ONE-SIDED LIMITS

Keeper 9

Honors Calculus

P. 20

FINDING ONE-SIDED LIMITS

One-Sided limits are the same as normal limits, we just restrict x so that it approaches from just one side.

$x \to a^+$ means x is approaching from the right

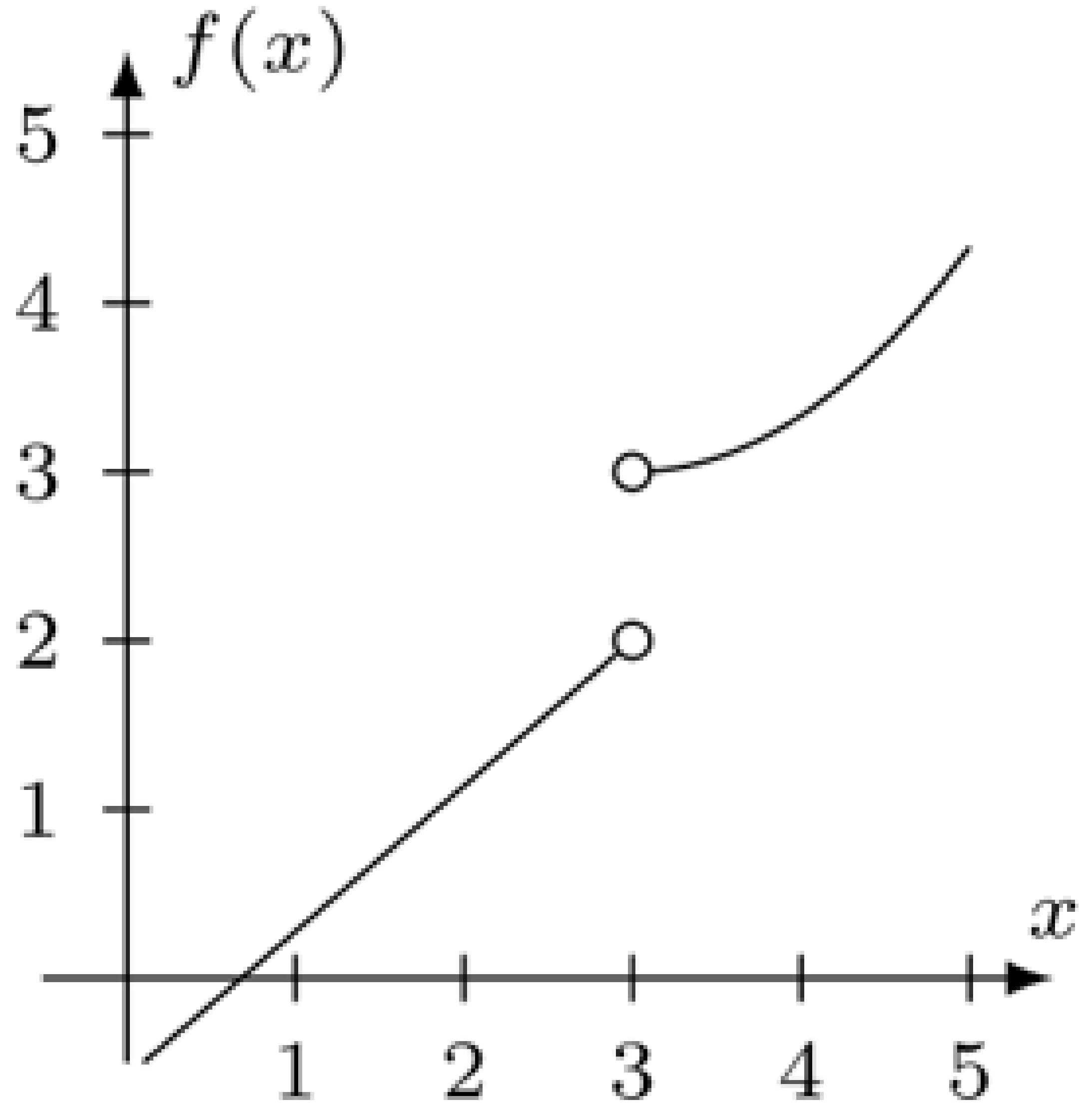
$x \to a^-$ means x is approaching from the left

REVIEW: FIND THE LIMIT

$$\lim_{x \to 3^+} f(x) = 3$$

$$\lim_{x \to 3^-} f(x) = 2$$

$$\lim_{x \to 3} f(x) = \text{DNE}$$



FIND THE LIMIT: EX 1

$$\lim_{x\to 5^+} \frac{x-5}{x^2-25}$$

$$\frac{5-5}{(5)^2-25} = \frac{0}{0}$$

DNE by direct subst. but we can simplify

$$\lim_{x\to 5^+} \frac{\cancel{x-5}}{(x+5)\cancel{(x-5)}}$$

hole in graph

$$\lim_{x\to 5^+} \frac{1}{x+5} = \frac{1}{5+5} = \boxed{\frac{1}{10}}$$

FIND THE LIMIT: EX 2

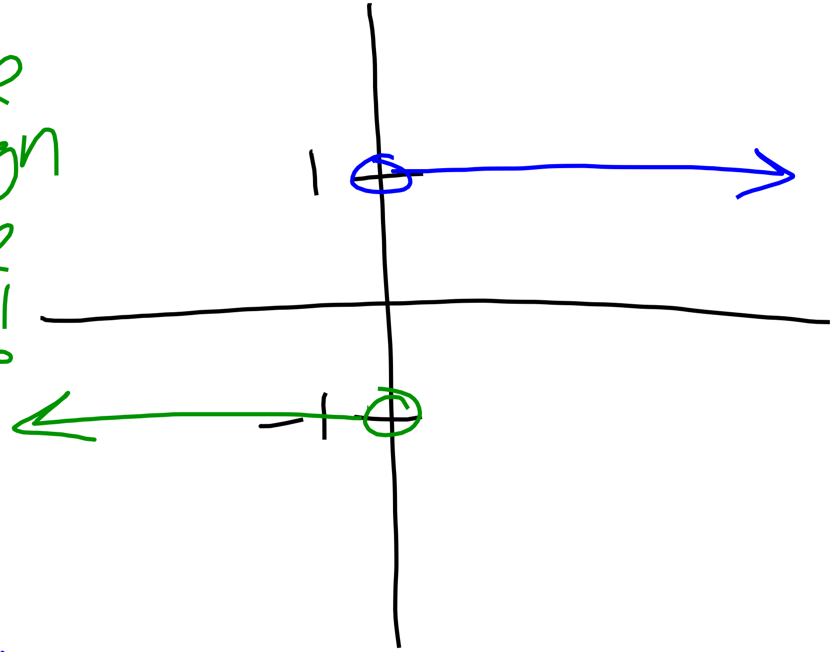
you can simplify the fraction & take the sign of the limit!

$$\lim_{x\to 0^-} \frac{|x|}{x}$$

$$\boxed{-1}$$

x	y
0	DNE
-.1	-1
-1	-1
-2	-1

x	y
0	DNE
1	1
2	1
3	1



FIND THE LIMIT: EX 3

$$\lim_{x\to0^+}\frac{|x|}{x} = 1$$

$$\lim_{x\to 2^{\textcircled{-}}}\frac{3x+6}{|5x+10|}$$

$$\frac{-3\cancel{(x+2)}}{5\cancel{(x+2)}}$$

EX 4

$$\lim_{x \to 3^-} \frac{1}{x-3}$$

$\frac{1}{3-3} = \frac{1}{0}$ DNE

$+\infty$ or $-\infty$

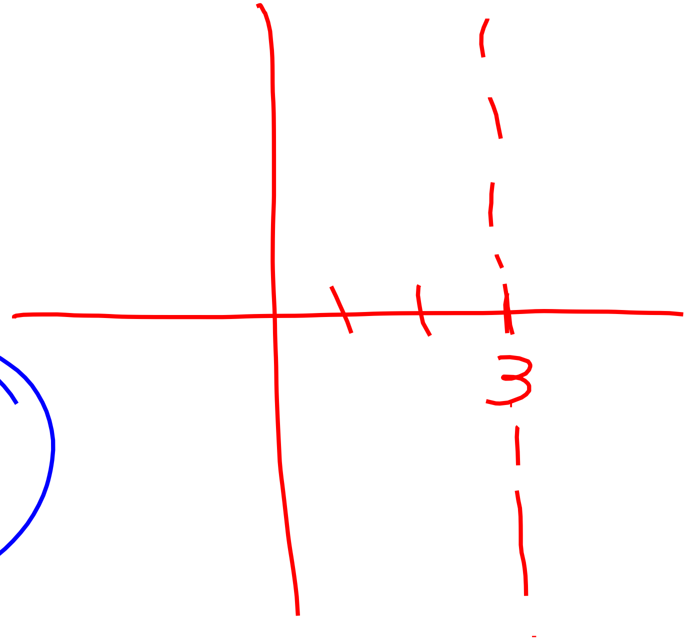
$x - 3 \neq 0$

VA: $x = 3$

x	y
2	-1
2.9	-10
2.99	-100
2.999	-1000

$-\infty$

$= \boxed{-\infty}$



EX 5

$+\infty$ or $-\infty$

$$\lim_{x\to -3^+} \frac{-5x^2-1}{x^2-9}$$

-2.999 is on the right side of -3

$(x+3)(x-3)$

VA: $x=-3+3$

$$\frac{-5(-2.999)^2-1}{(-2.999)^2-9}$$

$=7662.9$ which is really big so

$$\boxed{+\infty}$$

EX 6

$$f(x) = \begin{cases} x^2 + 1, & x \overset{\text{left}}{<} 0 \\ -2x + 4, & 0 \overset{\text{right}}{\leq} x \overset{\text{left}}{<} 2 \\ (x-2)^2 + 1, & x \underset{\text{right}}{>} 2 \end{cases}$$

a) $\lim_{x \to 0^-} f(x) = (0)^2 + 1 = \boxed{1}$

left

$x < 0$

b) $\lim_{x \to 0} f(x) = \boxed{DNE}$

both

$x \to 0^- = \underline{1} \quad x \to 0^+ = -2(0) + 4 = \underline{4}$

c) $\lim_{x \to 2^+} f(x) = (2-2)^2 + 1 = \boxed{1}$

$x > 2$

d) $\lim_{x \to 2^-} f(x) = -2(2) + 4 = \boxed{0}$

$x < 2$

e) $\lim_{x \to 1} f(x) = -2(1) + 4 = \boxed{2}$

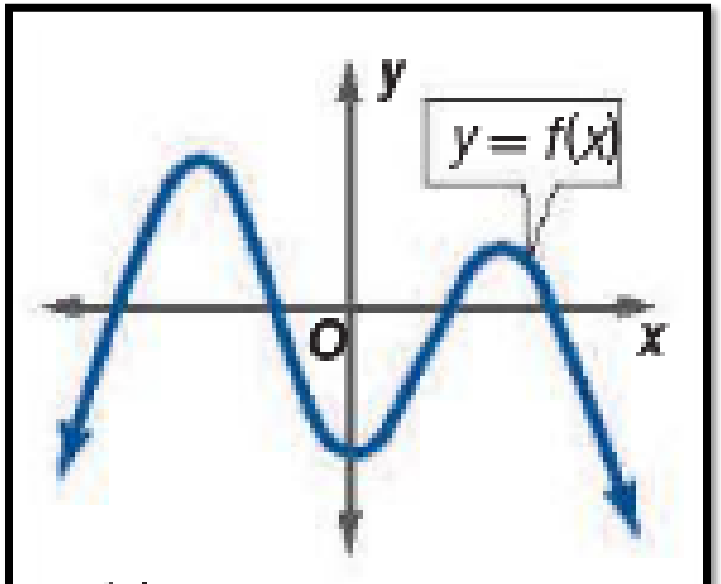
both

FIND THE LIMIT

$$\lim_{x \to 5^+} \frac{3x - 15}{|4x - 20|}$$

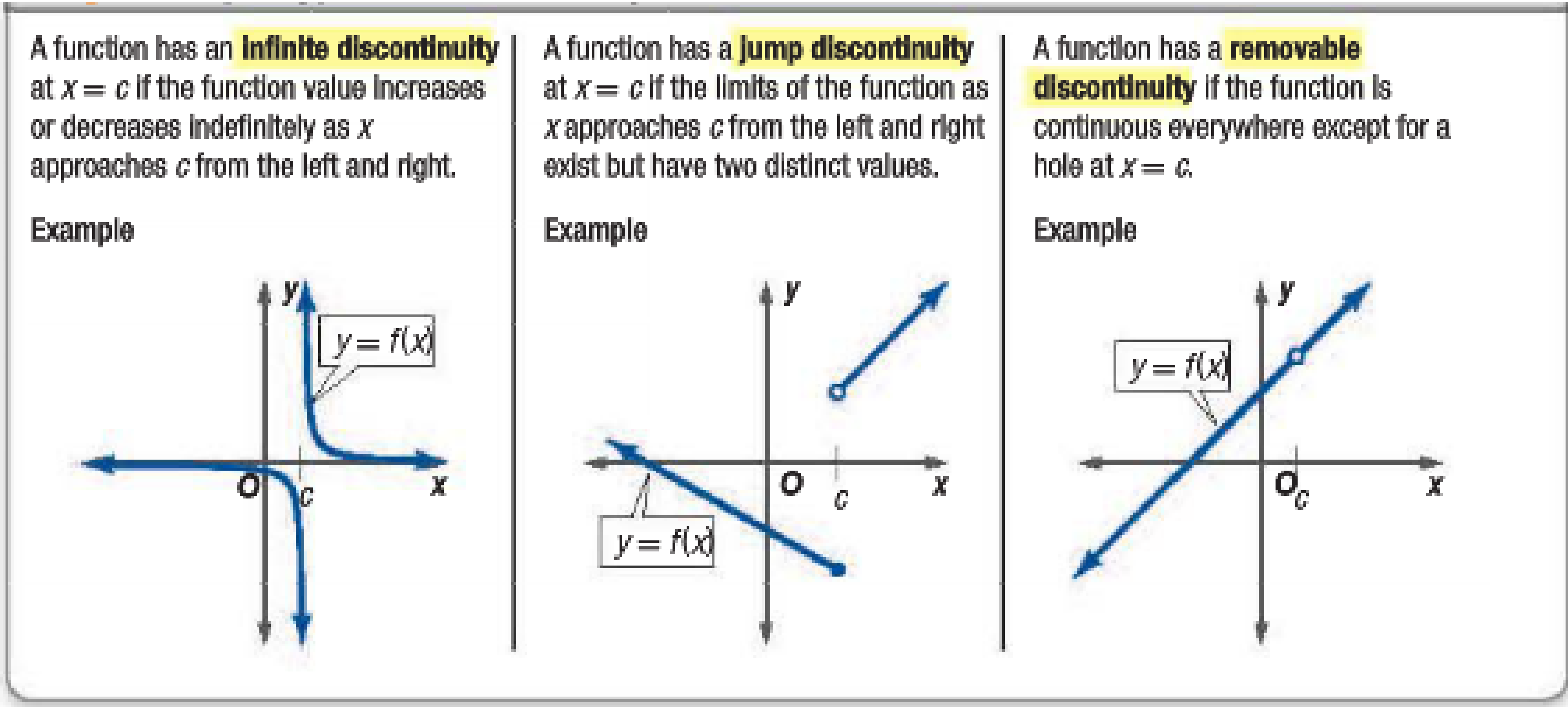
CONTINUOUS FUNCTIONS

The graph of a continuous function has no breaks, holes, or gaps. You can trace the graph of a continuous function without lifting your pencil.



$f(x)$ is continuous for all x.

TYPES OF DISCONTINUITY



A function has an **infinite discontinuity** at $x = c$ if the function value increases or decreases indefinitely as x approaches c from the left and right.

Example

A function has a **jump discontinuity** at $x = c$ if the limits of the function as x approaches c from the left and right exist but have two distinct values.

Example

A function has a **removable discontinuity** if the function is continuous everywhere except for a hole at $x = c$.

Example

CONTINUITY TEST

A function $f(x)$ is continuous at $x = c$ if it satisfies the following conditions.

$$\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = \lim_{x \to c} f(x) = f(c)$$

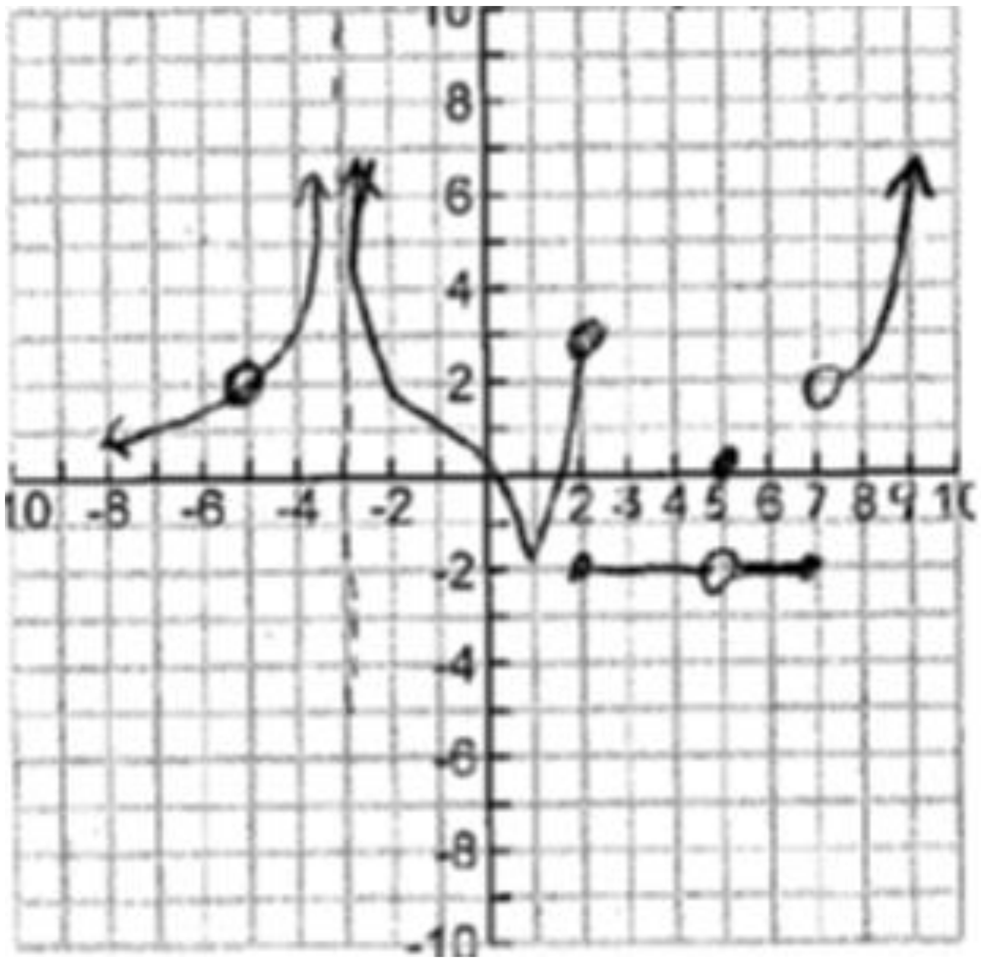
UNDERSTANDING CONTINUITY

a. Does $f(5)$ exist?

b. Does $\lim_{x \to 5} f(x)$ exist?

c. Is $f(x)$ continuous at $x = 5$? Justify.

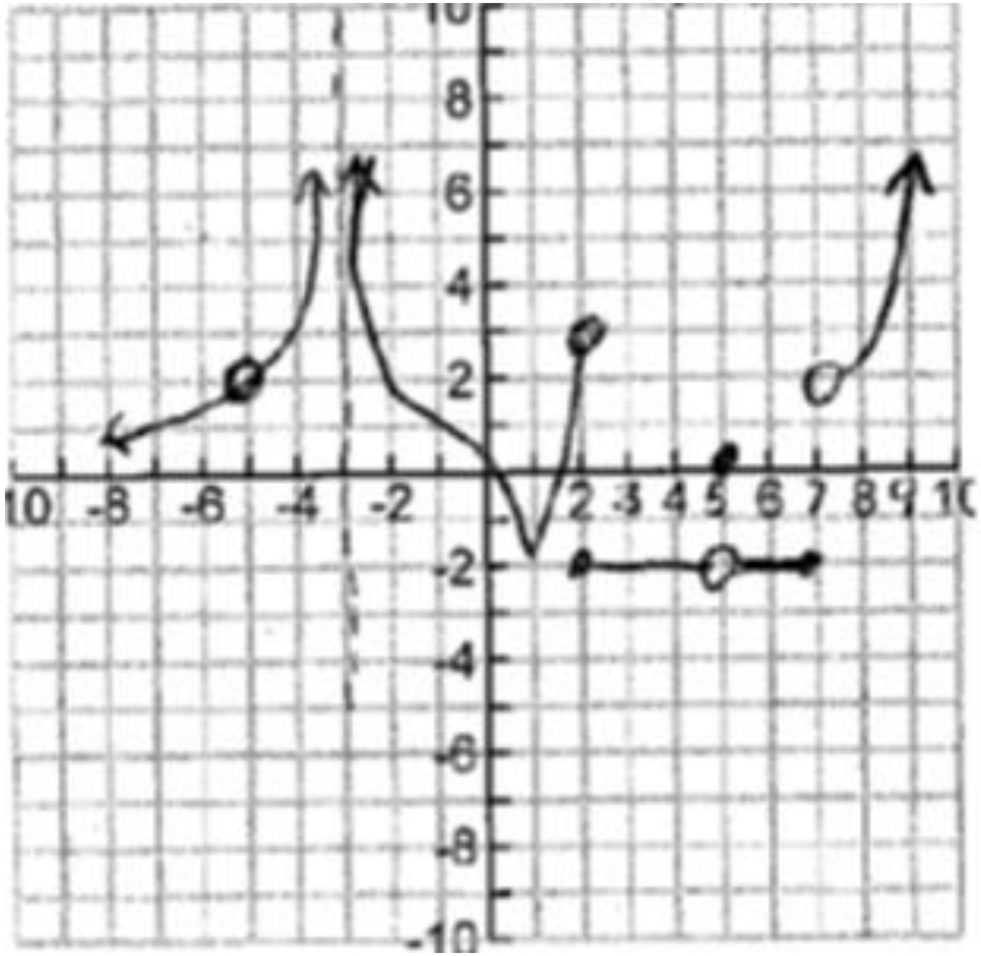
d. What new value should be assigned to $f(5)$ to remove the discontinuity?



UNDERSTANDING CONTINUITY

e. Does $f(2)$ exist?

f. Does $\lim_{x \to 2} f(x)$ exist?



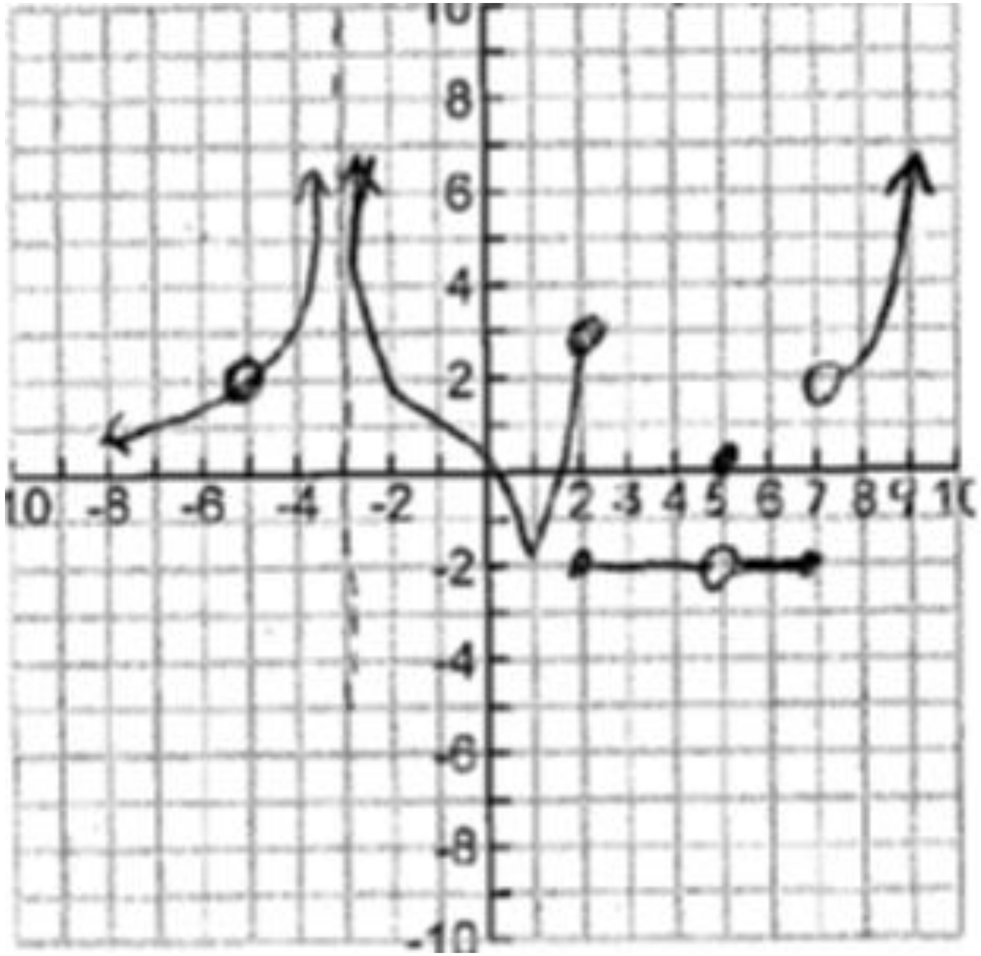
UNDERSTANDING CONTINUITY

g. Does $f(-5)$ exist?

h. Does $\lim_{x \to -5} f(x)$ exist?

i. Is $f(x)$ continuous at $x = -5$? Justify.

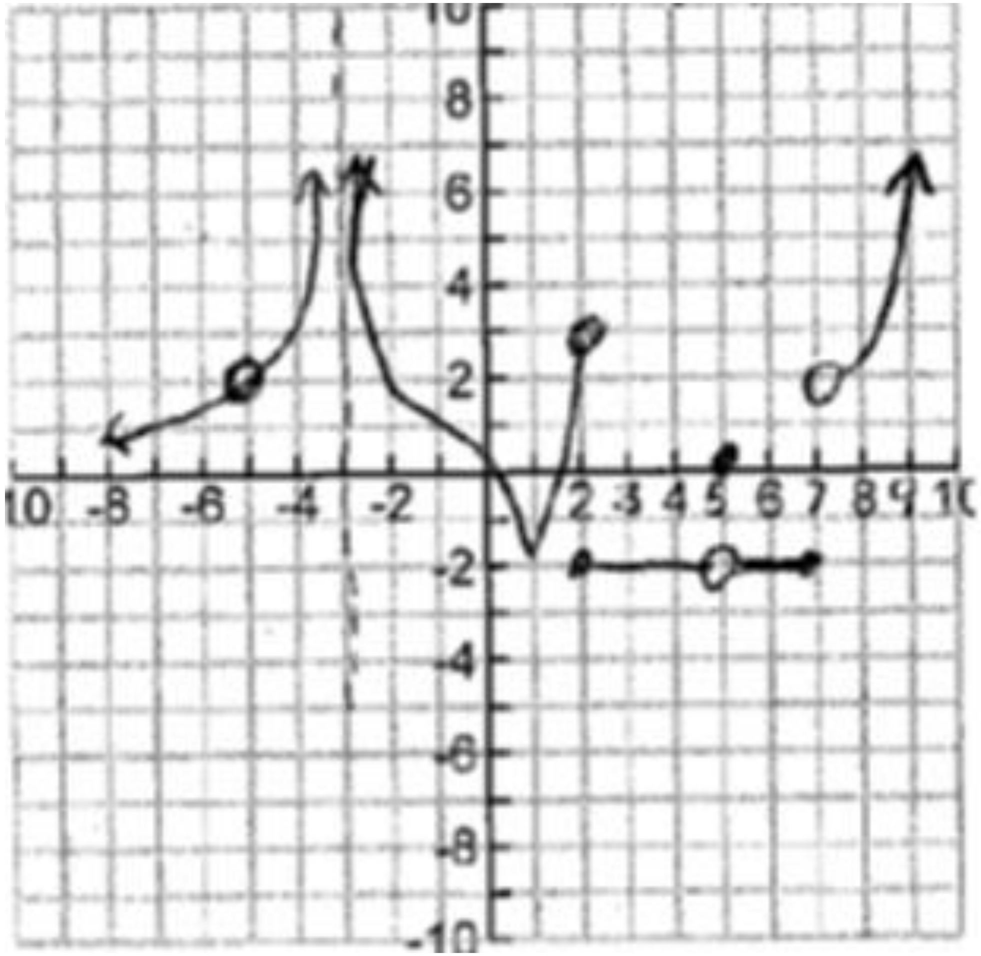
j. What new value should be assigned to $f(-5)$ to make $f(x)$ continuous at $x = -5$?



UNDERSTANDING CONTINUITY

k. Is $f(x)$ right continuous, left continuous, or neither at $x = 2$? How about for $x = 7$?

l. List all places where $f(x)$ is discontinuous and state the type of discontinuity.



IDENTIFY THE TYPE OF DISCONTINUITY IN THE FOLLOWING EQUATIONS

a. $h(x) = \dfrac{6}{x - 3}$

b. $p(x) = \begin{cases} 3x - 1, & if\ x \geq 1 \\ 4x - 2, & if\ x < 1 \end{cases}$

c. $m(x) = \begin{cases} 2x - 5, & if\ x \geq 2 \\ 3x, & if\ x < 2 \end{cases}$

d. $k(x) = \dfrac{6x - 2}{9x - 3}$

e. $j(x) = \dfrac{2x - 4}{x^2 - 2x}$

FINDING VALUES FOR DISCONTINUITY

Find a value for a so that $f(x)$ is continuous.

$$f(x) = \begin{cases} 2x+3, & if\ x \le 2 \\ ax+1, & if\ x > 2 \end{cases}$$

FINDING VALUES FOR DISCONTINUITY

Find a value for k so that $g(x)$ is continuous.

$$g(x) = \begin{cases} 4x - 7k, & if\ x \geq -3 \\ 2k + x, & if\ x < -3 \end{cases}$$

COMPLETE THE TABLE

	$f(x)$	**Discontinuity at:**	**Type of discontinuity (Be Specific):**
1.	$\frac{4}{x^2-1}$		
2.	$\begin{cases} x^2, & x \geq 0 \\ -3, & x < 0 \end{cases}$		
3.	$\frac{x^2-x-12}{x-4}$		
4.	$\frac{x-3}{x^2-9}$		