-5)$ to make $f(x)$ continuous at $x = -5$? 2



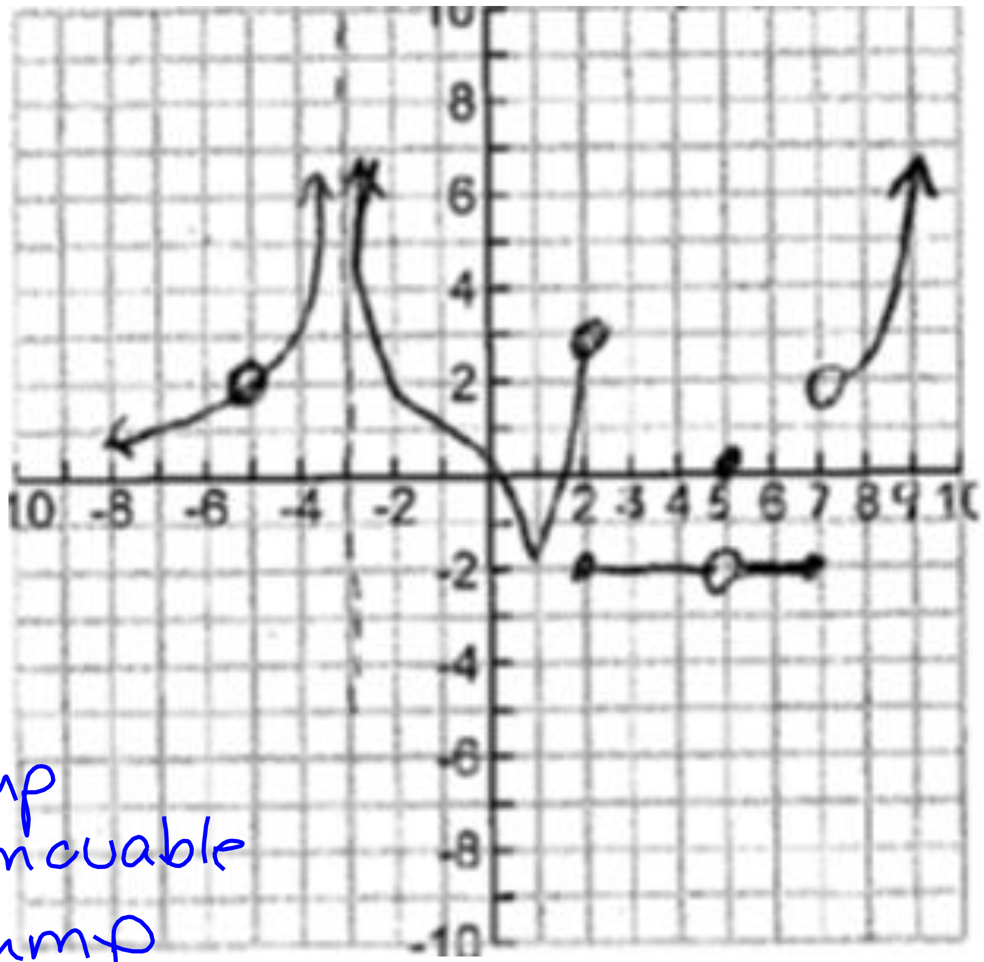
UNDERSTANDING CONTINUITY

look for included point

k. Is $f(x)$ right continuous, left continuous, or neither at $x = 2$? How about for $x = 7$?

right   left

l. List all places where $f(x)$ is discontinuous and state the type of discontinuity.



X=-5 removable
X=-3 infinite

X=2 jump
X=5 removable
X=7 jump

IDENTIFY THE TYPE OF DISCONTINUITY IN THE FOLLOWING EQUATIONS

a. $h(x) = \dfrac{6}{x-3}$

$x - 3 \neq 0$
$x = 3$ V.A.

$\boxed{\text{Infinite at } x = 3}$

b. $p(x) = \begin{cases} 3x - 1, & if\ x \geq 1 \\ 4x - 2, & if\ x < 1 \end{cases}$

$\lim_{x \to 1^+} 3(1) - 1 = 2$ ✓ $\lim_{x \to 1^-} 4(1) - 2 = 2$ ✓

$p(1) = 2$ ✓ $\boxed{\text{continuous}}$

c. $m(x) = \begin{cases} 2x - 5, & if\ x \geq 2 \\ 3x, & if\ x < 2 \end{cases}$

$\lim_{x \to 2^+} = 2(2) - 5 = -1$

$\lim_{x \to 2^-} = 3(2) = 6$

$\boxed{\text{Jump at } x = 2}$

d. $k(x) = \dfrac{6x - 2}{9x - 3} = \dfrac{2\cancel{(3x - 1)}}{3\cancel{(3x - 1)}} = \dfrac{2}{3}$

creates a hole

$3x - 1 = 0$

$\boxed{\text{Removable @ } x = 1/3}$

e. $j(x) = \dfrac{2x - 4}{x^2 - 2x}$ $\dfrac{2\cancel{(x-2)}}{x\cancel{(x-2)}} = \dfrac{2}{x}$

$\boxed{\text{Removable at } x = 2 \text{ \& Infinite at } x = 0}$

FINDING VALUES FOR DISCONTINUITY

Find a value for a so that $f(x)$ is continuous.

$$f(x) = \begin{cases} 2x + 3, & if\ x \leq 2 \\ ax + 1, & if\ x > 2 \end{cases}$$

plug in 2
for x

$2x + 3 = ax + 1$

$2(2) + 3 = a(2) + 1$

$7 = 2a + 1$

$a = 3$

FIND THE INTERVALS ON WHICH THE FUNCTION IS CONTINUOUS

1. $f(x) = \frac{x-3}{x^2-9} = \frac{\cancel{x-3}}{(x+3)(\cancel{x-3})} = \frac{1}{x+3}$

$x \neq 3$ hole

$x \neq -3$ VA

$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

2. $f(x) = \begin{cases} x^2, & x \geq 0 \\ -3, & x < 0 \end{cases}$

$(-\infty, 0) \cup (0, \infty)$

3. $f(x) = x^2 - x - 12$

$(-\infty, \infty)$