THE FUNDAMENTAL THEOREM OF CALCULUS cont.

Keeper 30

Honors Calculus

THE FUNDAMENTAL THEOREM OF CALCULUS

Part 1:

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\int_a^x f(t)\ dt = f(x)$$

THE LONG WAY…

1. $\frac{d}{dx}\int_1^x t^2\,dt$

$\frac{d}{dx}\ \frac{t^3}{3}\Big|_1^x$

$\frac{d}{dx}\left(\frac{x^3}{3}-\frac{1^3}{3}\right)$

$\frac{d}{dx}\left(\frac{1}{3}x^3-\frac{1}{3}\right)$

$1x^2-0=x^2$

2. $\frac{d}{dx}\int_3^x \sin t\ dt$

$\frac{d}{dx}\ -\cos t\Big|_3^x$

$\frac{d}{dx}\left(-\cos x\ \cancel{+} -\cos 3\right)$

$\sin x+0$

$\sin x$

SHORT CUT…

- When x is the upper limit and a constant is your lower limit, you can just plug in x for t.

EXAMPLES WITH SHORT CUT…

1. $\frac{d}{dx}\int_{-3}^{x}\sqrt{t^2+1}\,dt$

$\sqrt{x^2+1}$

2. $\frac{d}{dx}\int_{2}^{x}\csc^2 t\,dt$

$\csc^2 x$

EXAMPLE 3

$$\frac{d}{dx}\int_x^2 \sin t^2\, dt$$

$$-\frac{d}{dx}\int_2^x \sin t^2\, dt$$

$$-\sin x^2$$

MORE SOPHISTICATED USE OF FTC…

1. $$\frac{d}{dx}\int_{\frac{\pi}{2}}^{x^3} \cos t \, dt$$

$$\frac{d}{dx} \sin t \Big|_{\pi/2}^{x^3} = \frac{d}{dx}\left(\sin(x^3) - \sin \pi/2\right)$$

chain rule

$$\cos(x^3)\,3x^2 - 0$$

$$3x^2 \cos(x^3)$$

SHORT CUT…

When something other than just x is the upper limit and a constant is still your lower limit, then…

- Plug the upper limit into the function for t
- Take the derivative of the upper limit.

EXAMPLE WITH SHORT CUT

$$1.\ \frac{d}{dx}\int_3^{x^2}\sqrt{t^2-4\sin t}\ \ dt$$

$$\sqrt{(x^2)^2-4\sin x^2}\cdot 2x$$

$$2x\sqrt{x^4-4\sin x^2}$$

$$2.\ \frac{d}{dx}\int_{x^3}^{3}\frac{1}{t^2}\,dt$$

$$-\frac{d}{dx}\int_{3}^{x^3}\frac{1}{t^2}\,dt = -\frac{1}{(x^3)^2}\cdot\frac{3x^2}{1}$$

plug in upper

deriv. of upper

$$-\frac{1}{x^6}\cdot\frac{3x^2}{1} = \frac{-3x^2}{x^6} = \frac{-3}{x^4}$$

EXAMPLE – CAN'T IGNORE THE LOWER WHEN IT ISN'T A CONSTANT

$$\frac{d}{dx}\int_{3x}^{4x^2} \frac{4t}{1+t^2}\,dt$$

Short cut w/ upper – Short cut w/ lower

$$\frac{4(4x^2)}{1+(4x^2)^2}\cdot\frac{8x^1}{1} - \frac{4(3x)}{1+(3x)^2}\cdot 3$$

$$\frac{128x^3}{1+16x^4} - \frac{36x}{1+9x^2}$$