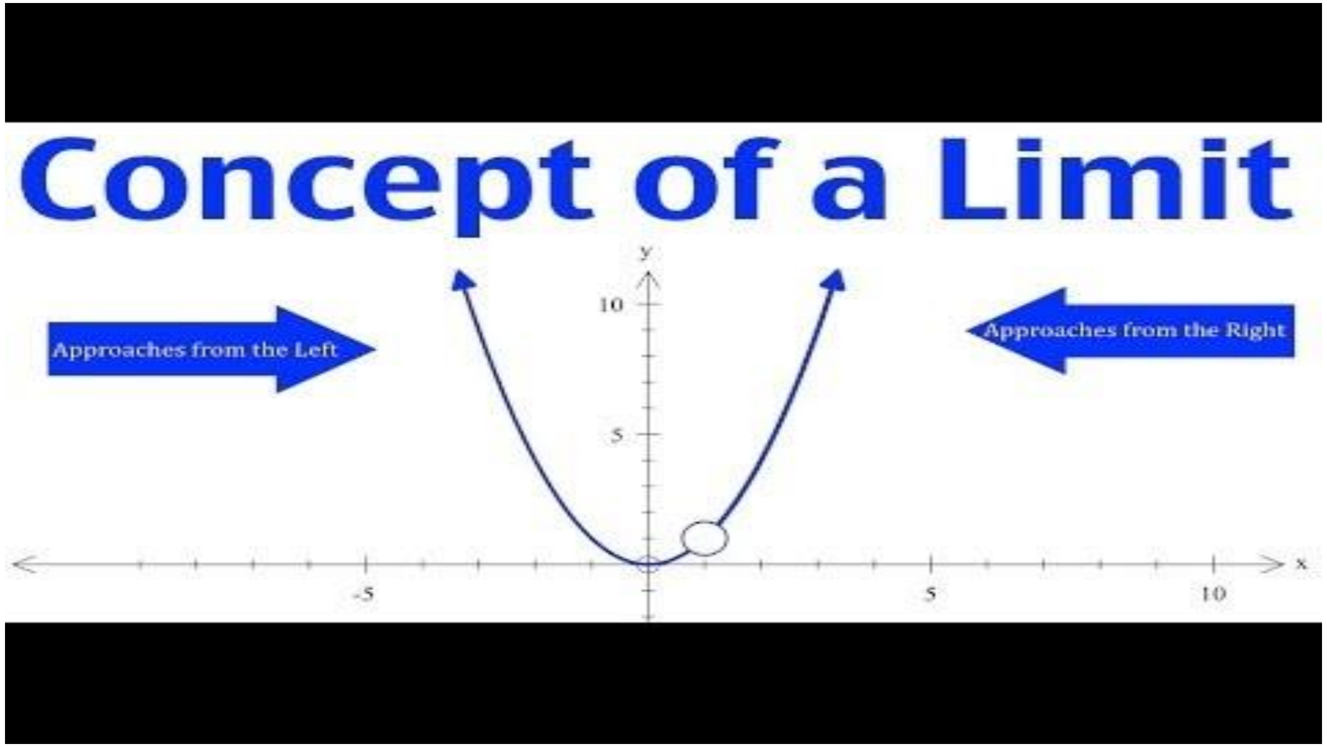
FINDING LIMITS FROM GRAPHS

Keeper 7

Honors Calculus

WHAT IS A LIMIT?



DEFINITION OF A LIMIT

If $f(x)$ becomes arbitrarily close to a unique number L as x approaches c from either side, then the **limit** of $f(x)$ as x approaches c is L. This is written as

$$\lim_{x \to c} f(x) = L$$

↑ no sign

(from both sides)

$\lim_{x \to c^-}$ (from left side)

$\lim_{x \to c^+}$ (from right side)

EVALUATING LIMITS FROM A GRAPH

a. $\lim_{x \to 0^-} f(x)$ left

0

b. $\lim_{x \to 0^+} f(x)$ right

2

c. $\lim_{x \to 0} f(x)$ both

DNE

d. $\lim_{x \to 2^-} f(x)$ left

-2

e. $\lim_{x \to 2^+} f(x)$ right

0

f. $\lim_{x \to 2} f(x)$ both

DNE

g. $\lim_{x \to 4^-} f(x)$ left

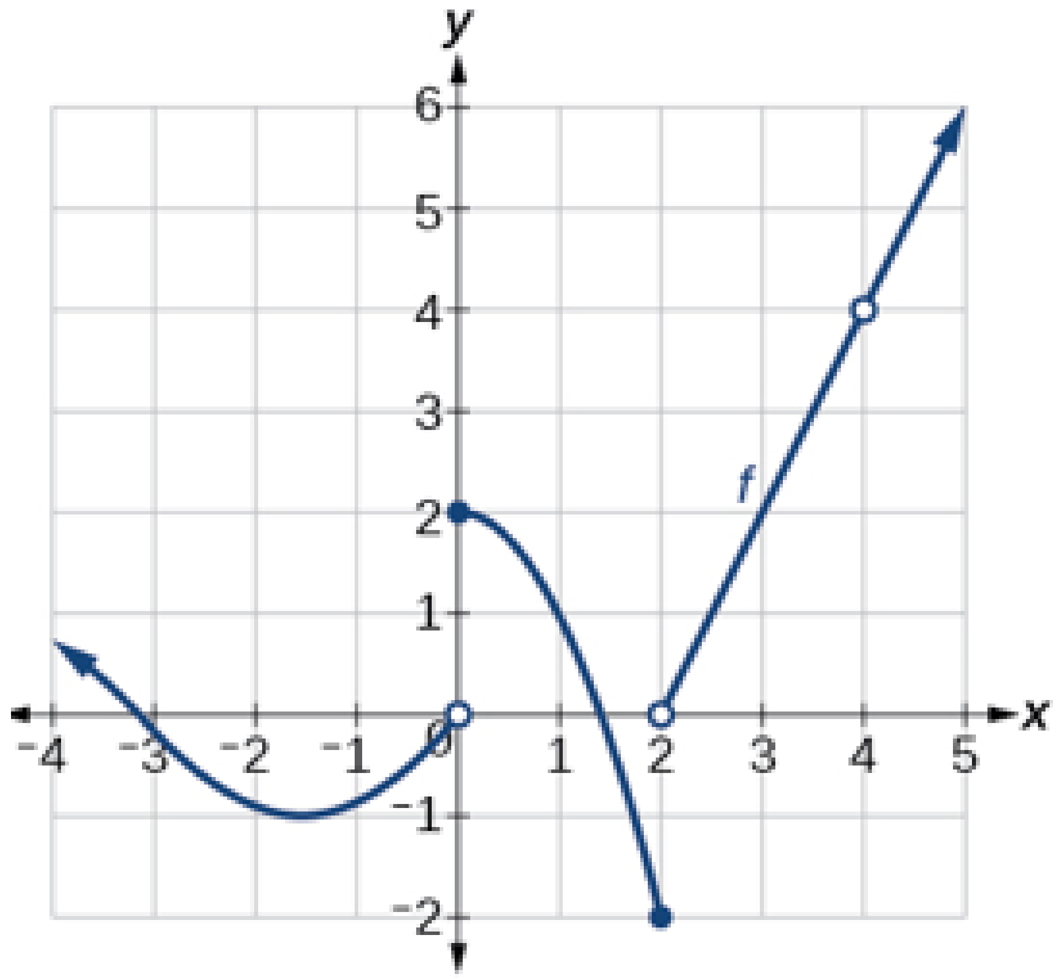
4

h. $\lim_{x \to 4^+} f(x)$ right

4

i. $\lim_{x \to 4} f(x)$ both

4



EVALUATING LIMITS FROM A GRAPH

a. $\lim_{x \to -1^-} h(x)$

1

b. $\lim_{x \to -1^+} h(x)$

−2

c. $\lim_{x \to -1} h(x)$

DNE

d. $h(-1)$

(−1,1)

1

e. $\lim_{x \to 3^-} h(x)$

2

f. $\lim_{x \to 3^+} h(x)$

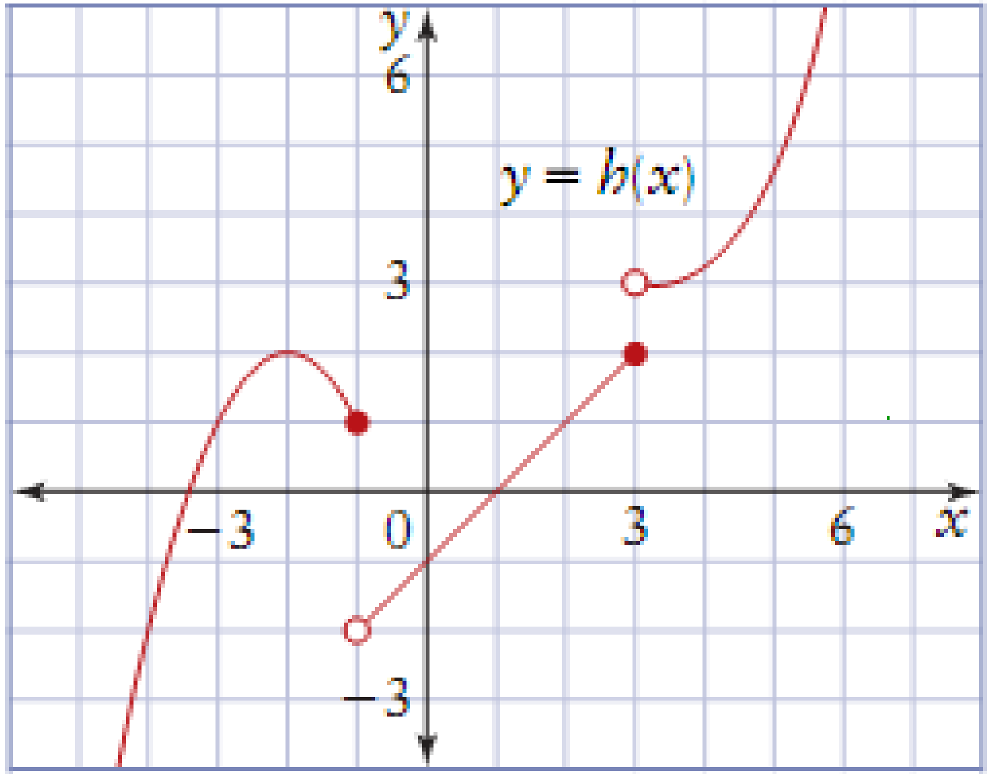
3

g. $\lim_{x \to 3} h(x)$

DNE

h. $h(3)$

2



EVALUATING LIMITS FROM A GRAPH

1. $\lim_{x \to 3} g(x)$ both sides

   -2

2. $\lim_{x \to 0} g(x)$

   -1

3. $\lim_{x \to -3} g(x)$

   3

4. $\lim_{x \to 1^+} g(x)$

   2

5. $\lim_{x \to 1^-} g(x)$

   -1

6. $\lim_{x \to 1} g(x)$

   DNE

7. $\lim_{x \to -2^+} g(x)$

   -1

8. $\lim_{x \to 4} g(x)$

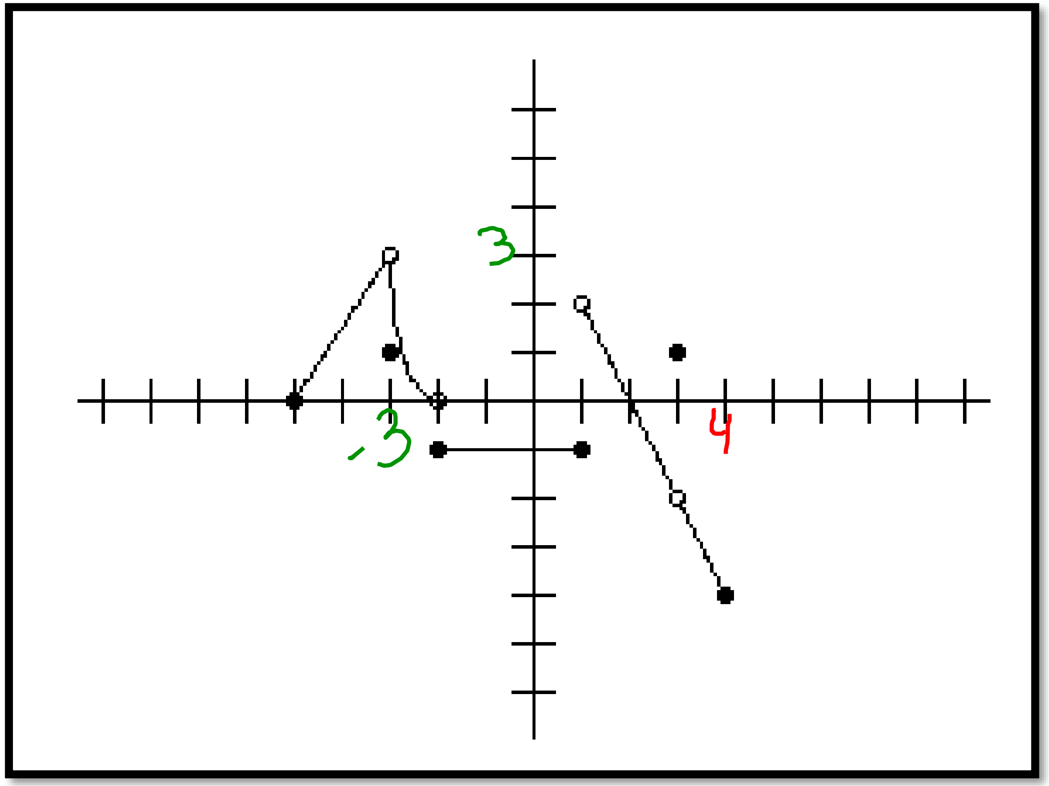
   -4

9. $\lim_{x \to 2} g(x)$

   0

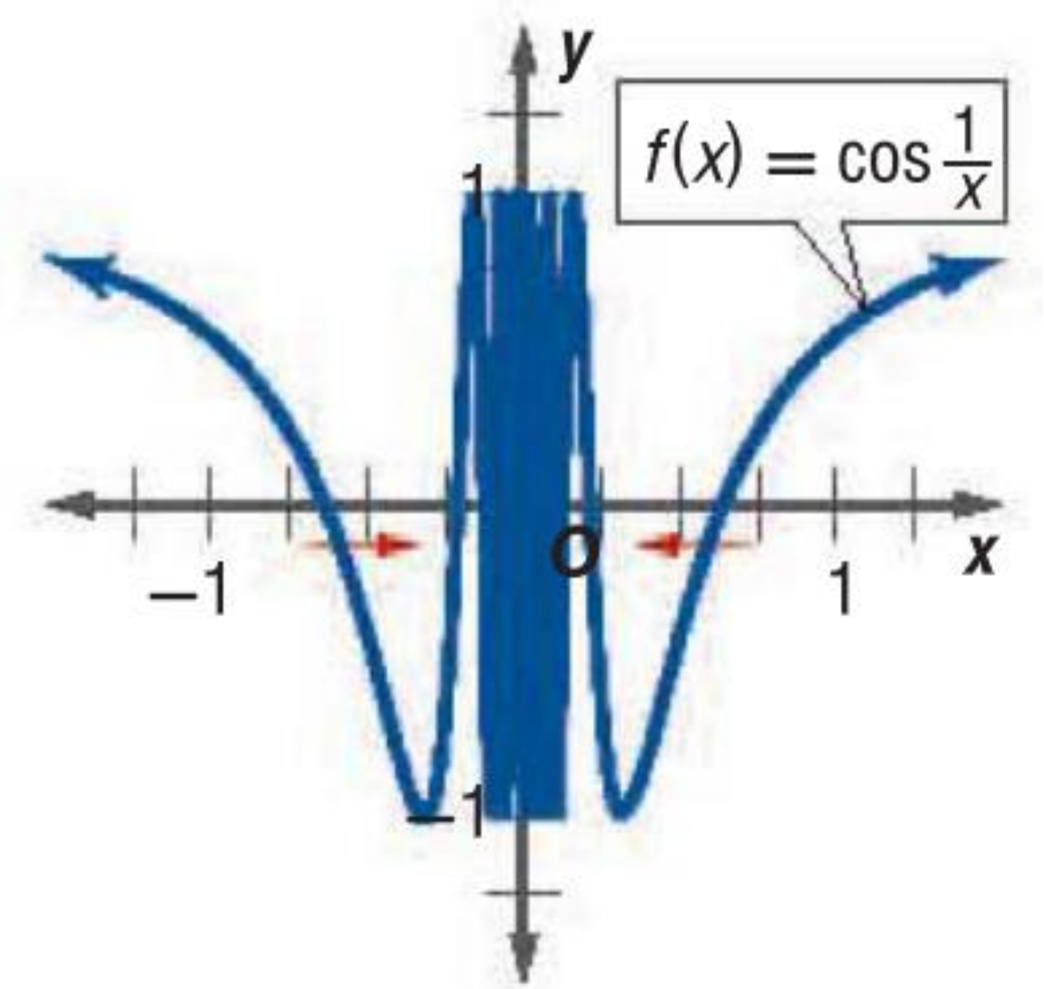
10. $\lim_{x \to -2^-} g(x)$

   0



EXAMPLE WITH OSCILLATION

$$\lim_{x \to 0} \cos \frac{1}{x}$$



CONDITIONS UNDER WHICH LIMITS DO NOT EXIST

The limit of $f(x)$ as $x \to c$ does not exist under any of the following conditions.

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side of c.
2. $f(x)$ increases or decreases without bound as x approaches c. $-\infty$ or ∞
3. $f(x)$ oscillates between two fixed values as x approaches c.

EVALUATE THE LIMIT

a. $\lim_{x\to -3^-} h(x)$

4

b. $\lim_{x\to -3^+} h(x)$

4

c. $\lim_{x\to -3} h(x)$

4

d. $h(-3)$

DNE

e. $\lim_{x\to 0^-} h(x)$

1

f. $\lim_{x\to 0^+} h(x)$

-1

g. $\lim_{x\to 0} h(x)$

DNE

h. $h(0)$

1

i. $\lim_{x\to 2} h(x)$

2

j. $h(2)$

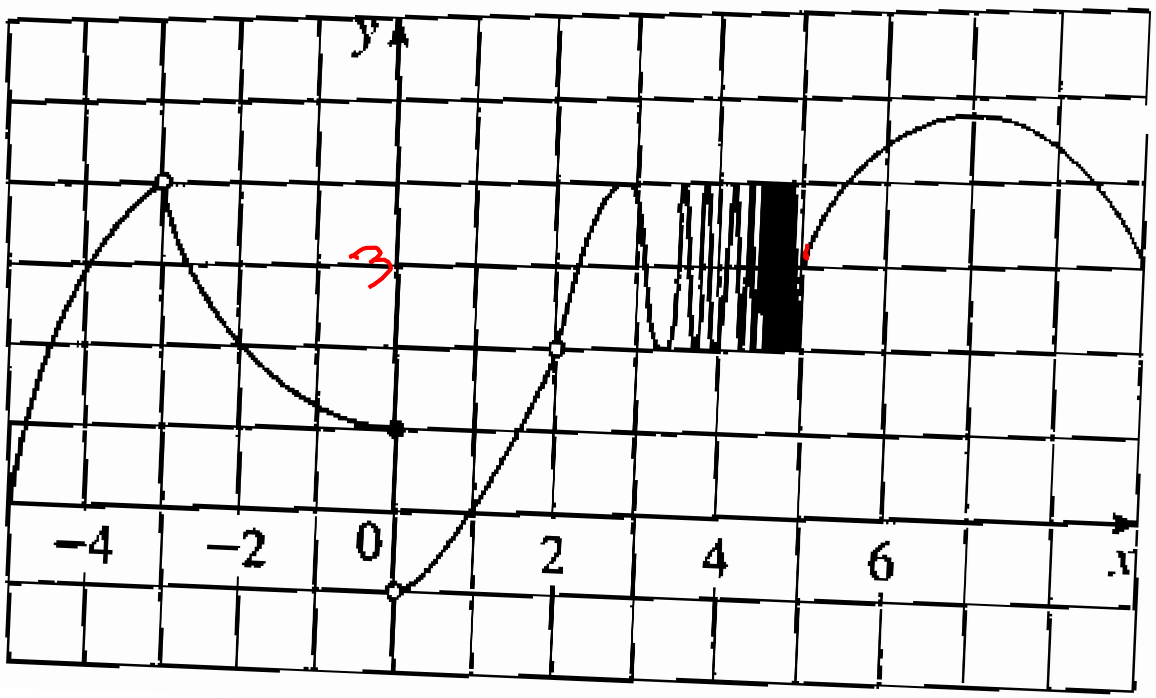
DNE

k. $\lim_{x\to 5^+} h(x)$

3

l. $\lim_{x\to 5^-} h(x)$

DNE



WARM UP – FINDING LIMITS FROM GRAPHS

a. $\lim_{x \to -4^-} h(x)$

b. $\lim_{x \to -4^+} h(x)$

c. $\lim_{x \to -4} h(x)$

d. $h(-4)$

e. $\lim_{x \to 2^-} h(x)$

f. $\lim_{x \to 2^+} h(x)$

g. $\lim_{x \to 2} h(x)$

h. $h(2)$

i. $\lim_{x \to 4} h(x)$

j. $\lim_{x \to -1} h(x)$

k. $\lim_{x \to -\infty} h(x)$

l. $\lim_{x \to -\infty} h(x)$

m. $h(4)$

n. $h(-1)$