THE FUNDAMENTAL THEOREM OF CALCULUS PART 2?: DEFINITE INTEGRALS

Keeper 29

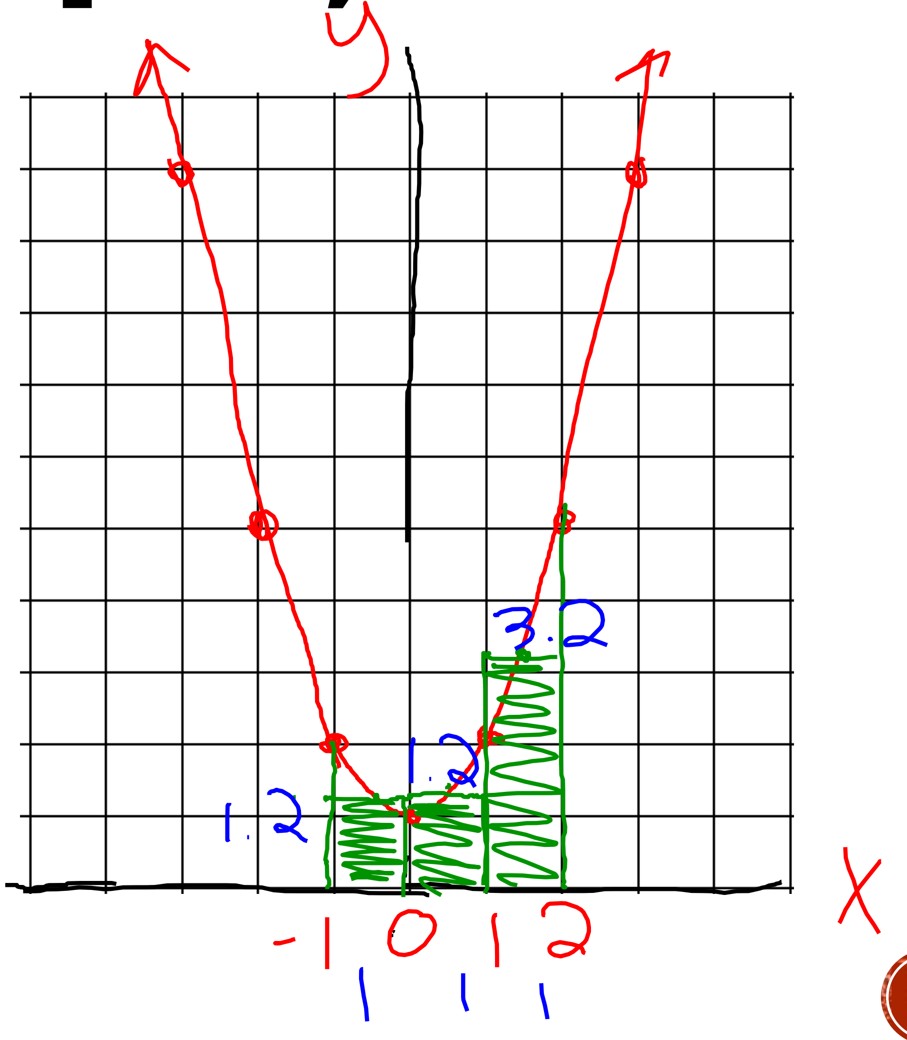
Honors Calculus

REIMANN SUMS TO APPROXIMATE AREA

Find the area under the graph of $y = x^2 + 1$ over the interval $[-1,2]$.

$$A = 1(1.2 + 1.2 + 3.2)$$

$$A = 5.6$$



THE FUNDAMENTAL THEOREM OF CALCULUS PART 2: DEFINITE INTEGRAL DEFINITION

If F(x) is the antiderivative of the continuous function *f(x)* over the interval [*a*, *b*], then the **definite integral** of *f* from *a* to *b* is

$$\int_a^b f(x)\ dx = F(b) - F(a).$$

STEPS FOR EVALUATING DEFINITE INTEGRALS

1. Find any antiderivative, $F(x)$, of $f(x)$.
2. Evaluate $F(x)$ using b and a, and compute $F(b) - F(a)$. The result is the area under the graph over the interval $[a, b]$.

NOW EVALUATE THE DEFINITE INTEGRAL

*This problem is read "the integral of $x^2 + 1$, with respect to x, from -1 to 2".

$$\int_{-1}^{2} x^2 + 1\,dx$$

$$\left.\frac{x^{2+1}}{3} + x\right|_{-1}^{2}$$

$$\left.\frac{x^3}{3} + x\right|_{-1}^{2}$$

$$F(b) - F(a)$$

$$\left(\frac{2^3}{3} + 2\right) - \left(\frac{(-1)^3}{3} - 1\right)$$

$$\frac{8}{3} + 2 + \frac{1}{3} + 1$$

$$\frac{8}{3} + \frac{6}{3} + \frac{1}{3} + \frac{3}{3} = 6$$

1. EVALUATE $\displaystyle\int_{-1}^{4} (x^2 - x)\, dx$

$$\frac{x^3}{3} - \frac{x^2}{2}\Bigg|_{-1}^{4} = \left(\frac{4^3}{3} - \frac{4^2}{2}\right) - \left(\frac{(-1)^3}{3} - \frac{(-1)^2}{2}\right)$$

$$+\left(-\frac{1}{3} + \frac{1}{2}\right)$$

$$\frac{64}{3} - 8 + \frac{1}{3} + \frac{1}{2}$$

$$85/6$$

2. EVALUATE $\int_0^3 e^x \, dx$

$$e^x \Big|_0^3 = e^3 - e^0 = e^3 - 1$$

3. EVALUATE $\int_1^e \left(1+2x-\frac{1}{x}\right) dx$ (assume $x > 0$).

$\ln e$

$\log_e e = 1$

$x + \frac{2x^{1+1}}{2}$

$\ln 1$

$\log_e 1 = x$

$e^x = 1$

$x = 0$

$x + x^2 - \ln|x| \Big|_1^e$

$(e + e^2 - \ln e) - (1 + 1^2 - \ln 1)$

$e + e^2 - 1 - 1 - 1 + 0 = e + e^2 - 3$

4. EVALUATE $\int_2^4 \left(2x^3 - 3x\right) dx$

$$\frac{\cancel{2}x^4}{2\cancel{4}} - \frac{3x^2}{2}\Bigg|_2^4$$

$$\left(\frac{(4)^4}{2} - \frac{3(4)^2}{2}\right) - \left(\frac{\cancel{2}(2)^4}{2\cancel{4}} - \frac{3(2)^2}{2}\right) =$$

$$128 - 24 - 8 + 6 = 102$$

5. EVALUATE $\int_0^{\ln 4} 2e^x dx$

$F(b) - F(a)$

$2e^x \Big|_0^{\ln 4} = 2e^{\ln 4} - 2e^0$

$2\cancel{\ln e}^4 \quad 2(1)$

$8 - 2 = 6$

6. EVALUATE $\int_1^5 \frac{x-1}{x}\,dx$

Rewrite: $\int_1^5 1-\frac{1}{x}\,dx$

Integrate: $x-\ln|x|\Big|_1^5$

Evaluate: $(5-\ln 5)-(1-\ln 1)$

$5-\ln 5-1+0=4-\ln 5$

7. EVALUATE $\int_{\pi}^{5} \cos x \, dx$

$\sin x \Big|_{\pi}^{5} = \sin 5 - \sin \pi$

$\sin 5 - 0 = \sin 5$

HW: p. 6-7

$\left(-\frac{2}{3}, \frac{2}{27}\right)$

- Talk about if the smaller number is on top

$$\int_{1}^{-2} x\,dx = -\int_{-2}^{1} x\,dx$$