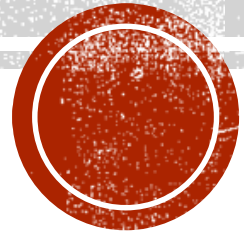
LINEAR MODELS AND RATES OF CHANGE

Honors Calculus

Keeper 3



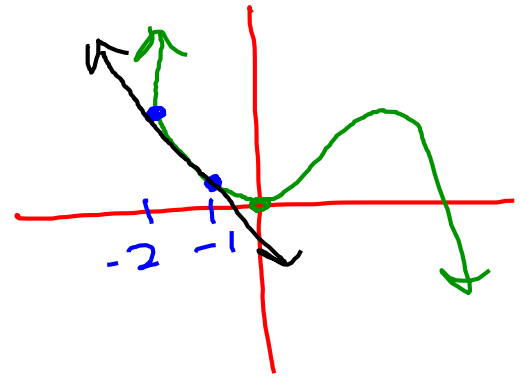
AVERAGE RATE OF CHANGE

The average rate of change between any two points on the graph of f is the slope of the line through those points.

The average rate of change on the interval $[x_1, x_2]$ is

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \text{ or } \frac{y_2 - y_1}{x_2 - x_1}$$

FIND AVERAGE RATE OF CHANGE



1. Find the average rate of change of $f(x) = -x^3 + 3x$ on each interval:

a. $[-2, -1]$ x_1 x_2

$f(x_1) = -(-2)^3 + 3(-2) = 2$

$f(x_2) = -(-1)^3 + 3(-1) = -2$

$(-2, 2)$ x_1 y_1

$(-1, -2)$ x_2 y_2

$m = \frac{-2-2}{-1--2} = \frac{-4}{1} = -4$

b. $[0,1]$ x_1 x_2

$f(0) = -(0)^3 + 3(0) = 0$

$f(1) = -(1)^3 + 3(1) = 2$

$(0,0)$ x_1 y_1

$(1,2)$ x_2 y_2

$m = \frac{2-0}{2-1} = 2$

FIND AVERAGE RATE OF CHANGE

2. Find the average rate if change on the interval $[-6,2]$.

x_1, x_2

$$f(x) = \frac{x+5}{x-4}$$

$f(-6) = \frac{-6+5}{-6-4} = \frac{-1}{-10}$

$f(2) = \frac{2+5}{2-4} = \frac{7}{-2}$

$\left(-6, +\frac{1}{10}\right)$
$x_1 \quad y_1$

$\left(2, -\frac{7}{2}\right)$
$x_2 \quad y_2$

$m = \frac{-\frac{7}{2} - \frac{1}{10}}{2+6} = \frac{-\frac{35}{10} - \frac{1}{10}}{8}$

$\frac{-\frac{36}{10}}{\frac{8}{1}} = -\frac{18}{5} \cdot \frac{1}{8} = \frac{-9}{20}$

FIND AVERAGE RATE OF CHANGE

3. Find the average rate if change on the interval $[-4,4]$.

$f(x) = \sqrt{x+8}$

$f(-4) = \sqrt{4} = 2$

$f(4) = \sqrt{12} = 2\sqrt{3}$

$(-4, 2)$ (x_1, y_1)

$(4, 2\sqrt{3})$ (x_2, y_2)

$$m = \frac{2\sqrt{3} - 2}{4 - -4}$$

$$m = \frac{2\sqrt{3} - 2}{8} = \frac{\sqrt{3} - 1}{4}$$

WRITING AN EQUATION OF A LINE

Slope-Intercept Form: Given the slope m and the y-intercept b,

$$y = mx + b$$

Point-Slope Form: Given the slope m and a point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

Standard Form: $\mathbf{Ax + By = C}$

A, B, & C are all integers & A is positive

1. Write the equation of the line that passes through $(2,3)$ and has a slope of $-\frac{1}{2}$.

x_1, y_1

$m = -\frac{1}{2}$

Pt-Slope: $y - 3 = -\frac{1}{2}(x - 2)$

$y = mx + b$

$3 = -\frac{1}{2}(2) + b$

$3 = -1 + b$

$b = 4$

$y = -\frac{1}{2}x + 4$

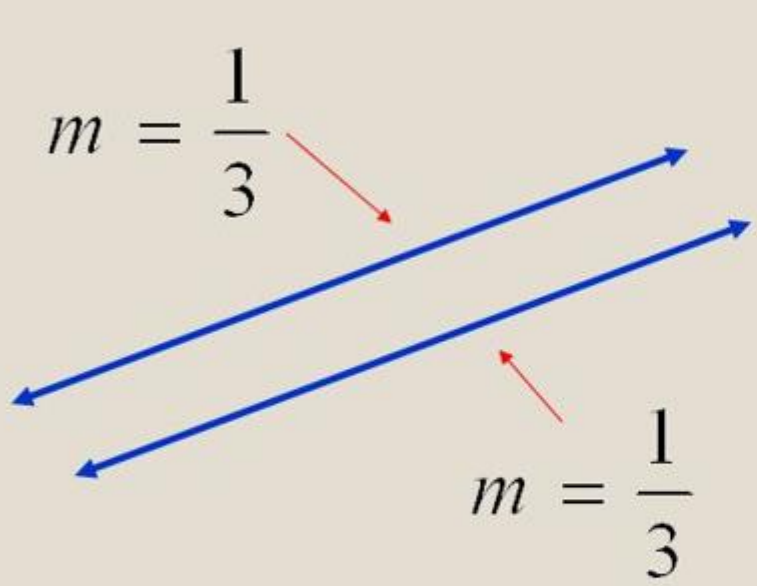
Slope-Int: $y - 3 = -\frac{1}{2}x + 1$

$y = -\frac{1}{2}x + 4$

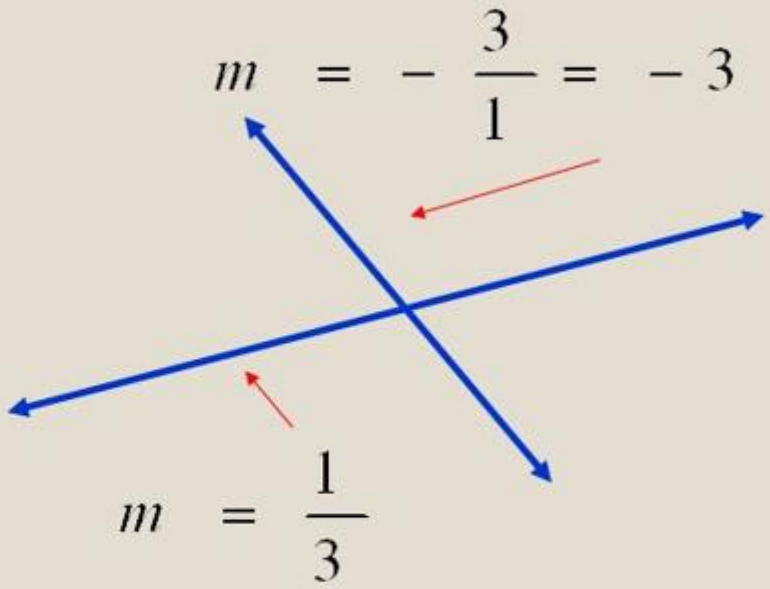
Standard Form: $2\left(+\frac{1}{2}x + y = 4\right)$

$x + 2y = 8$

PARALLEL AND PERPENDICULAR LINES



Parallel lines have the SAME slope.



Perpendicular lines have an OPPOSITE, RECIPROCAL slope.

2. Write the equation of the line that passes through $(3,2)$ and is perpendicular to the line $y = -3x + 2$

(x_1, y_1)

m

$m = -3$

$\perp m = \frac{1}{3}$

$(y - y_1) = m(x - x_1)$

$y - 2 = \frac{1}{3}(x - 3)$

or

$y = mx + b$

$2 = \frac{1}{3}(3) + b$

$b = 1$

or

$y = \frac{1}{3}x + 1$

3. Write the equation of the line that passes through $(3,2)$ and is parallel to the line $y = -3x + 2$

x y

m

$m = -3$

$b = 11$

$y = mx + b$

$2 = -3(3) + b$

$2 = -9 + b$

$b = 11$

$y = mx + b$

$y = -3x + 11$

4. Write the equation of the line that passes through $(-2, -1)$ and $(3,4)$.

x_1 y_1 x_2 y_2

① Find slope

$$m = \frac{4+1}{3+2} = 1$$

② Choose either pt for pt-slope form or find b for slope-int.

$$y+1 = 1(x+2)$$

or

$$y-4 = 1(x-3)$$

or

$$y+1 = x+2$$

$$y = x+1$$

$$y-4 = x-3$$

5. Find the standard form of the equations of the lines that pass through the point $(2, -1)$ and is perpendicular to the line $2x - 3y = 5$

(x, y)

① Find slope

$2x - 3y = 5$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{5}{-3}$

$y = \frac{2}{3}x - \frac{5}{3}$

$m = -\frac{3}{2}$

$y + 1 = -\frac{3}{2}(x - 2)$

$y + 1 = -\frac{3}{2}x + 3$

$y = -\frac{3}{2}x + 2$

$2\left(\frac{3}{2}x + y = 2\right) =$ $3x + 2y = 4$