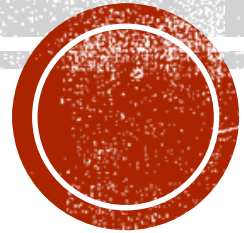
THE ANTIDERIVATIVE

(Indefinite Integrals)



Keeper 28

Honors Calculus

CONNECTION TO DERIVATIVES

Find the derivative of…

- $\frac{d}{dx}x^2 = 2x$
- $\frac{d}{dx}x^2 + 1 = 2x$
- $\frac{d}{dx}x^2 + \pi = 2x$
- $\frac{d}{dx}x^2 + c = 2x$

Antiderviative

What is 2x the derivative of?

x^2 , x^2+1, $x^2+\pi$ or

$x^2 + C$

constant

*must have $+C$

THEOREM: ANTIDERIVATIVE

THEOREM 1

The **antiderivative** of $f(x)$ is the set of functions $F(x) + C$ such that

$$\frac{d}{dx}[F(x) + C] = f(x)$$

The constant C is called the **constant of integration.**

NOTATION

$\int f(x)dx$ is used to represent the antiderivative of $f(x)$.

BASIC INTEGRATION FORMULAS

- $\int k\,dx = kx + C$
- $\int x^r\,dx = \frac{x^{r+1}}{r+1} + C,\ provided\ r \neq -1$
- $\int x^{-1}dx = \int \frac{1}{x}dx = \ln|x| + C$
- $\int be^{ax}dx = \frac{b}{a}e^{ax} + C$

$\frac{1}{x} \cdot 1 = \frac{1}{x} = x^{-1}$

EXAMPLES - EVALUATE

1. $\int 8dx$

$\frac{8x^{0+1}}{1} = 8x + C$

2. $\int 3x^2 dx$

$\frac{3x^{2+1}}{3} + C$

$x^3 + C$

EXAMPLES - EVALUATE

3. $\int x^7 dx$

$\frac{x^{7+1}}{8} + C$

$\frac{1}{8}x^8 + C$

4. $\int x^{99} dx$

$\frac{x^{99+1}}{100} + C$

$\frac{1}{100}x^{100} + C$

EXAMPLES - EVALUATE

5. $\int \sqrt{x}\,dx$

rewrite: $\int x^{1/2}\,dx$

integrate: $\frac{1}{\frac{3}{2}} x^{\frac{1}{2}+1} + C$

$1 \div \frac{3}{2}$

$1 \cdot \frac{2}{3}$

$\frac{2}{3}$

Simplify: $\frac{2}{3}x^{3/2} + C$

6. $\int \frac{1}{x^3}\,dx$

rewrite $\int x^{-3}\,dx$

integral $\frac{x^{-3+1}}{-2} + C$

Simplify: $-\frac{1}{2}x^{-2}$

$-\frac{1}{2x^2} + C$

EXAMPLES - EVALUATE

7. $\int \sqrt[6]{x}\,dx$

$\int x^{1/6}\,dx$

$\frac{x^{1/6+1}}{7/6} + C$

$\frac{1}{6} + \frac{6}{6} = \frac{7}{6}$

$1 \div \frac{7}{6}$

$1 \cdot \frac{6}{7}$

$\frac{6}{7}x^{7/6} + C$ or $\frac{6\sqrt[6]{x^7}}{7} + C$

8. $\int \frac{1}{x^4}\,dx$

$\int x^{-4}\,dx$

$\frac{1x^{-4+1}}{-3} + C$

$-\frac{1}{3x^3} + C$

EXAMPLES - EVALUATE

9. $\int e^x dx$

$\int 1 e^{1x}$

$\frac{1}{1} e^{1x}$

$e^x + C$

10. $\int e^{4x} dx$

$\int 1 e^{4x}$

$\frac{b}{a}$ $\frac{1}{4} e^{4x} + C$

$\frac{1}{4} e^{4x} \cdot 4$

EXAMPLES - EVALUATE

11. $\int \left(\frac{1}{x}\right) dx$

$\ln|x| + C$

$\frac{x^{-1+1}}{0}$ not possible

So memorize this one!

MORE ANTIDERIVATIVE RULES

$$\int kf(x)dx = k\int f(x)dx$$

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

EXAMPLES - EVALUATE

$x^{-1} = \frac{1}{x}$

12. $\int (3x^5 + 7x^2 + 8)dx$

$3\int x^5 dx + 7\int x^2 dx + \int 8\,dx$

$\frac{3x^{5+1}}{6} + \frac{7x^{2+1}}{3} + 8x$

$\frac{1}{2}x^6 + \frac{7}{3}x^3 + 8x + C$

13. $\int \frac{4 + 3x + 2x^4}{x} dx$

rewrite $\int \frac{4}{x} + \frac{3x}{x} + \frac{2x^4}{x}\,dx$

$\int 4x^{-1} + 3 + 2x^3\,dx$

$4\int x^{-1}dx + \int 3dx + 2\int x^3 dx$

$4\ln|x| + 3x + \frac{2x^{3+1}}{4} + C$

$4\ln|x| + 3x + \frac{1}{2}x^4 + C$

EXAMPLES - EVALUATE

$\frac{x}{x^2} = \frac{1}{x}$

14. $\int \frac{x^2 + 7x + 2}{x^2}\,dx$

$\int \left(\frac{x^2}{x^2} + \frac{7x}{x^2} + \frac{2}{x^2}\right) dx$

$\int \left(1 + 7 \cdot \frac{1}{x} + 2x^{-2}\right) dx$

$x + 7\ln|x| + \frac{2x^{-2+1}}{-1} + C$

$x + 7\ln|x| - \frac{2}{x} + C$

15. $\int \frac{\pi}{x}\,dx$

$\int \pi \cdot \frac{1}{x}\,dx$

$\pi \int \frac{1}{x}\,dx$

$\pi \ln|x| + C$

TRIG INTEGRALS

$\sec^2 x = \tan x$ ←

P S S T

→ $\sec x = \sec x \tan x$

$\int \cos x \, dx = \sin x + C$
$\int \sin x \, dx = -\cos x + C$
$\int \sec^2 x \, dx = \tan x + C$
$\int \csc^2 x \, dx = -\cot x + C$
$\int \sec x \tan x \, dx = \sec x + C$
$\int \csc x \cot x \, dx = -\csc x + C$

EXAMPLE

$\sec^2 x \rightarrow \tan x$

PSST

16. $\int 2\pi \sin x \, dx$

$2\pi \int \sin x \, dx$

$2\pi(-\cos x) + C$

$-2\pi \cos x + C$

17. $\int \sec^2 x \, dx$

$\tan x + C$

EXAMPLE

$\frac{d}{dx}\sec x = \sec x \tan x$

PSST

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

18. $\int \sec x \tan x \, dx$

$\sec x + C$

19. $\int (5\cos x + 2\sin x)dx$

$5\sin x + 2(-\cos x) + C$

$5\sin x - 2\cos x + C$

EXAMPLE

$$20. \int 3\csc^2 x \, dx$$

SST

$\frac{d}{dx} \sec^2 x = \tan x$

$c \cdot c = -c$

$\frac{d}{dx} \csc^2 x = -\cot x$

$3(-\cot x) + C$

$-3\cot x + C$

OTHER COMMON INTEGRAL

$\int \frac{1}{x}\,\mathrm{dx}$ or $\int x^{-1}dx = \ln\lvert x\rvert + c$
$\int e^x dx = e^x + c$
$\int a^x dx = \frac{a^x}{\ln a} + c$

EXAMPLE – INTEGRATE THE FOLLOWING

$$21. \int 3^x dx$$

e^x

$\frac{e^x}{\ln e} + C$

$e^x + C$

$\frac{3^x}{\ln 3} + C$

HW:
p. 3-4
1-22